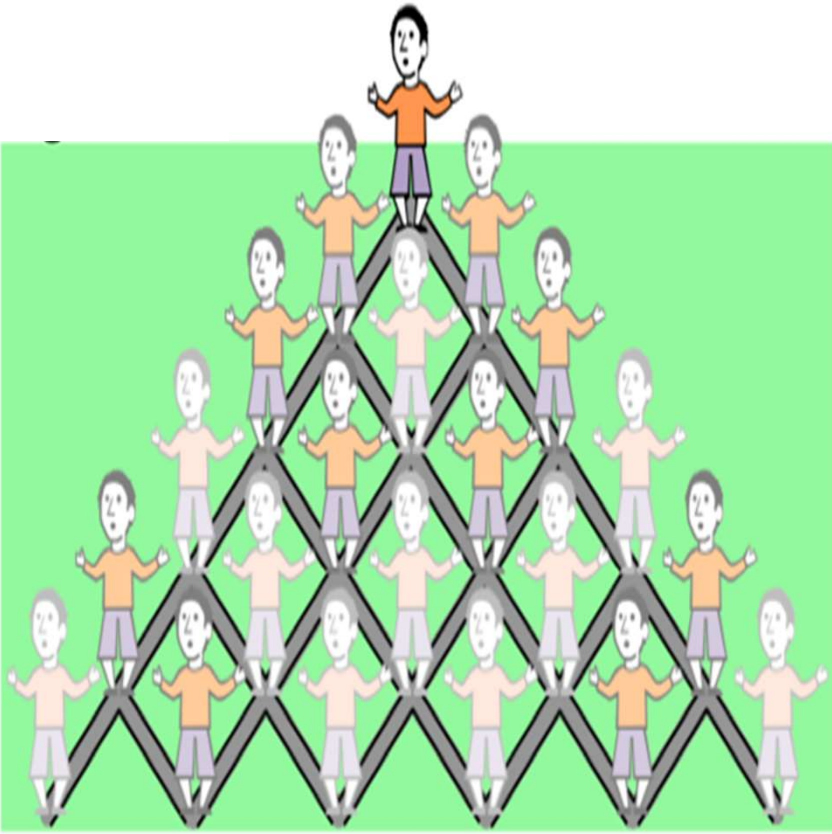


Experimental Investigation of Superdiffusion via Coherent Disordered Quantum Walks

Geraldi, A., A. Laneve, L.D. Bonavena, L. Sansoni, J. Ferraz, A. Fratalocchi, F. Sciarrino, Á. Cuevas, and P. Mataloni. "Experimental Investigation of Superdiffusion via Coherent Disordered Quantum Walks." *Physical Review Letters* 123, no. 14 (2019).

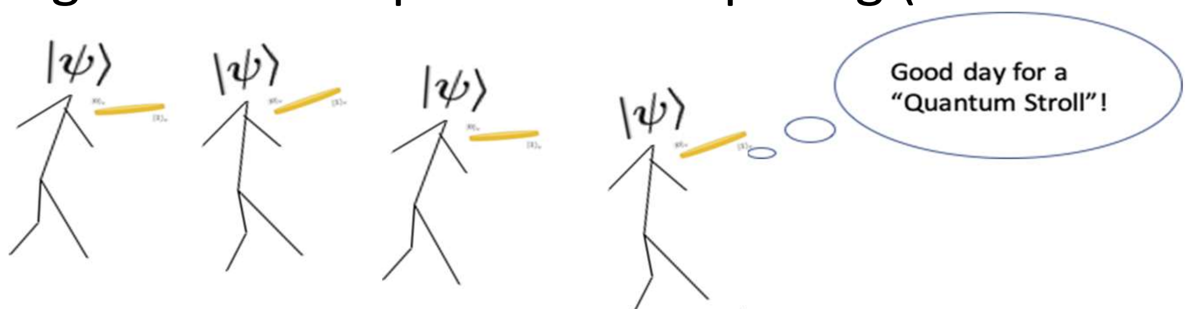
Ian Osborne, Aaron Ouellette, Randy Owen, Samuel Peng, and Ujaan Purakayastha

Why care about quantum walks?

QWs can be used to efficiently model coherent energy transport processes

By introducing disorder, they can also model various superdiffusive processes (various biological systems, many-body localization problems, etc)

QWs also appear in various algorithms for quantum computing (ex. Grover's search algorithm)



The main goal of this paper was to create an experimental setup to study the superdiffusion process by introducing disorder into a quantum walk

Superdiffusive Regime

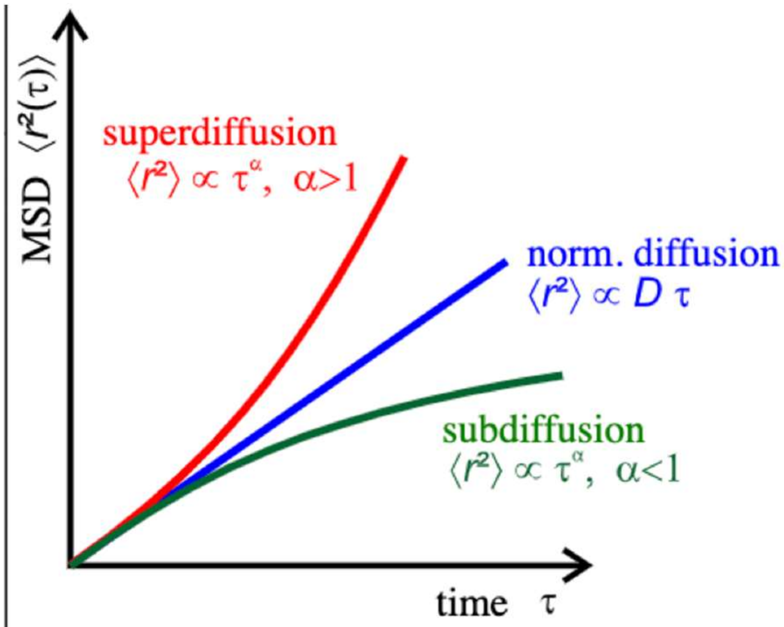
Anomalous diffusion [1]: Non-linear relationship between the time and the mean squared displacement (variance).

- **Subdiffusion ($\alpha < 1$):** cell nucleus, plasma membrane and cytoplasm [2]

- **Superdiffusion ($\alpha > 1$):** Dusty Plasma [3]

- **Ballistic diffusion ($\alpha = 2$):** quantum walk (QW)

- **Hyper-ballistic ($\alpha > 2$):** disorder is dynamic [4], QW with time-dependent jumps [5]



[1] https://en.wikipedia.org/wiki/Anomalous_diffusion

[2] Saxton, Michael J. *Biophysical Journal*. **92** (4): 1178–1191 (2007)

[3] Super Bin Liu and J. Goree. Phys. Rev. Lett. **100**, 055003 (2008)

[4] Peccianti, Marco and Morandotti, *Nature Physics*. **8** (12): 858–859. (2012)

[5] hyper : Marcelo A. Pires, Giuseppe Di Molfetta & Sílvio M. Duarte Queirós *Scientific Reports* **9**, 19292 (2019)

Superdiffusive Regime

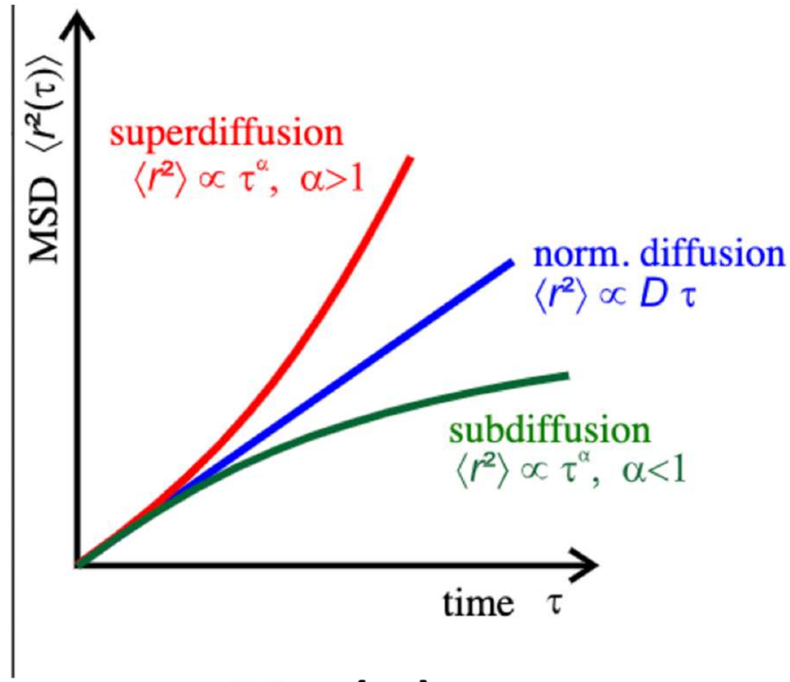
Anomalous diffusion [1]: Non-linear relationship between the time and the mean squared displacement (variance).

- **Subdiffusion (α < 1):** cell nucleus, plasma membrane and cytoplasm [2]

- **Superdiffusion (α > 1):** Dusty Plasma [3]

- **Ballistic diffusion (α = 2):** quantum walk (QW)

- **Hyper-ballistic (α > 2):** disorder is dynamic [4], QW with time-dependent jumps [5]



$$Var(\boldsymbol{\tau}) \propto \boldsymbol{\tau}^{\alpha}$$

Superdiffusive:

$$1 < \alpha < 2$$

CRW (diffusive) QW (ballistic)

[1] https://en.wikipedia.org/wiki/Anomalous_diffusion

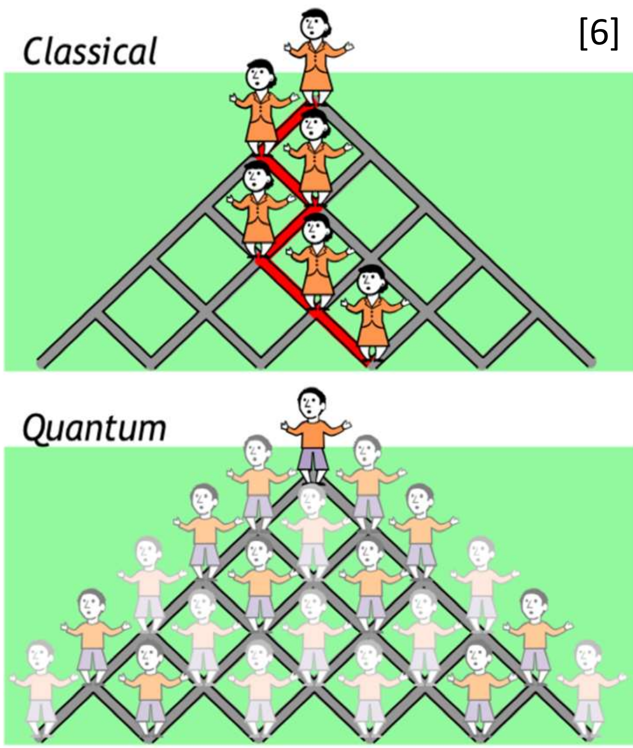
[2] Saxton, Michael J. *Biophysical Journal*. **92** (4): 1178–1191 (2007)

[3] Super Bin Liu and J. Goree. Phys. Rev. Lett. **100**, 055003 (2008)

[4] Peccianti, Marco and Morandotti, *Nature Physics*. **8** (12): 858–859. (2012)

[5] hyper : Marcelo A. Pires, Giuseppe Di Molfetta & Sílvio M. Duarte Queirós *Scientific Reports* **9**, 19292 (2019)

Quantum vs Random Walk



[6]

Classical Random Walk (CRW): [7]

- Randomness arises from the stochastic transitions between states.

Quantum Walk (QW): [7]

- Randomness arises from the superposition of quantum states, non-random and reversible unitary evolutions, or the collapse of the wavefunction when states are measured.

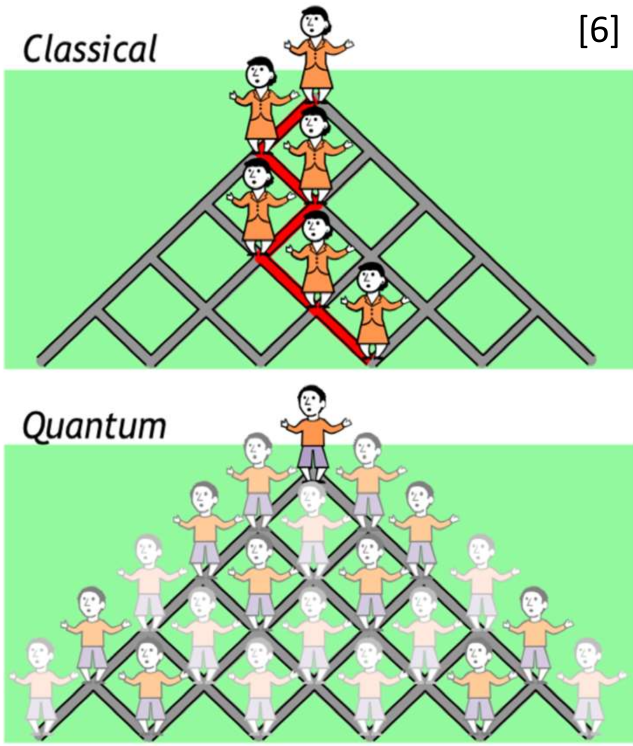
- Continuous time: Schrödinger equation $\mathbf{i}\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}$

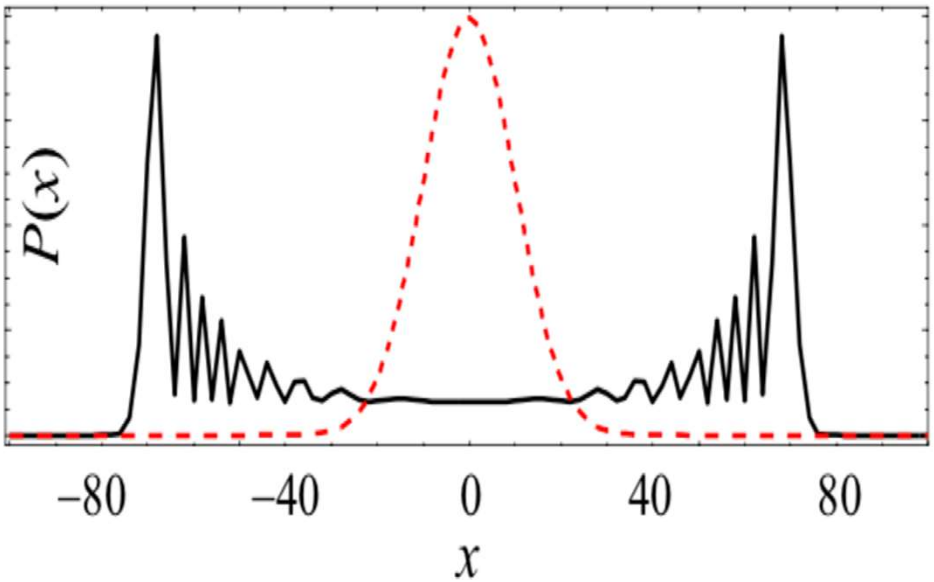
- Discrete time: product of a "coin flip" operator and a shift operator $|\Psi\rangle = |s\rangle \otimes |\psi\rangle$

[6]K. Manouchehri, et al. *Solid State Implementation of Quantum Random Walks on General Graphs,* AIP Conference Proceedings (2008): n. pag. Crossref. Web.

[7] https://en.wikipedia.org/wiki/Quantum_walk

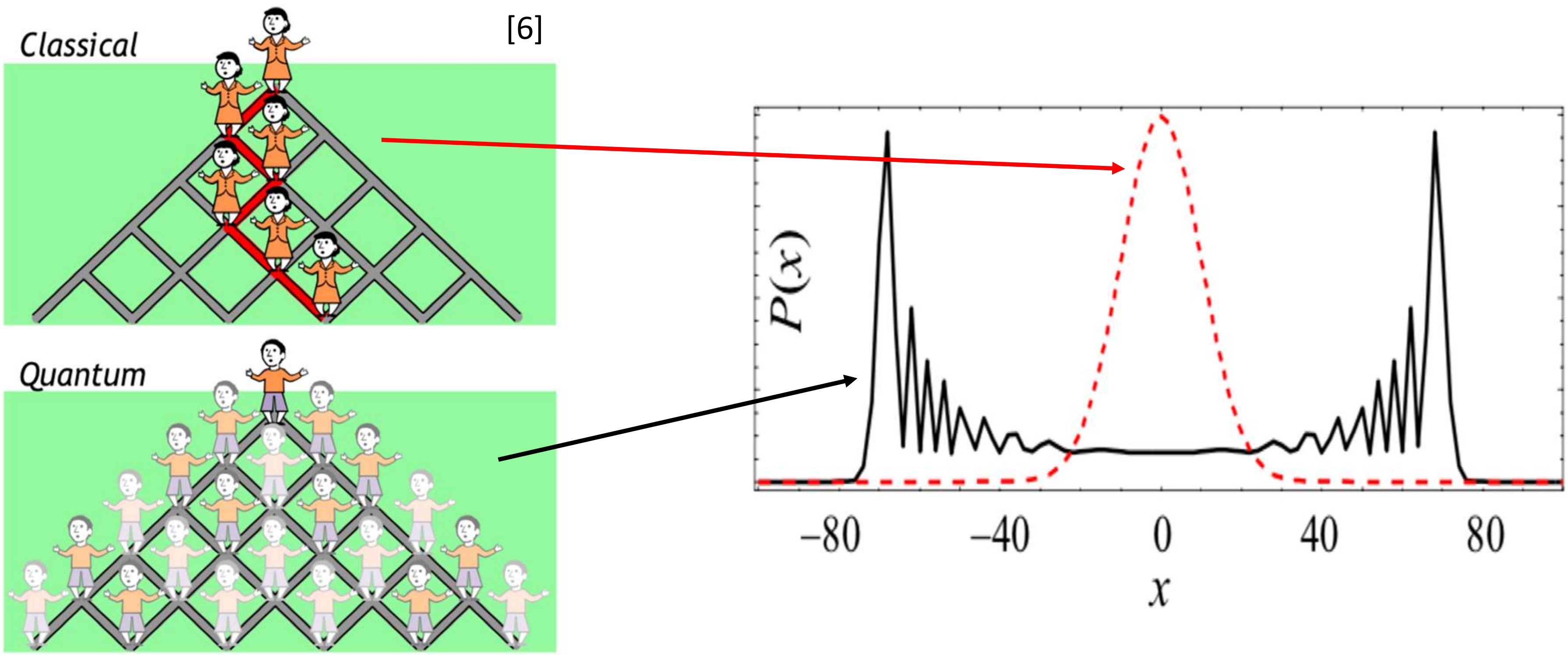
Quantum vs Random Walk





[6]K. Manouchehri, et al. *Solid State Implementation of Quantum Random Walks on General Graphs,* AIP Conference Proceedings (2008): n. pag. Crossref. Web.

Quantum vs Random Walk

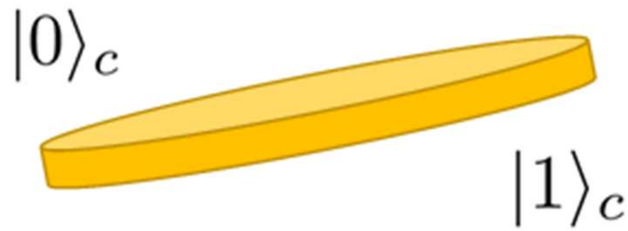


[6]K. Manouchehri, et al. *Solid State Implementation of Quantum Random Walks on General Graphs,* AIP Conference Proceedings (2008): n. pag. Crossref. Web.

Introducing Disorder in the Quantum Walk Model

$$|\psi\rangle = \sum_{ij} P_{ij} |i\rangle_p \otimes |j\rangle_c$$

Introducing Disorder in the Quantum Walk Model



$$|\psi\rangle = \sum_{ij} P_{ij} |i\rangle_p \otimes |j\rangle_c$$

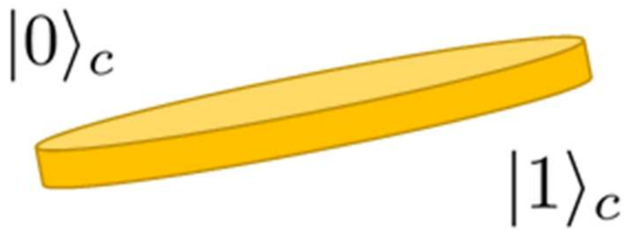
position $i \in \mathbb{Z}$

coin $j \in \{0, 1\}$

Probability

Two degrees of freedom

Introducing Disorder in the Quantum Walk Model



$$|\psi\rangle = \sum_{ij} P_{ij} |i\rangle_p \otimes |j\rangle_c$$

position $i \in \mathbb{Z}$
coin $j \in \{0, 1\}$

Probability — Two degrees of freedom

Ordered Quantum Walk (ballistic)

$$|\psi'(n+1)\rangle = \hat{S}(\hat{I} \otimes \hat{C})|\psi(n)\rangle$$

Step Operator — Coin operator

$$|i\rangle_p \xrightarrow{|1\rangle_c} |i+1\rangle$$

$$|i\rangle_p \xrightarrow{|0\rangle_c} |i-1\rangle$$

Introducing Disorder in the Quantum Walk Model

$|0\rangle_c$

$|1\rangle_c$

$$|\psi\rangle = \sum_{ij} P_{ij} |i\rangle_p \otimes |j\rangle_c$$

position $i \in \mathbb{Z}$
coin $j \in \{0, 1\}$

Probability

Two degrees of freedom

Ordered Quantum Walk (ballistic)

$$|\psi'(n+1)\rangle = \hat{S}(\hat{I} \otimes \hat{C})|\psi(n)\rangle$$

Step Operator

Coin operator

$$|i\rangle_p \xrightarrow{|1\rangle_c} |i+1\rangle$$

$$|i\rangle_p \xrightarrow{|0\rangle_c} |i-1\rangle$$

Introduce disorder with a random operator

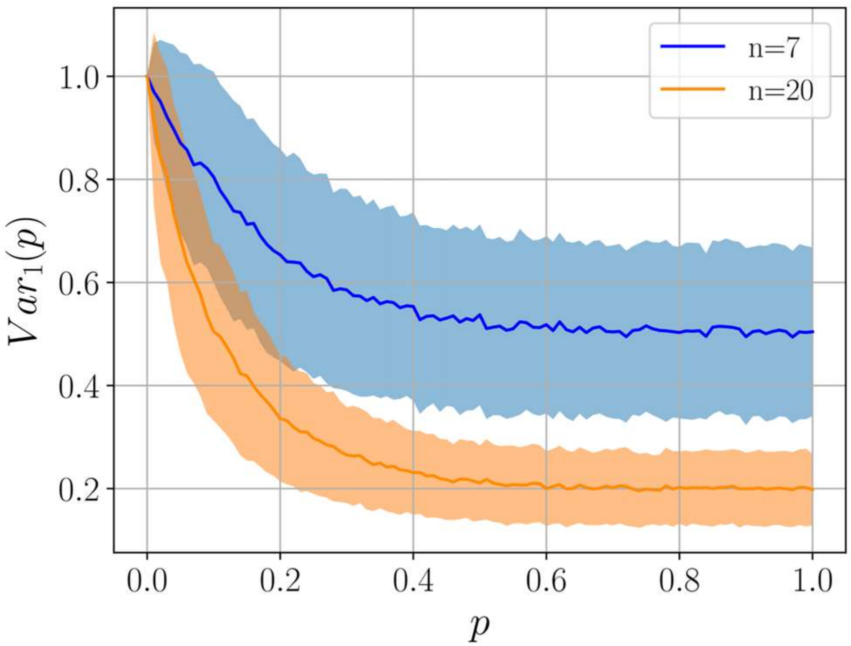
$$|\psi'(n+1)\rangle = \hat{P}_n \hat{S}(\hat{I} \otimes \hat{C})|\psi(n)\rangle$$

Adds a phase to each state randomly

1-p: $|i\rangle_p \otimes |j\rangle_c \rightarrow e^{i\phi} |i\rangle_p \otimes |j\rangle_c \qquad \phi \in \{0, \pi\}$

p: $|i\rangle_p \otimes |j\rangle_c \rightarrow |i\rangle_p \otimes |j\rangle_c$

Effect of Randomness on the Quantum Walk



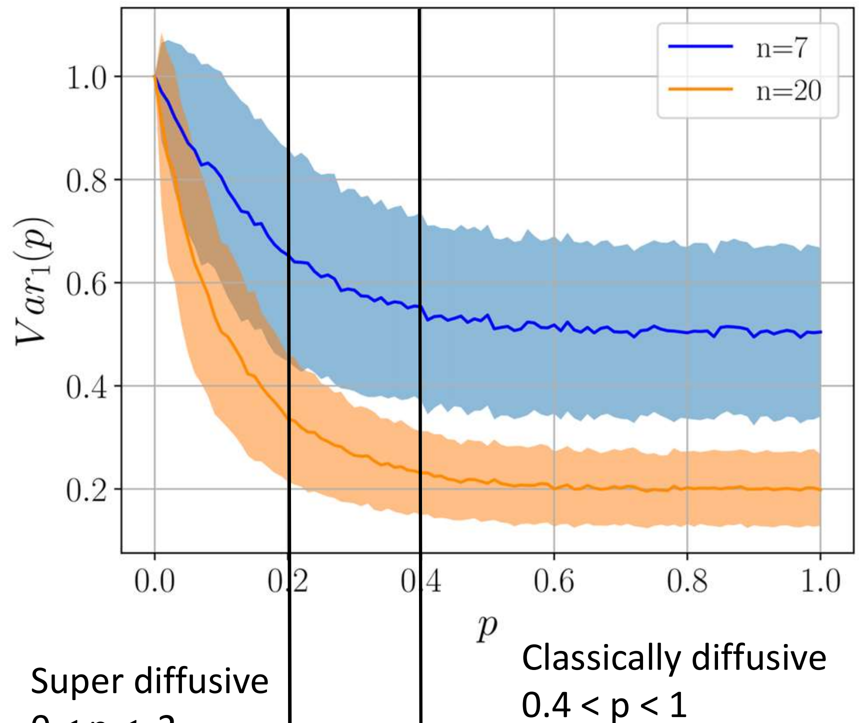
$$\mathrm{Var}_1(n) = \sum_{i=-n}^{n} i^2 P_i(n) - \left[\sum_{i=-n}^{n} i P_i(n)\right]^2$$

$P_i(n)$: Probability of finding the walker on site i at step n

Simulated evolution of 1000 different phase maps for each p

Rapid suppression of quantum effects with increasing p

Effect of Randomness on the Quantum Walk



Super diffusive
0 < p < .2

Classically diffusive
0.4 < p < 1

$$\text{Var}_1(n) = \sum_{i=-n}^{n} i^2 P_i(n) - \left[\sum_{i=-n}^{n} i P_i(n)\right]^2$$

$P_i(n)$: Probability of finding the walker on site i at step n

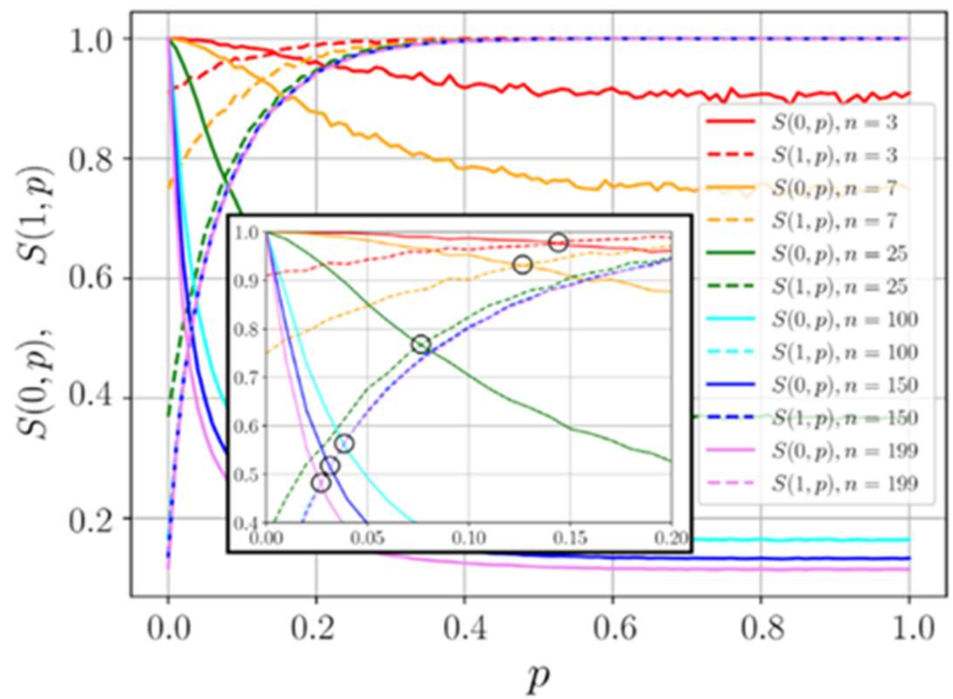
Simulated evolution of 1000 different phase maps for each p

Rapid suppression of quantum effects with increasing p

Effect of Randomness on the Quantum Walk

S(0, p) : Similarity between the probability distribution and that of the Ordered Quantum Walk (no disorder)

S(1, p) : Similarity between the probability distribution and that of the Classical Random Walk (complete disorder)

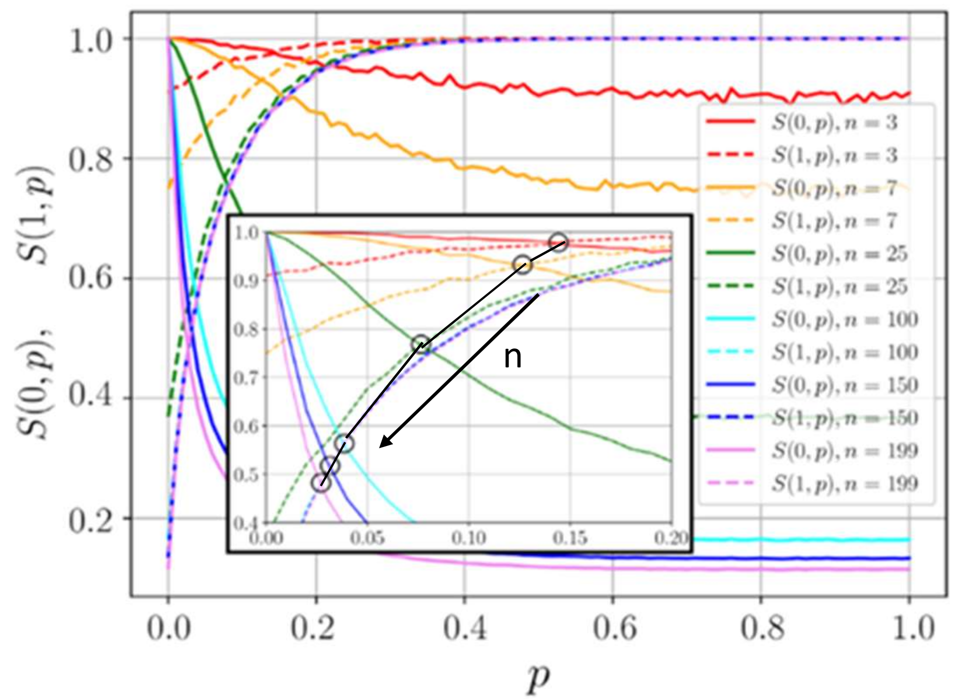


Effect of Randomness on the Quantum Walk

S(0, p) : Similarity between the probability distribution and that of the Ordered Quantum Walk (no disorder)

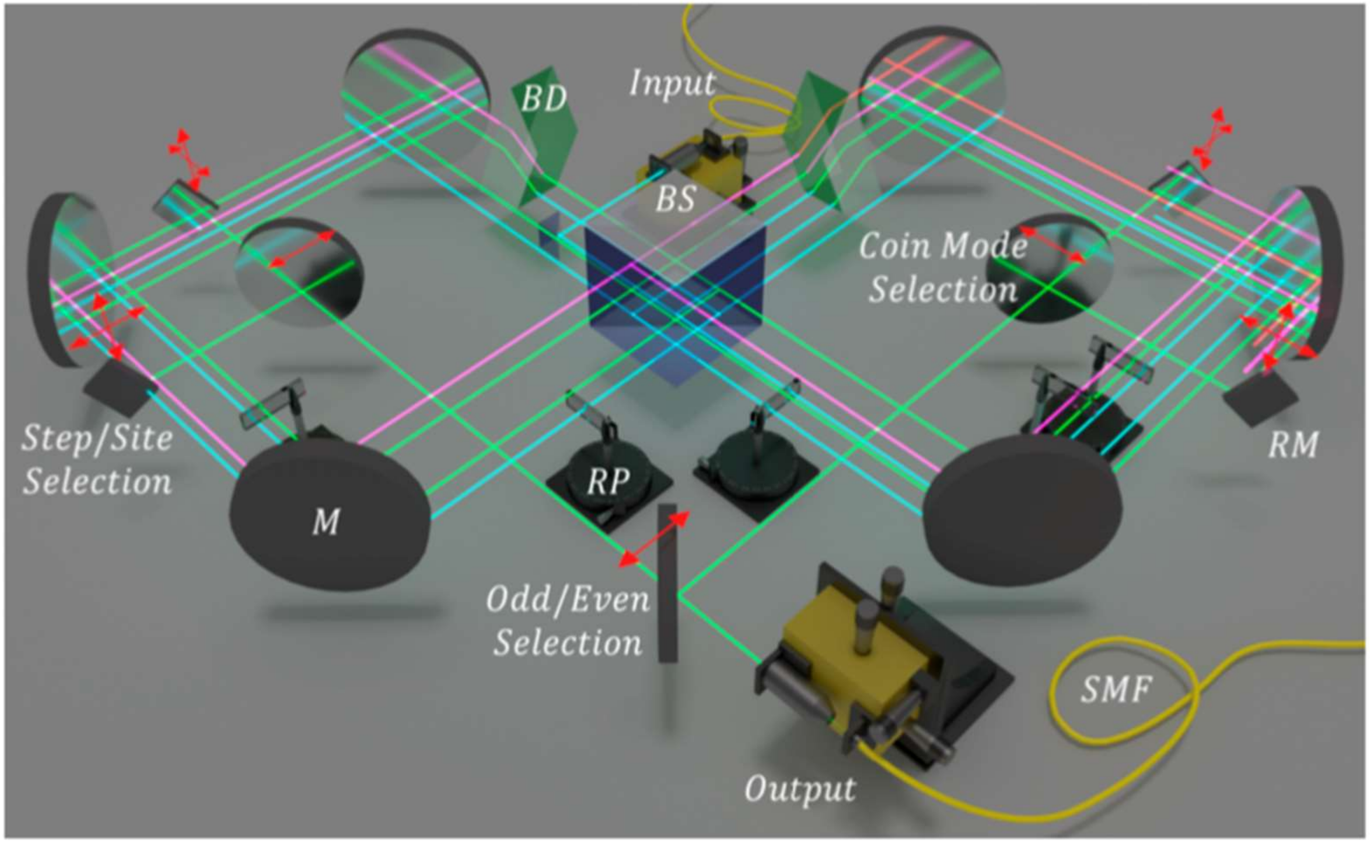
S(1, p) : Similarity between the probability distribution and that of the Classical Random Walk (complete disorder)

Crossing points show the system is more classical-like than quantum-like for a greater range of p values with increasing n.



Experimental Implementation

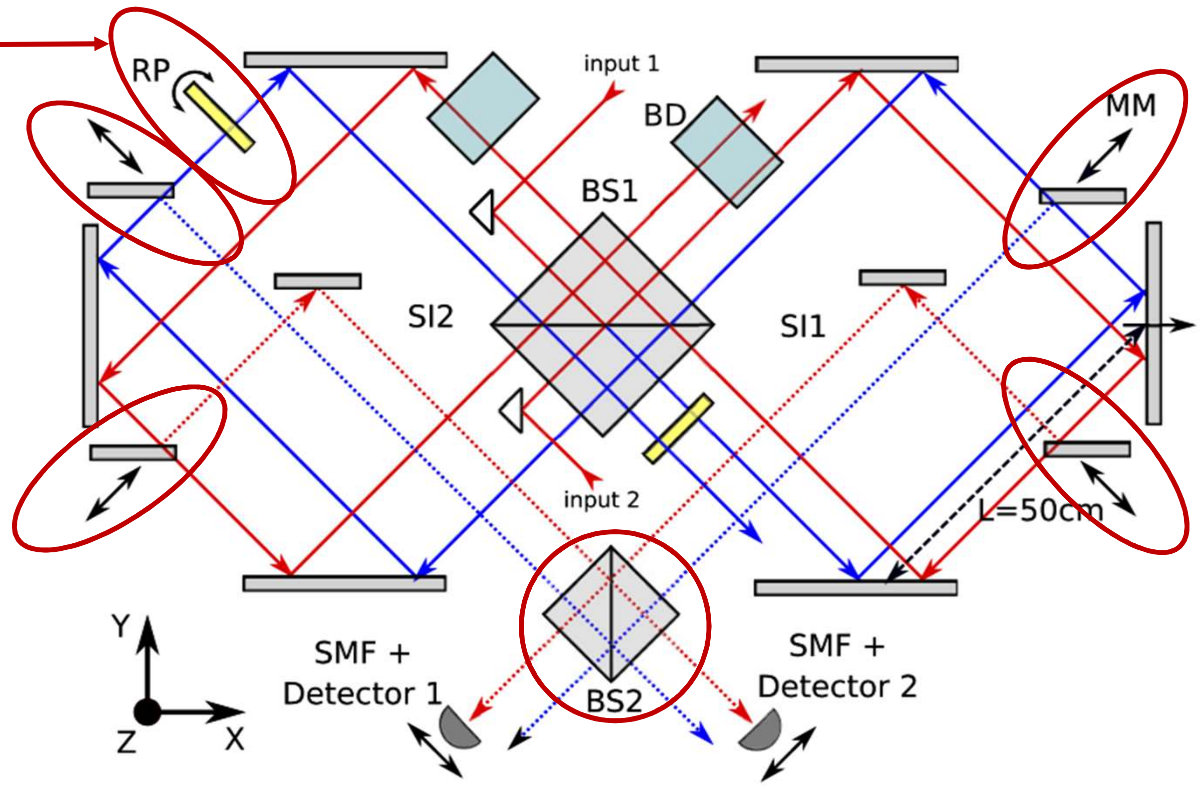
[8]



[8] Geraldi,et. al. A novel bulk-optics scheme for quantum walk with high phase stability, Condens. Matter 4, 14 (2019).

Disordered QW on Bulk Optical Setup

Phase shifts: Disorder!



For two-photon measurements

BS Network: 2 Multipass Sagnac Interferometers

Odd n: SI1, Even n: SI2

Entire network realized by utilizing BD’s

MM’s extract modes at selected step

Variance shows superdiffusive dynamics!

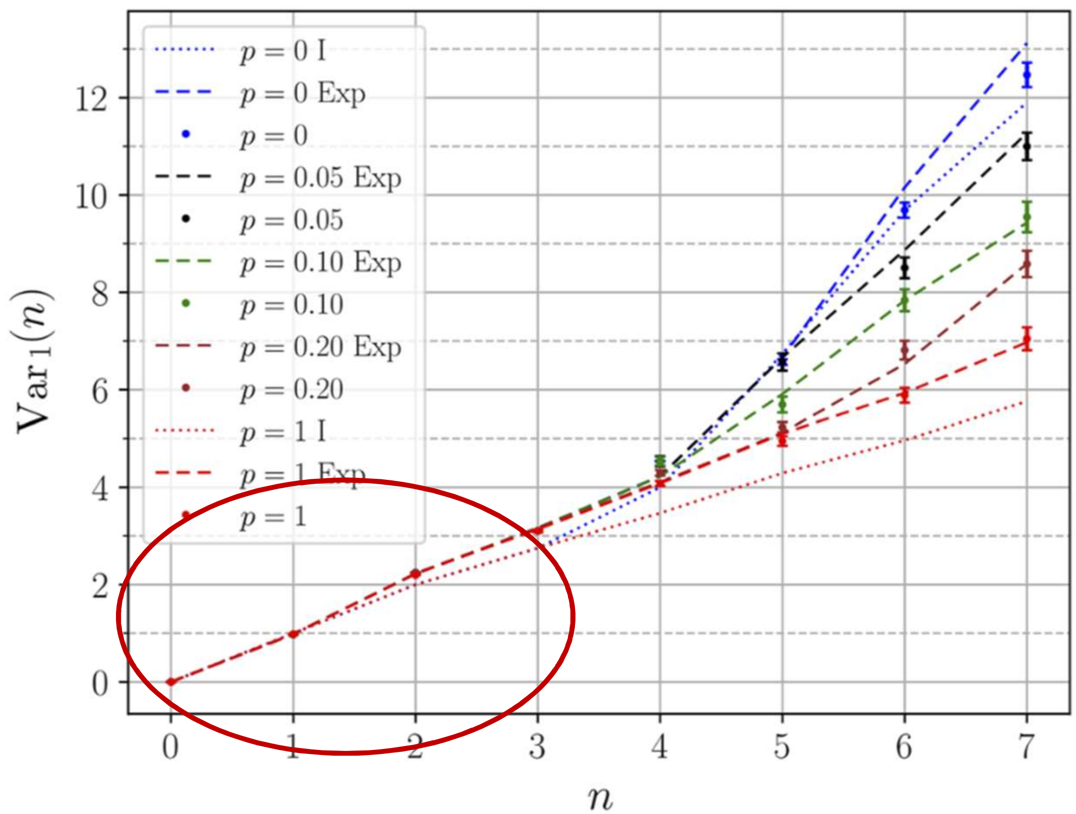
Dashed lines (Exp): average of 3 phase maps for each p

Dotted lines (I): ideal 50:50 BS

BS1 had R = 45%

p	β_{theo}	β_{fit}
0	1.69	1.64 ± 0.10
0.05	1.540	1.433 ± 0.067
0.10	1.414	1.277 ± 0.061
0.20	1.198	1.160 ± 0.060
1	0.921	0.961 ± 0.022

$1 < \beta < 2$: Superdiffsuive



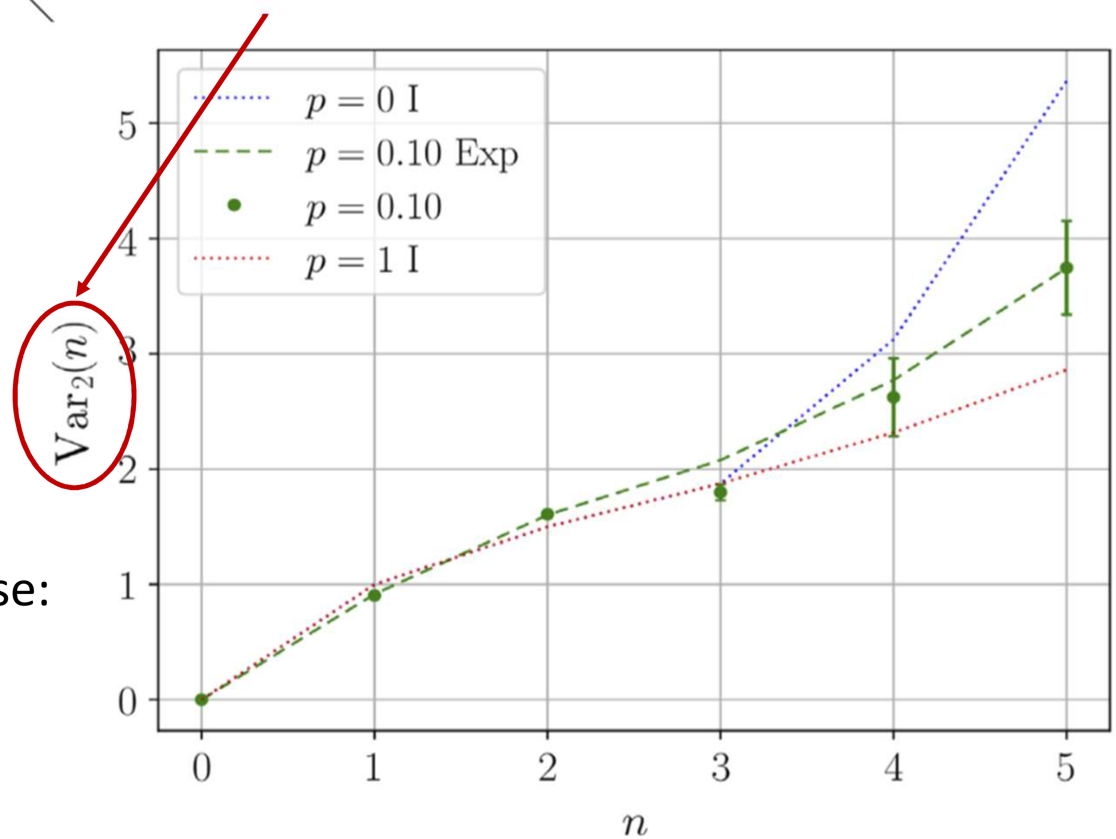
Two Walkers

$$\mathrm{Var}_2(n) = \sum_{i,j=-n}^{n} \left(\frac{i+j}{2}\right)^2 P_{i,j}(n) - \left(\sum_{i,j=-n}^{n} \frac{i+j}{2} P_{i,j}(n)\right)^2$$

Two indistinguishable photons through the two input ports of BS1

$n \leq 5$: losses, growth in # of modes that photons can travel

Error bars larger than one photon case: phase variation is double counted

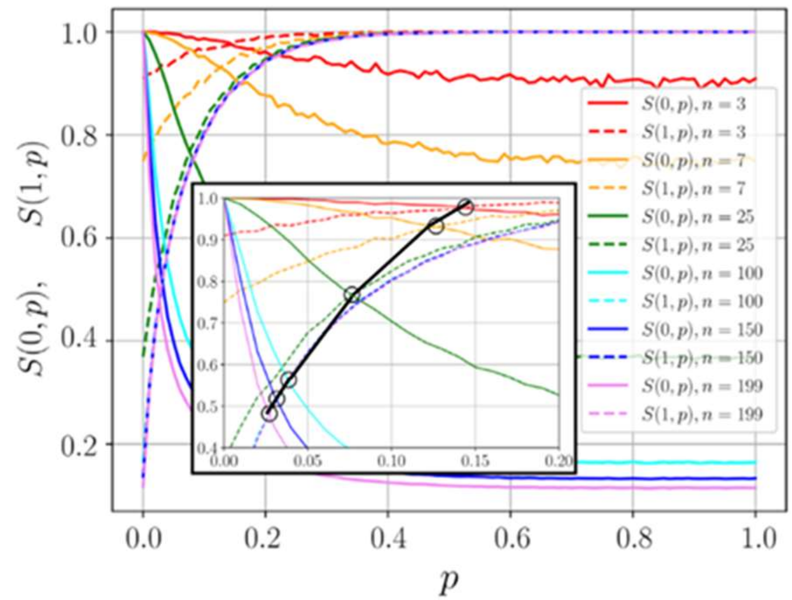


Conclusions

1. Modeled and simulated disordered random walks in the superdiffusive regime
   a. Found the disorder parameter where the random walk goes from quantum to classical
2. Experimentally demonstrated superdiffusive quantum walk with a bulk optic setup
   a. Able to experimentally replicate controlled disordered quantum random walks with up to 7 steps
   b. Obtained $\boldsymbol{\beta}$ values from 0.96 to 1.6, showing both superdiffusive and non-superdiffusive behavior

Critiques

1. This paper could have easily fit the quantum/classical crossing points
   a. We felt it was an important topic worth exploring more
2. The experimental setup could be more clear
   a. It more-or-less expects the reader to have read their previous papers
   b. A few more sentences of clarification here would have made a large difference.
   c. Did not mention the source of the single photons



Summary

- Theoretical model: introducing varying extent of disorders to the quantum state formed by the product of a coin operator and a shift operator.

- Experimentally demonstrated the variances of QWs versus steps with different p values.

- The excellent agreement between the experimental results and the theoretical predictions manifests different superdiffusive processes.

- Finally, this research paves the way for the further understanding of decoherent processes of QWs.

(Some details should be addressed)