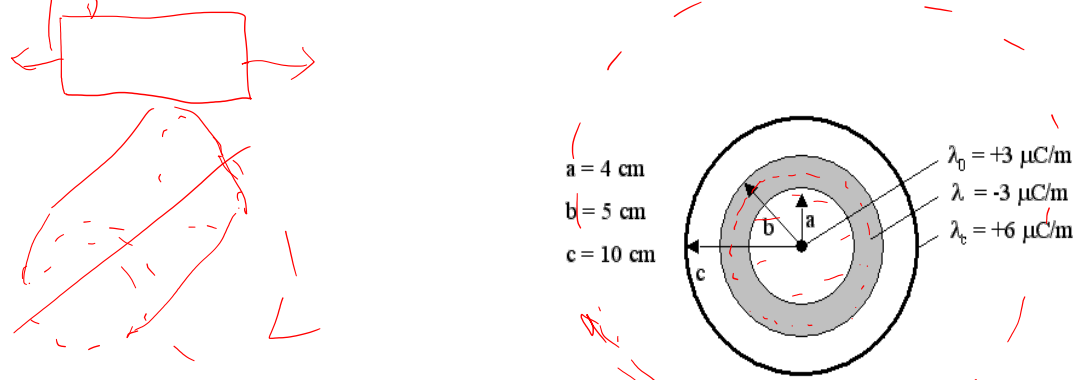


This and the next two questions are about the following situation:

Consider an infinite line with charge density  $\lambda_0 = +3 \mu\text{C/m}$ , shown in the center of the figure below. Concentric with the line is a hollow thick-walled cylinder (shaded), made of conducting material. The hollow cylinder carries a charge per unit length of density  $\lambda = -3 \mu\text{C/m}$ . Finally, a thin nonconducting cylindrical shell is concentric with the other two objects, and carries a charge per unit length  $\lambda_c = +6 \mu\text{C/m}$ . The dimensions of the objects are shown in the figure; all three have (effectively) infinite length.



$$\int \vec{E} \cdot d\vec{s} = \frac{Q_{enc}}{\epsilon_0} = \frac{\lambda L}{\epsilon_0}$$

$$E \int ds = E \cdot 2\pi r L$$

$$E = \frac{\lambda}{2\pi r \epsilon_0}$$

1. What is the surface charge density  $\sigma_b$  on the outer surface of the thick conducting shell?

- $\sigma_b > 0$
- $\sigma_b = 0$
- $\sigma_b < 0$

$\lambda_{outer} = 0$

$Q_{outer} = \lambda_{outer} L$

$= \sigma_b 2\pi r L$

$\sigma_b 2\pi r L = \lambda_{outer} L$

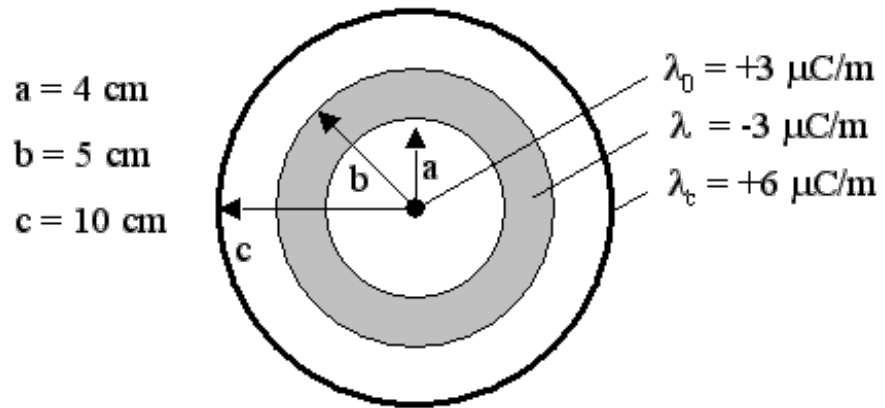
$\sigma_b = \frac{\lambda_{outer}}{2\pi r}$

2. What is the magnitude of the electric field at a radius  $r = 20 \text{ cm}$ ?

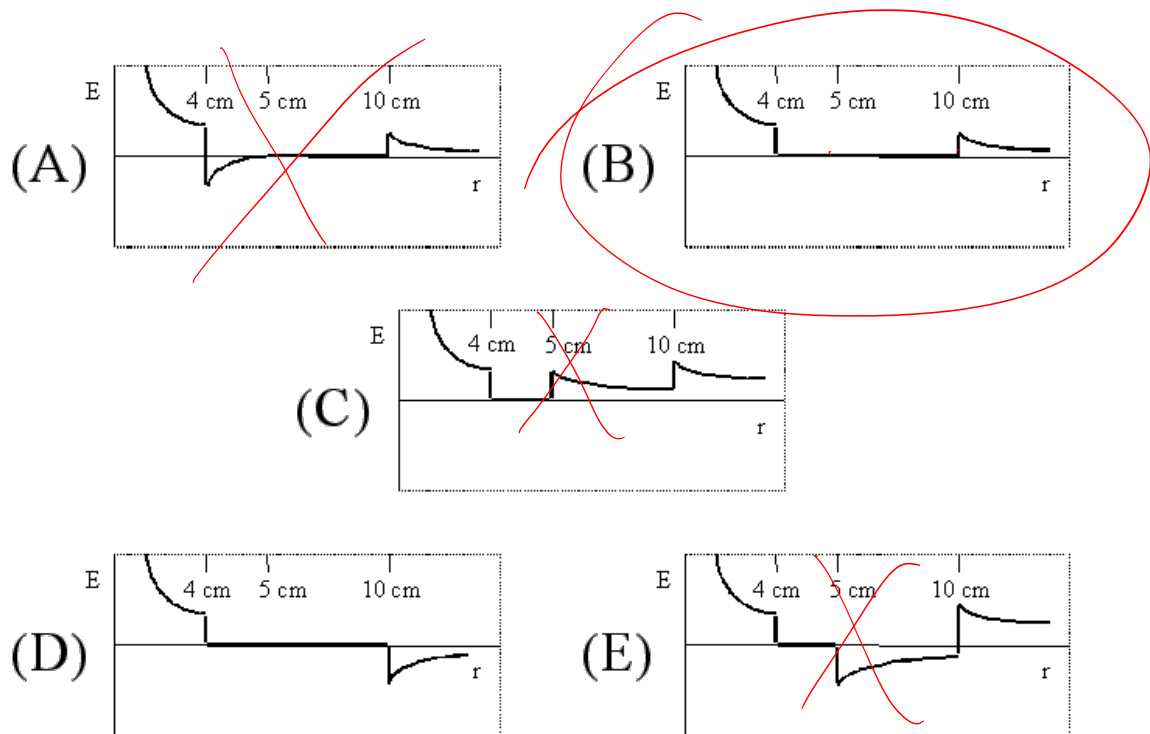
- $0.54 \times 10^6 \text{ N/C}$
- $1.35 \times 10^6 \text{ N/C}$
- $0.34 \times 10^6 \text{ N/C}$

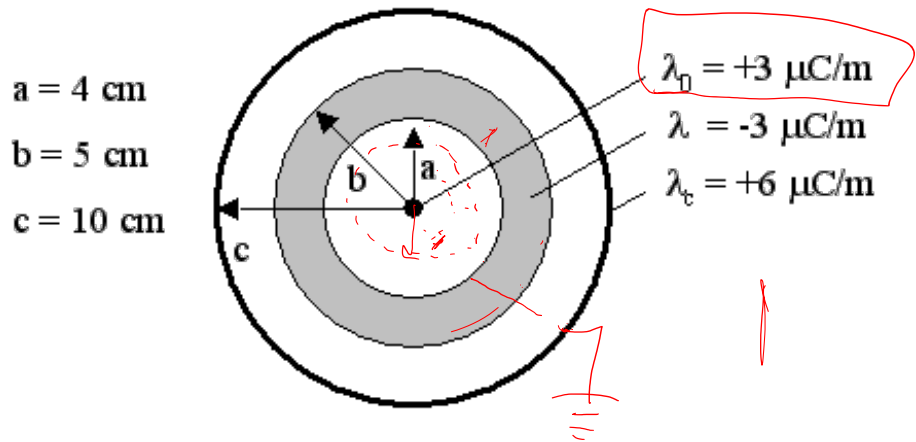
$E(r) = \frac{\lambda_{total}}{2\pi r \epsilon_0}$

$\lambda_{total} = \frac{+6 \mu\text{C/m}}{2\pi \cdot 0.2 \text{ m} \cdot 8.85 \times 10^{-12} \text{ F/m}}$



3. Which of the following graphs best represents the radial dependence of the electric field?





4. The outermost shell is now 'grounded', i.e., connected to a large reservoir that we will define as  $V = 0$ . What is the potential at a distance 2 cm from the axis?

- a. -37,400 V
- b. 37,400 V**
- c. 675,000 V

5. If the conductor were transformed into an insulator with a uniform charge density  $\rho$  (but still the same line-charge density), how would the answer to the previous problem change?

- a.  $V(2 \text{ cm})$  would increase**
- b.  $V(2 \text{ cm})$  would decrease
- c.  $V(2 \text{ cm})$  would stay the same

$V(\text{place don't know}) = V(\text{place know}) - \int_{\text{know}}^{\text{don't know}} E \cdot dl$   
 $= 0 - \int_c^b \frac{\lambda}{2\pi\epsilon_0 r} dr - \int_b^a \frac{\lambda}{2\pi\epsilon_0 r} dr - \int_a^h \frac{\lambda}{2\pi\epsilon_0 r} dr$   
 $= - \int_a^h \frac{\lambda}{2\pi\epsilon_0 r} dr = - \frac{\lambda}{2\pi\epsilon_0} \ln \frac{h}{a}$   
 $= - \frac{3 \mu\text{C/m}}{2\pi\epsilon_0} (\ln 2 - \ln 4)$   
 $= \frac{3 \mu\text{C/m}}{2\pi\epsilon_0} \ln 2 = 37,400$

because  
 $E(a < r < b) > 0$

5. A 12-volt battery is connected to a copper wire of length  $L$  and diameter  $d$ . If we stretch the wire to double its length, but keep the volume constant, what will happen to the power dissipated in the wire?

- a. Power dissipated will quadruple
- b. Power dissipated will double
- c. Power dissipated will stay the same
- d. Power dissipated will halve (go down by 2)
- e. Power dissipated will quarter (go down by 4)

$$P = IV = \left(\frac{V}{R}\right)V = \frac{V^2}{R}$$

$$V = IR$$

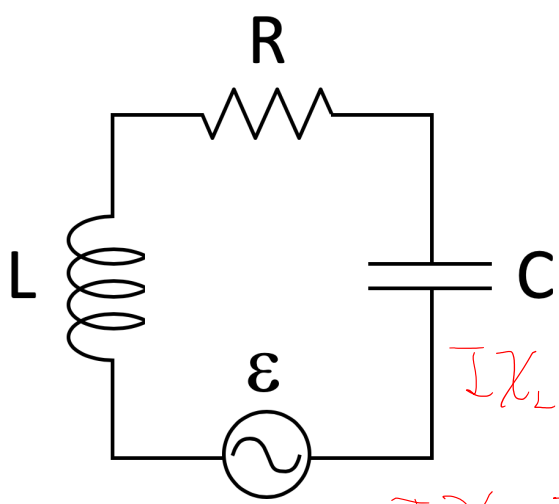
$$R = \rho \frac{L}{A} \quad R' = \rho \frac{L'}{A'} \quad L' = 2L$$

$$V_{\text{vol}} = L \cdot A = L' \cdot A'$$

$$A' = \frac{LA}{L'}$$

$$R' = \rho \frac{(L')^2}{LA} = \rho \frac{4L^2}{LA} = 4 \frac{\rho L}{A} = 4R$$

$$\Rightarrow P' = \frac{V^2}{R'} = \frac{V^2}{4R} = \frac{P}{4}$$



$$X_L = \omega L = 125,600 \Omega$$

$$X_C = \frac{1}{\omega C} = 15,924 \Omega$$

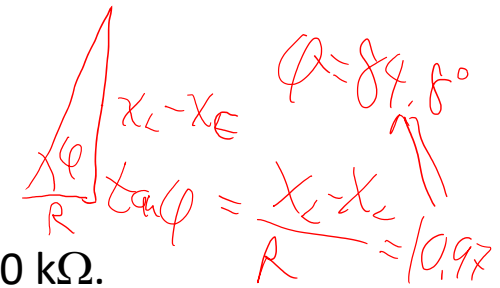
Consider the LRC circuit shown, where  $L = 20 \text{ mH}$ ,  $C = 10 \text{ pF}$ , and  $R = 10 \text{ k}\Omega$ .

The AC voltage generator is supplying a maximum voltage of  $100 \text{ V}$  at a frequency of  $1 \text{ MHz}$ .

$$IX_L = V_L \uparrow$$

$$IX_C = V_C \downarrow$$

$$I, V_R = IR$$



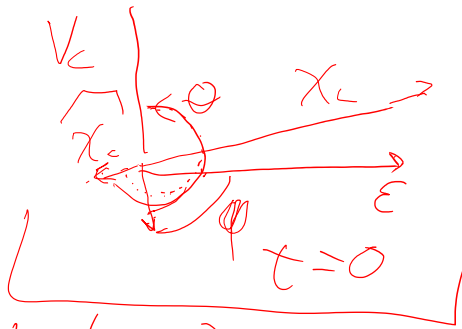
$$\omega = 2\pi f$$

$$= 6,280,000 \text{ rad/s}$$

6. Assuming the generator voltage is 0 at  $t = 0$ , when will the voltage across the capacitor be maximum?

- $1 \mu\text{s}$
- $2.8 \mu\text{s}$
- $0.74 \mu\text{s}$

cannot be determined



$V_C$  phasor needs to rotate

$$\theta = 90^\circ + \phi + 90^\circ = 180^\circ + 84.8^\circ = 264.8^\circ$$

$$= \omega t = 2\pi f t$$

$$t = \frac{264.8^\circ}{2\pi f} = \frac{264.8^\circ}{560^\circ \cdot 10^6 \text{ s}^{-1}}$$

7. What will be the maximum energy stored in the capacitor, if we suitably adjust the generator frequency?

- $1 \mu\text{J}$
- a.  $0.65 \mu\text{J}$
- b.  $0.21 \mu\text{J}$

$$I_{\text{max}} = \frac{\epsilon_{\text{max}}}{R}$$

$$X_{C,0} = \frac{1}{\omega_0 C} = 44,721 \Omega$$

$$X_{L,0} = \omega_0 L = "$$

when  $\omega = \omega_0 = \frac{1}{\sqrt{LC}}$

$$= 2\pi f_0$$

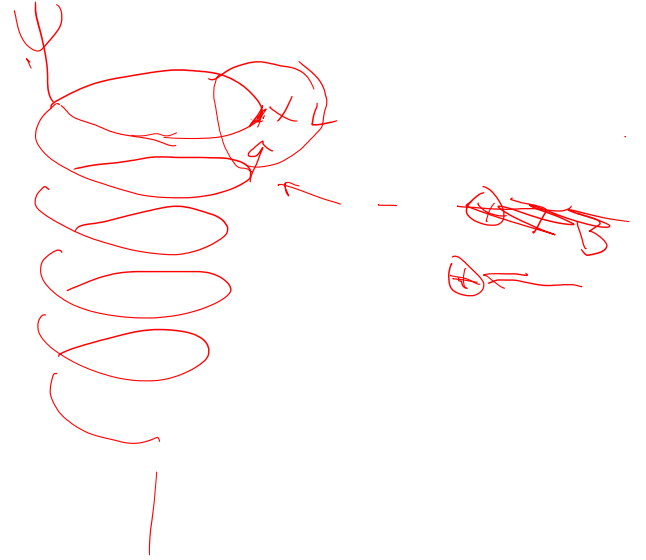
$$f_0 = 356,062 \text{ Hz}$$

$$V_C = IX_C = \frac{\epsilon_{\text{max}}}{R} X_C = 100 \text{ V} \frac{44,726}{10,000} = 447 \text{ V}$$

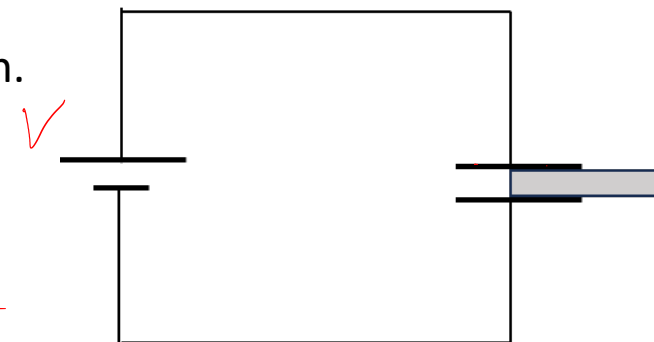
8. We run a current through a metal spring (but electrically insulated so that the turns do not touch each other).

What will happen to the spring:

- The coils will be repelled from each other, and the spring will expand/lengthen.
- The coils will be attracted to each other, and the spring will shorten/contract.
- Nothing



9. Consider a physical capacitor, hooked up to a 9-volt battery. There is a dielectric slab half-way inserted between the plates of the capacitor, as shown.



Which of the following statements is false?

- The capacitor will store more energy than if the slab were not present T
- The charge densities on the left and right halves of the capacitor will be different. T
- The slab will experience a force trying to pull it all the way into the capacitor F
- The slab will experience a force trying to push it all the way out of the capacitor T
- The value of the capacitance will increase by approximately  $(1 + \kappa)/2$ , where  $\kappa$  is the dielectric constant of the slab. T

$$Q = CV$$

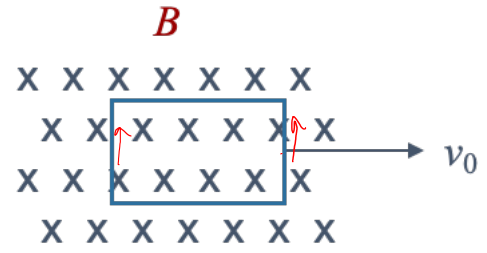
$$\frac{1}{2} \frac{Q^2}{C}$$

$$= \frac{1}{2} CV^2$$

$$C = \kappa \epsilon_0 \frac{A}{d}$$

$$C_{\text{eff}} = \frac{C}{2} (1 + \kappa)$$

Consider a region of space with a static magnetic field (pointing into the page).



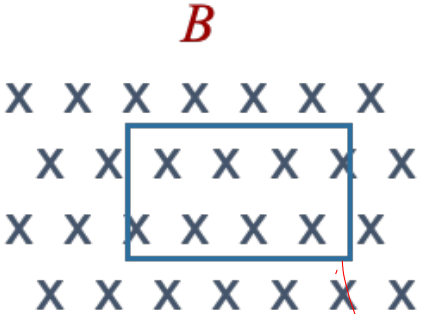
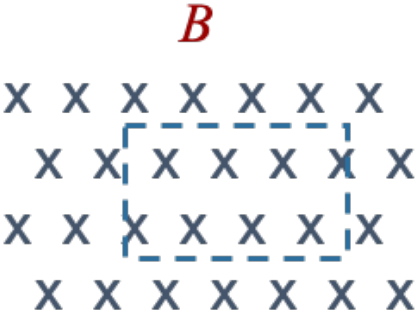
10. If we pull a conductive loop to the right as shown, what force will we need to apply, assuming the loop has area  $A$ ?

- $F = v_0 A B$
- No force, unless we use a battery to drive a current  $I$  clockwise in the loop.
- No force

No net force in a uniform field



Consider a region of space in which a magnetic field (pointing into the page) is increasing.

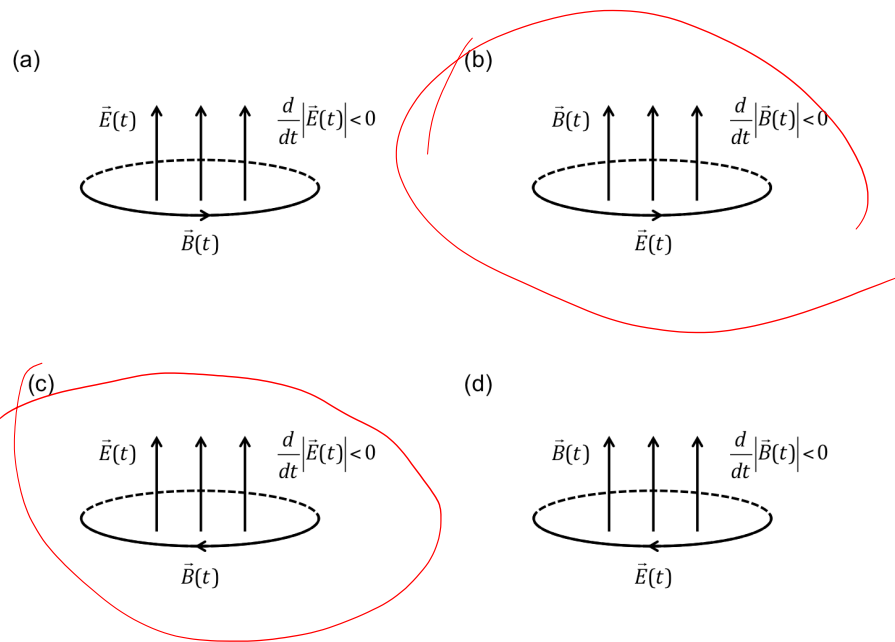


$$\mathcal{E}_{\text{emf}} = - \oint \vec{E} \cdot d\vec{\ell} = \frac{d\Phi}{dt}$$

11. Compare the emf around the loop shown, when the region is in vacuum (left), or when there is a physical conductor along the loop.

- $emf_{\text{vacuum}} = emf_{\text{conductor}}$
- $emf_{\text{vacuum}} < emf_{\text{conductor}}$
- $emf_{\text{vacuum}} > emf_{\text{conductor}}$

Induced current will circulate CCW to oppose  $\frac{d\Phi}{dt}$



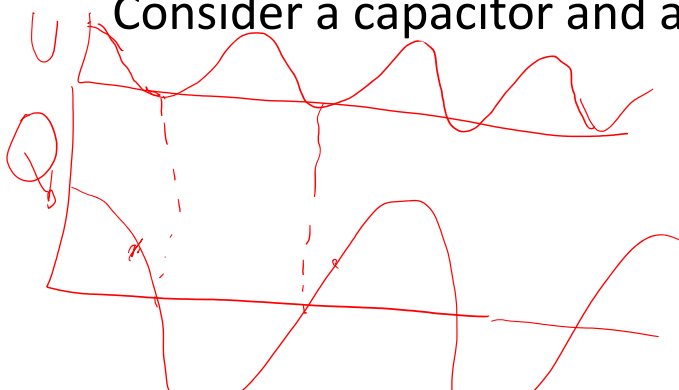
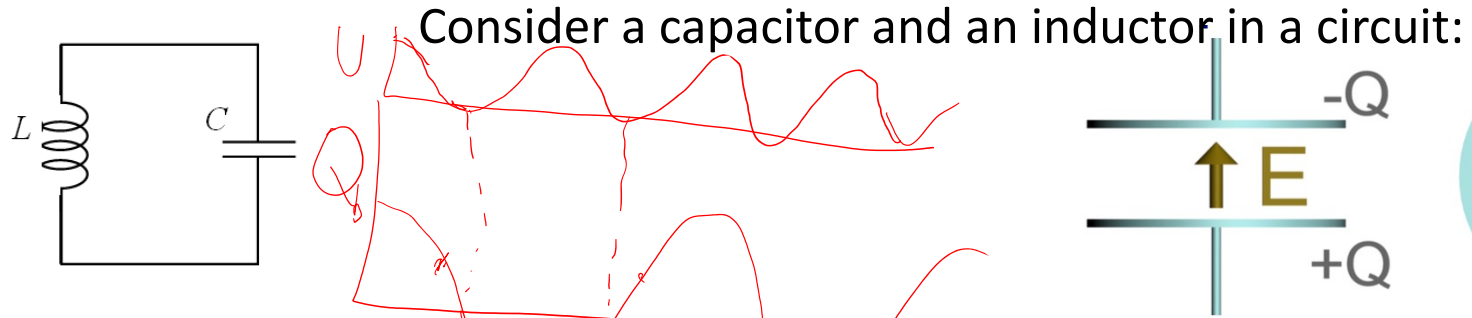
$$\int E \cdot dl = - \frac{d\Phi_B}{dt}$$

$$\int B \cdot dl = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

12. Which of the above represent a possible correct physical relationship between  $E$  and  $B$  fields?

- Top two.
- Bottom two
- Top left, bottom right
- Top right, bottom left
- Top right, and neither of the left are correct.





Handwritten notes:

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

$$I_D = \frac{dQ}{dt} = \frac{d\Phi_E}{dt}$$

Here is a detailed look at the capacitor at some instant of time  $t$ , with charge  $Q$  and electric and magnetic fields  $E$  and  $B$ . Right is a top down view of the capacitor.

13. At this time the energy stored in the electric field is:

- Increasing in time
- Constant in time
- Decreasing in time

14. At this time, what is the direction of the Poynting vector (in the right diagram, i.e., top view)?

- Out of the screen
- Radially inward
- Radially outward

Handwritten equation:

$$\vec{S} = \vec{E} \times \vec{B}$$

Handwritten notes:

CCW  $B \rightarrow E$  increasing  
 $\Rightarrow Q$  increasing  
 $\Rightarrow U$  increasing

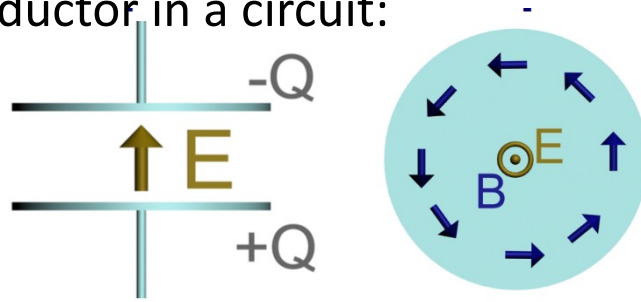
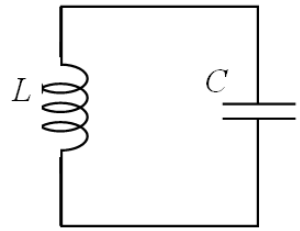
15. When the capacitor is fully charged, what is the direction of the Poynting vector (in the right diagram, i.e., top view)?

- The magnitude of the Poynting vector is zero, so it has no direction
- Radially inward
- Radially outward

Handwritten notes:

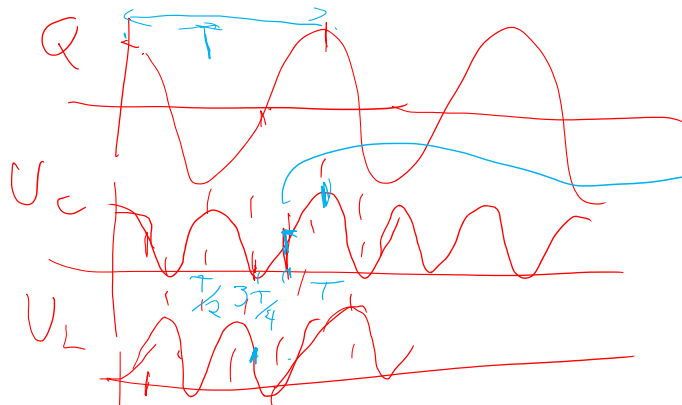
No current  $\rightarrow$  no  $\frac{dQ}{dt}$   
 $\Rightarrow$  No  $\frac{dE}{dt} \Rightarrow$  No  $\frac{d\Phi_E}{dt} \Rightarrow B = 0$

Consider a capacitor and an inductor in a circuit:



16. Assuming the capacitor is fully charged (with charge +Q on the bottom plate) at time  $t = 0$ , when is the next time that the energy of the system will be evenly divided between the capacitor and the inductor, and the configuration of fields inside the capacitor will be as shown in the top view above? Assume  $C = 2 \text{ mF}$  and  $L = 2 \text{ nH}$ .

- a.  $12.6 \mu\text{s}$
- b.  $11.0 \mu\text{s}$
- c.  $9.42 \mu\text{s}$
- d.  $6.28 \mu\text{s}$
- e.  $3.14 \mu\text{s}$



Handwritten notes and equations:

- $T = \frac{1}{f}$
- $\omega = 2\pi f = \frac{1}{\sqrt{LC}}$
- $T = \frac{1}{f} = \frac{1}{\frac{\omega}{2\pi}} = \frac{2\pi}{\omega} = 2\pi\sqrt{LC}$

17. Assuming that the maximum voltage on the capacitor is  $V_{\text{max}}$ , what is the magnitude of the magnetic field inside the inductor at the time of the previous question

- a.  $|B| = V_{\text{max}}/(\text{sqrt}2 d c)$ , where  $d$  is the separation of the capacitor plates, and  $c$  is the speed of light.
- b.  $|B| = \text{sqrt}(C) V_{\text{max}}/2$ , where  $C$  is the capacitor value
- c. Cannot be determined from the information given.

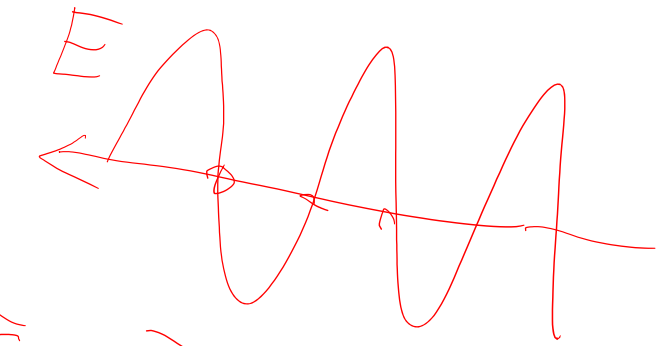
Handwritten equation:  $\frac{1}{2} L I^2 = U_{\text{mag}}$

Handwritten equation:  $U_{\text{mag}} = \frac{1}{2} \frac{B^2}{\mu_0} V$

18. Consider an electromagnetic plane wave whose electric field is given by  $E = x \cos(kz + \omega t) + y \sin(kz + \omega t)$

$B_{in} = -\hat{y}$   $B = +\hat{x}$

Circular in  $(-\hat{z})$



What is the correct description of the magnetic field in this case?

- a.  $B = x \cos(kz + \omega t) + y \sin(kz + \omega t)$
- b.  $B = x \sin(kz + \omega t) - y \cos(kz + \omega t)$
- c.  $B = x \sin(kz + \omega t) + y \sin(kz + \omega t)$
- d.  $B = x \sin(kz - \omega t) - y \cos(kz - \omega t)$
- e.  $B = x \sin(kz - \omega t) + y \sin(kz - \omega t)$



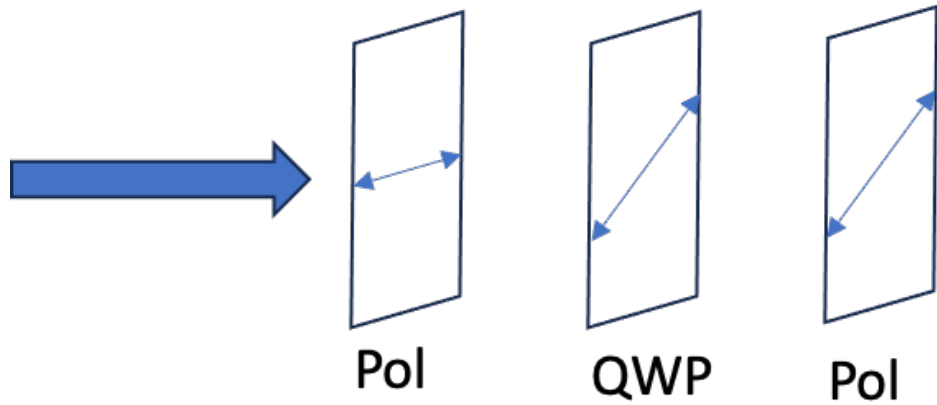
$$S = \frac{E \times B}{\mu_0}$$

19. Linear polarized light has nulls, i.e., for a (plane) wave polarized along  $x$  and propagating along  $z$ , at one instant of time there are locations along  $z$  where  $|E| = 0$ .

What can we say about the magnitude of the electric field for circularly polarized light of the same total intensity, also propagating along  $z$ ?

- a.  $|E| > 0$  everywhere
- b.  $|E| = 0$  occurs with the same spacing (along  $z$ ) as it does for linearly polarized light, and  $|E_{max,circular}| = |E_{max,linear}|$ , assuming the light has the same intensity,  $I_{circular} = I_{linear}$ .
- c.  $|E| = 0$  occurs with the same spacing (along  $z$ ) as it does for linearly polarized light, and  $|E_{max,circular}| < |E_{max,linear}|$ , assuming the light has the same intensity,  $I_{circular} = I_{linear}$ .

Consider the following optical setup, with light of intensity  $I_0$  incident on a horizontal polarizer, then a quarter wave plate (with fast axis at +45deg) and a final polarizer with transmission axis also at 45 deg.



Unpol  $\rightarrow$  H  $\frac{I}{2}$   $\rightarrow$  Circular  $\frac{I}{2}$   $\rightarrow$  H  $\frac{I}{4}$   $I_0$

$I_0 \cos^2(\theta_2 - \theta_1)$

20. Which of the following will have the largest intensity at the end?

- Circular polarized light
- Unpolarized light
- Diagonally polarized light
- Unpolarized and diagonal will both have same final intensity, greater than with circular input.
- All these inputs will produce the same final intensity.

21. Assuming now that the input light is unpolarized. One of the three elements will now be rotated by 45 deg.

Which one should be rotated to increase the transmitted intensity?

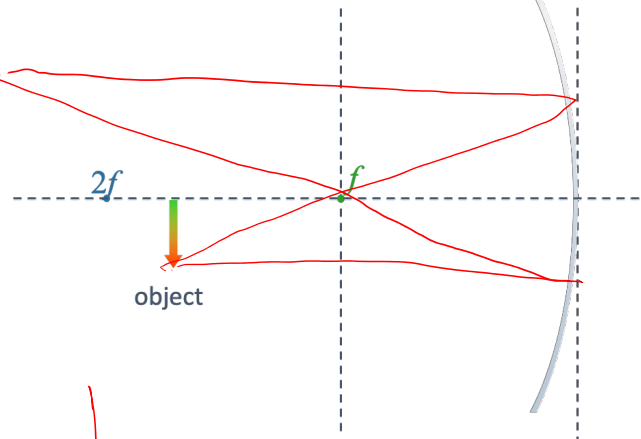
- The first polarizer should be rotated to +45deg.
- The quarter waveplate fast axis should be rotated to 0 (horizontally)
- The second polarizer should be rotated to +45 deg.

Nothing rotated Un  $\rightarrow$  H  $\frac{I}{2}$   $\rightarrow$  Circular  $\frac{I}{2}$   $\rightarrow$  Diagonal  $\frac{I}{4}$   $I_0$

Unpo  $\rightarrow$  Diag  $\frac{I}{2}$   $\rightarrow$  Diagonal  $\frac{I}{2}$   $\rightarrow$  Diagonal  $\frac{I}{2}$   $I_0$

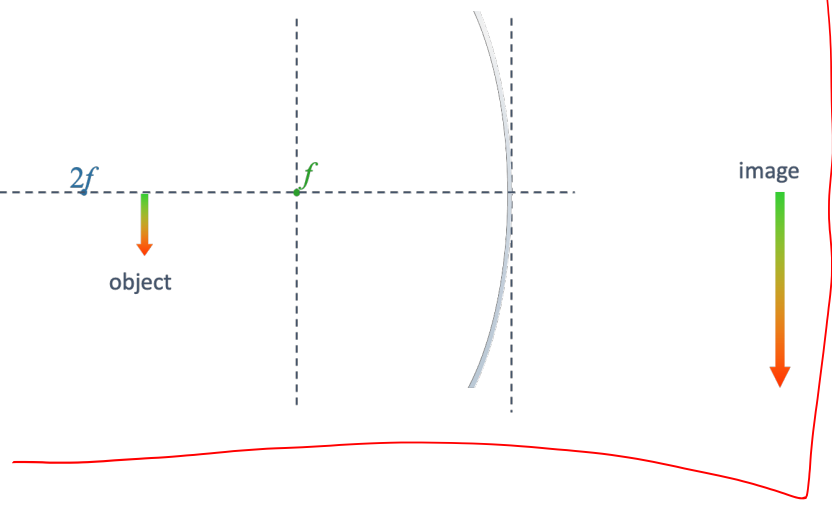
Unpol  $\rightarrow$  Hor  $\frac{I}{2}$   $\rightarrow$  Hor  $\frac{I}{2}$   $\rightarrow$  Diag  $\frac{I}{4}$   $I_0$

Consider the following optical setup.

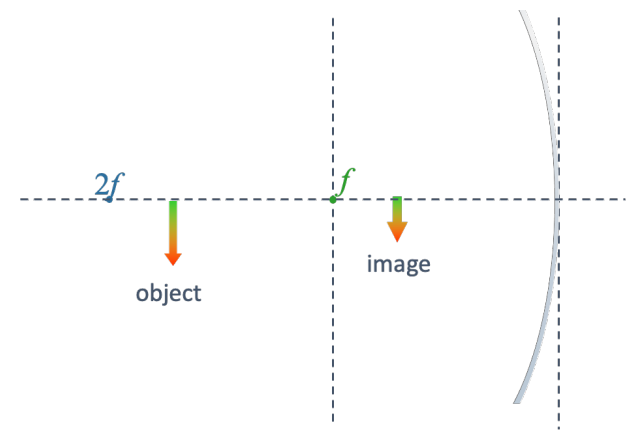


22. Which of the following properly describes where the image will be?

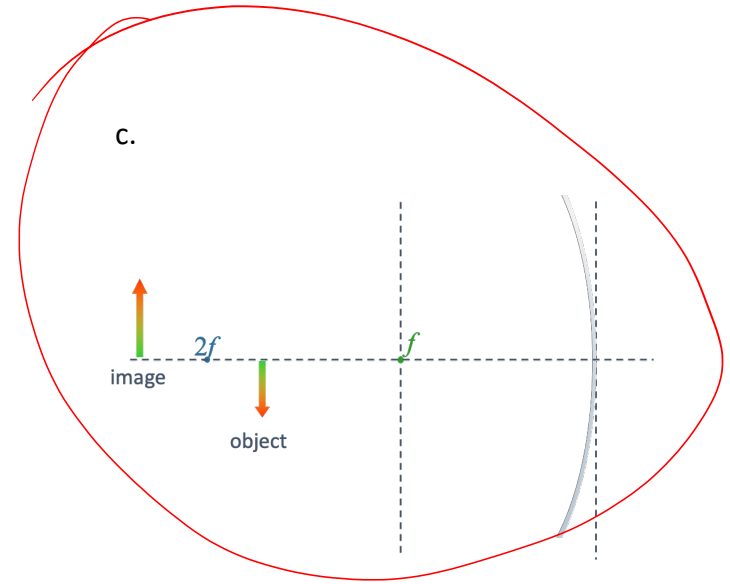
a.



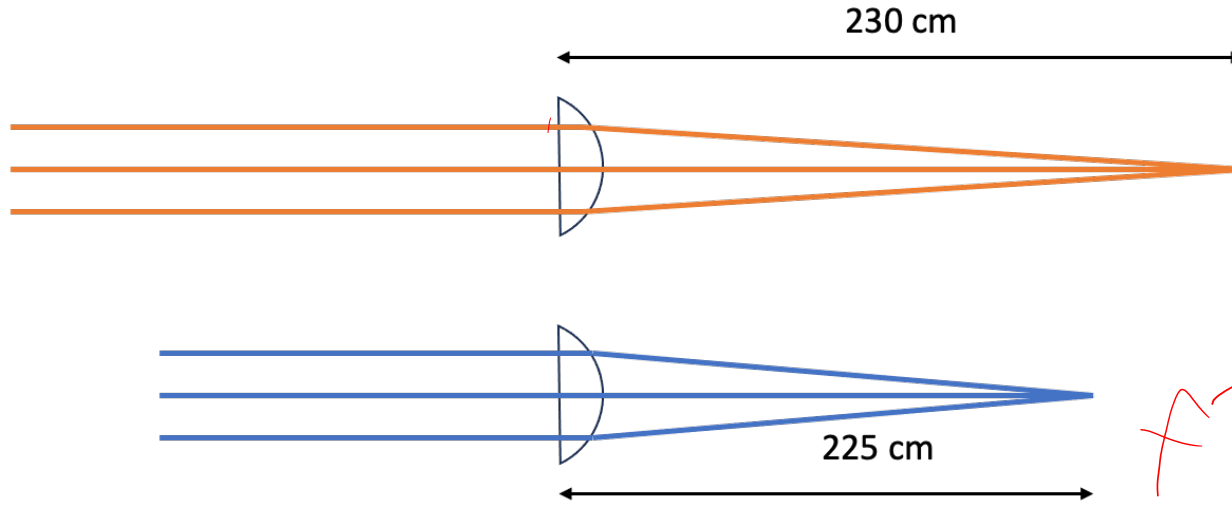
b.



c.



Consider the following optical setup. Light of wavelength 600 nm is incident on a plano-convex lens, made of material whose index of refraction at 600 nm is 1.52. This light is observed to come to a focus 230 cm from the lens (assume thin lens approximation). We now shine in light of wavelength 450 nm, and observe the focus has moved close to the lens by 5 cm.



$$\frac{1}{f} = (n-1) \frac{1}{R}$$

$$f = 230 \text{ cm}$$

$$R = (n-1) f$$

$$f' = 225 \text{ cm}$$

$$(n'-1) = \frac{R}{f'}$$

$$n' = 1 + (n-1) \frac{f}{f'}$$

$$= 1 + (1.52-1) \frac{230}{225} = 1.532$$

23. What is the index of refraction of the lens material at 450 nm?

- 1.532
- 1.525
- ~~1.515~~
- ~~1.508~~
- Cannot be determined from the information given.

$$\frac{1}{f} = (n-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$



# Lenses

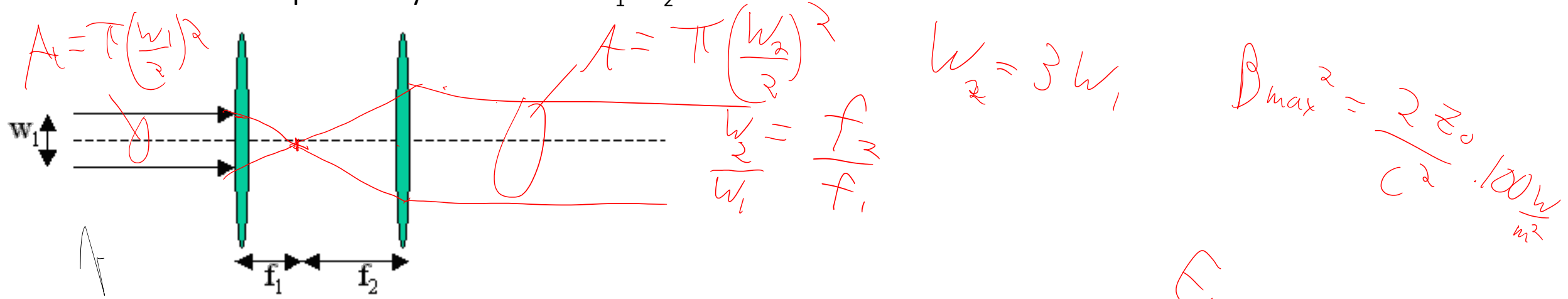
Object	left	$s > 0$
	right	$s < 0$
image	right	$s' > 0$
	left	$s' < 0$
Converging		$f > 0$
Diverging		$f < 0$
$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$		

Light propagates  $L \rightarrow R$

# Mirrors

> Same		
image	left	(in front of mirror) $s' > 0$
	right	(virtual behind) $s' < 0$
Concave	)	$f > 0$
Convex	(	$f < 0$

Consider the following simple optical system, used to change the diameter of a laser beam. Here  $f_2$ , the focal length of the second lens is three times as big as the focal length  $f_1$  of the first lens ( $f_2 = 3 f_1$ ), and the two lenses are separated by a distance  $d = f_1 + f_2$ .



24. Assuming the incident laser has a diameter  $W_1 = 4 \text{ mm}$ , and an intensity  $10 \text{ mW/cm}^2$ , what is the maximum value of the magnetic field to the right of the second lens?

- 823.8 T
- $2.75 \mu\text{T}$
- $1.58 \mu\text{T}$

Handwritten calculations for question 24:

$$I_2 = I_1 \left(\frac{W_1}{W_2}\right)^2 = \frac{I_1}{3^2}$$

$$I_1 = \frac{0.01 \text{ W}}{\text{cm}^2} \times \left(\frac{100 \text{ cm}}{1 \text{ m}}\right)^2 = 100 \frac{\text{W}}{\text{m}^2}$$

$$I_2 = \frac{100 \text{ W/m}^2}{9} = \frac{1}{9} \frac{E_{max}^2}{2Z_0}$$

$$I_2 = \frac{1}{9} \frac{c^2 B_{max}^2}{2Z_0}$$

25. If we now change the wavelength of the laser from 600 nm to 450 nm, how will the intensity of the beam just after the second lens change?

- Intensity will increase
- Intensity will decrease
- Intensity will not change.

Handwritten calculation for question 25:

$$I_2 = \frac{1}{9} \frac{I_1}{\lambda}$$

$$I_2 = \frac{1}{9} \frac{c^2 B_{max}^2}{2Z_0}$$