Phys 211 Formula Sheet

Kinematics

 $\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$ $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t + \mathbf{a}t^2/2$ $\mathbf{v}^2 = \mathbf{v}_0^2 + 2\mathbf{a}(\mathbf{x}-\mathbf{x}_0)$

 $g = 9.81 \text{ m/s}^2 = 32.2 \text{ ft/s}^2$

 $\mathbf{v}_{A,B} = \mathbf{v}_{A,C} + \mathbf{v}_{C,B}$

Uniform Circular Motion $a = v^2/r = \omega^2 r$ $v = \omega r$ $\omega = 2\pi/T = 2\pi f$

$$\label{eq:product} \begin{split} & \textit{Dynamics} \\ & \textbf{F}_{net} = \textbf{ma} = d\textbf{p}/dt \\ & \textbf{F}_{A,B} = \textbf{-F}_{B,A} \end{split}$$

F = mg (near earth's surface) $F_{12} = -Gm_1m_2/r^2 \text{ (in general)}$ (where G = 6.67x10⁻¹¹ Nm²/kg²) $F_{spring} = -k \Delta x$

Friction

 $\begin{aligned} f &= \mu_k N \text{ (kinetic)} \\ f &\leq \mu_S N \text{ (static)} \end{aligned}$

Work & Kinetic energy $W = \int \mathbf{F} \cdot \mathbf{dl}$ $W = \mathbf{F} \cdot \Delta \mathbf{r} = F \Delta r \cos \theta$ (constant force)

 $W_{grav} = -mg\Delta y$ $W_{spring} = -k(x_2^2 - x_1^2)/2$

$$\begin{split} K &= m v^2 / 2 = p^2 / 2m \\ W_{\rm NET} &= \Delta K \end{split}$$

Potential Energy

 $U_{grav} = mgy \text{ (near earth surface)}$ $U_{grav} = -GMm/r \text{ (in general)}$ $U_{spring} = kx^{2}/2$ $\Delta E = \Delta K + \Delta U = W_{nc}$

Power

P = dW/dtP = **F**·**v** (for constant force)

System of Particles

$$\begin{split} \mathbf{R}_{CM} &= \Sigma m_i \mathbf{r}_i / \Sigma m_i \\ \mathbf{V}_{CM} &= \Sigma m_i \mathbf{v}_i / \Sigma m_i \\ \mathbf{A}_{CM} &= \Sigma m_i \mathbf{a}_i / \Sigma m_i \\ \mathbf{P} &= \Sigma m_i \mathbf{v}_i \\ \Sigma \mathbf{F}_{EXT} &= \mathbf{M} \mathbf{A}_{CM} = \mathbf{d} \mathbf{P} / \mathbf{d} t \end{split}$$

Impulse $\mathbf{I} = \int \mathbf{F} dt$ $\Delta \mathbf{P} = \mathbf{F}_{av} \Delta t$

Collisions:

If $\Sigma \mathbf{F}_{EXT} = 0$ in some direction, then $\mathbf{P}_{before} = \mathbf{P}_{after}$ in this direction: $\Sigma m_i \mathbf{v}_i$ (before) = $\Sigma m_i \mathbf{v}_i$ (after)

In addition, if the collision is elastic: * E_{before} = E_{after} * Rate of approach = Rate of recession * The speed of an object in the Center-of-Mass reference frame is unchanged by an elastic collision.

Rotational kinematics

 $\left. \begin{array}{l} s = R\theta, \, v = R\omega, \, a = R\alpha \\ \theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \\ \omega = \omega_0 + \alpha t \\ \omega^2 = \omega_0^2 + 2\alpha\Delta\theta \end{array} \right\}$

Rotational Dynamics

$$\begin{split} I &= \Sigma m_i r_i^2 \\ I_{parallel} &= I_{CM} + MD^2 \\ I_{disk} &= I_{cylinder} = {}^1/_2 MR^2 \\ I_{hoop} &= MR^2 \\ I_{solid-sphere} &= {}^2/_5 MR^2 \\ I_{spherical shell} &= {}^2/_3 MR^2 \\ I_{rod-end} &= {}^1/_2 ML^2 \\ I_{rod-end} &= {}^1/_3 ML^2 \\ \tau &= I\alpha \text{ (rotation about a fixed axis)} \\ \tau &= r \times F \text{, } |\tau| = rFsin\phi \end{split}$$

 $\label{eq:kappa} \begin{array}{l} \textit{Work \& Energy} \\ K_{rotation} = {}^{1}\!/_{2}I\omega^{2} \,, \\ K_{translation} = {}^{1}\!/_{2}MV_{cm}^{2} \\ K_{total} = K_{rotation} + K_{translation} \\ W = \tau\theta \end{array}$

Statics $\Sigma \mathbf{F} = 0$, $\Sigma \tau = 0$ (about any axis)

Angular Momentum:

$$\begin{split} \mathbf{L} &= \mathbf{r} \times \mathbf{p} \\ \mathbf{L}_z &= \mathbf{I} \omega_z \\ \mathbf{L}_{tot} &= \mathbf{L}_{CM} + \mathbf{L}^* \\ \boldsymbol{\tau}_{ext} &= d\mathbf{L}/dt \\ \boldsymbol{\tau}_{cm} &= d\mathbf{L}^*/dt \\ \boldsymbol{\Omega}_{precession} &= \boldsymbol{\tau} / L \end{split}$$

Simple Harmonic Motion: $d^{2}x/dt^{2} = -\omega^{2}x$ (differential equation for SHM)

 $x(t) = A\cos(\omega t + \phi)$ $v(t) = -\omega A\sin(\omega t + \phi)$ $a(t) = -\omega^2 A\cos(\omega t + \phi)$

$$\begin{split} \omega^{2} &= k/m \text{ (mass on spring)} \\ \omega^{2} &= g/L \text{ (simple pendulum)} \\ \omega^{2} &= mgR_{CM}/I \text{ (physical pendulum)} \\ \omega^{2} &= \kappa/I \text{ (torsion pendulum)} \end{split}$$

General harmonic transverse waves: $y(x,t) = A\cos(kx - \omega t)$ $k = 2\pi/\lambda$, $\omega = 2\pi f = 2\pi/T$ $v = \lambda f = \omega/k$

Waves on a string:

$$v^{2} = \frac{F}{\mu} = \frac{(\text{tension})}{(\text{mass per unit length})}$$

$$\overline{P} = \frac{1}{2} \mu v \omega^2 A^2$$

$$\frac{d\overline{E}}{dx} = \frac{1}{2} \mu \omega^2 A^2$$

$$\frac{d^2 y}{dx^2} = \frac{1}{v^2} \frac{d^2 y}{dt^2}$$
Wave Equation

Fluids:

$$\rho = \frac{m}{V} \qquad p = \frac{F}{A}$$

$$A_1 v_1 = A_2 v_2$$

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$F_{B} = \rho_{liquid} g V_{liquid}$$

$$F_2 = F_1 \frac{A_2}{A_1}$$