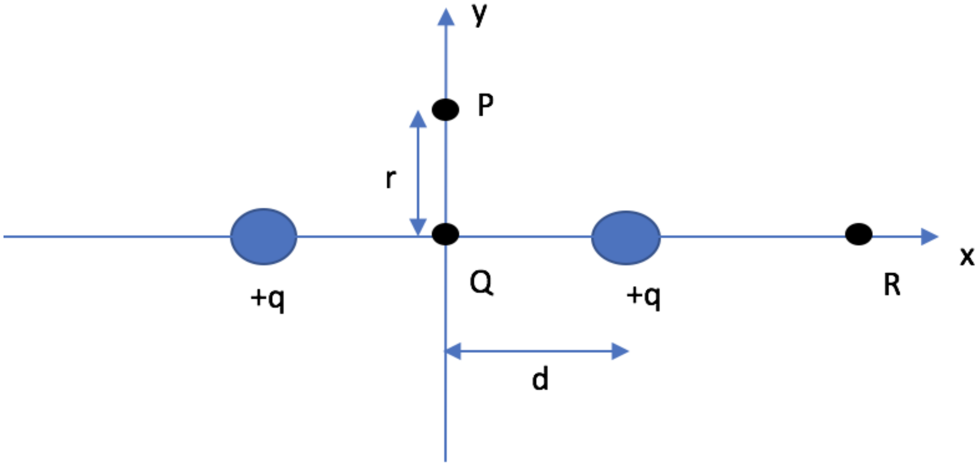
The next three questions pertain to the situation described below.

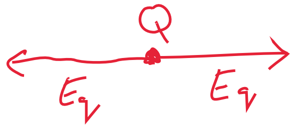
Consider the collection of two charges separated in the x-axis by a distance 2d:



1) Where should we place a negatively charged test particle so that it is not subject to a force?

At point Q, the net electric field is zero

a. Point R
[✓ b.] Point Q
c. Point P



2) Now place the negative test charge at point Q. Which statement below is correct?

Displace charge slightly in +y direction, net force is in the -y direction

a. The test charge is stable to a slight perturbation in the x direction
[✓ b.] The test charge is stable to a slight perturbation in the y direction
c. The test charge is stable to a slight perturbation in both x and y directions

3) Write an expression for the force on a negatively charged test particle of charge -q at location P. ($\widehat{x}$ and $\hat{y}$ are unit vectors in the x and y directions.)

a. $-\frac{2kq^2}{(d^2+r^2)}$

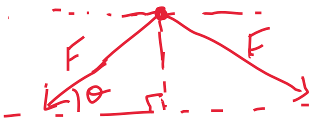
[✓ b.] $-\frac{2kq^2 r}{(d^2+r^2)^{3/2}}\hat{y}$

c. $-\frac{2kq^2 d}{(d^2+r^2)^{3/2}}\hat{y}$

d. $-\frac{2kq^2 dr}{(d^2+r^2)^2}\widehat{x}$

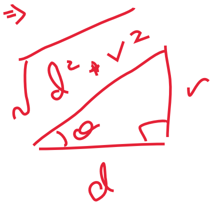
e. $\frac{2kq^2 r}{(d^2+r^2)^{3/2}}\widehat{x}$

At P ⇒



by Coulomb's law ⇒

$$|F| = \frac{Kq^2}{d^2+r^2}$$



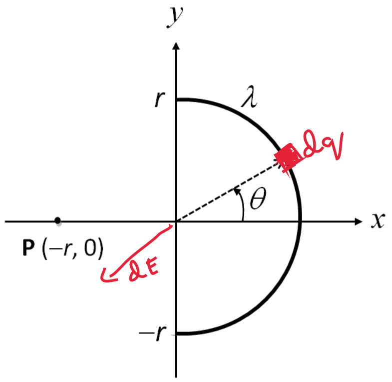
Note that the horizontal components cancel
Vertical components add and point in $-\hat{y}$ direction

Net force $= -2F\sin\theta\,\hat{y}$

$$= -\frac{2Kq^2}{d^2+r^2}\,\frac{r\,\hat{y}}{(d^2+r^2)^{1/2}} = -\frac{2Krq^2}{(d^2+r^2)^{3/2}}\hat{y}$$

The next three questions pertain to the situation described below.

A uniformly charged semi-circular line with linear charge density λ is placed in the x-y plane as shown in the Figure on the right. Its radius is r and its total charge Q = 15 μC.



length of the line = $\frac{\cancel{2}\pi r}{\cancel{2}}$

so $\lambda = \frac{Q}{\pi r} = \frac{15\mu C}{\pi (5/100)m}$ ← pay attention to the units.

$\lambda \simeq 95\,\mu C/m$

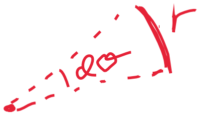
4) If r = 5 cm, what is λ?

a. λ = 48 μC/m
[✓ b.] λ = 95 μC/m
c. λ = 150 μC/m

5) What is the correct expression for the x-component of the total electric field at the origin, E_x, due to this charge?

Consider a charge element dq at position θ shown above

$dq = \lambda r d\theta \rightarrow dE_x = -\frac{K\,dq}{r^2}\cos\theta$



$dE_x = -\frac{K\lambda}{r}\cos\theta\, d\theta$

$E_x = -\int_{-\pi/2}^{\pi/2} \frac{K\lambda}{r}\cos\theta\, d\theta$

a. $E_x = -\int_{-\pi/2}^{\pi/2} \frac{k\lambda}{r^2} cos\theta\, d\theta$
b. $E_x = -\int_{-\pi/2}^{\pi/2} \frac{k\lambda}{r} cos\theta sin\theta\, d\theta$
c. $E_x = 0$
d. $E_x = -\int_{-\pi/2}^{\pi/2} \frac{k\lambda}{r} sin\theta\, d\theta$
[✓ e.] $E_x = -\int_{-\pi/2}^{\pi/2} \frac{k\lambda}{r} cos\theta\, d\theta$

6) What kind of point charge should be placed at point **P**(-r, 0) in order to make the net electric field at the origin vanish?

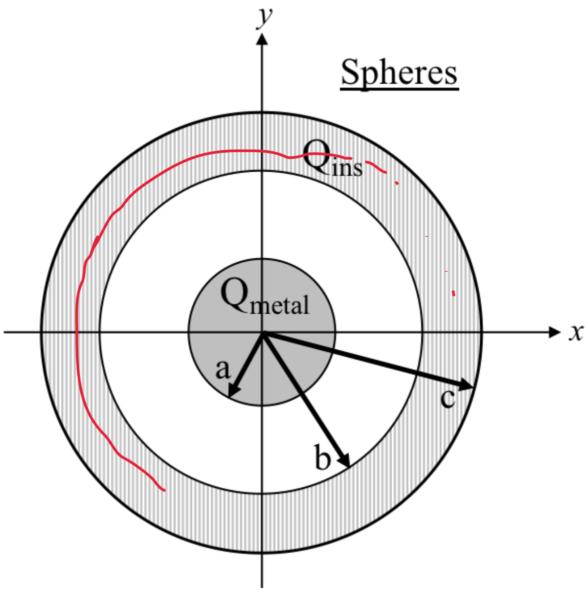
Note that $E_y = -\int_{-\pi/2}^{\pi/2} \frac{K\lambda}{r}\sin\theta\, d\theta = 0$

a. There is no such charge
[✓ b.] Positive charge
c. Negative Charge

The electric field above is only along the x-direction and since $\lambda > 0$, E_x points in the negative x-direction. So we need to place a positive charge at point P to make net electric field to be zero.

The next four questions pertain to the situation described below.

A metal conducting sphere of radius a is centered on the origin. Concentric with it is a spherical shell made of insulating material of inner radius b and outer radius c. A total positive charge Q_{metal} is placed on the inner metal sphere, while a total negative charge Q_{insul} is uniformly distributed over the volume of the outer insulating sphere. The values of all parameters are given in the figure below. The figure is not drawn to scale.



a = 5 cm
b = 9 cm
c = 17 cm
$Q_{metal} = 6$ μC
$Q_{insul} = -4$ μC

7) Calculate the surface charge density on the inner metal sphere at a = 5 cm

$$\sigma = \frac{Q_{metal}}{4\pi a^2} = \frac{6\mu C}{4\pi (5/100)^2} = 190\,\mu C/m^2$$

[✓ a.] $\sigma = 190$ μC/m²
b. $\sigma = 2 \times 10^4$ μC/m²
c. $\sigma = 19$ μC/m²

8) Calculate the magnitude of the electric field E at a radius of 7 cm from the origin.

at $r < b$, Use Gauss' law, $\oint \vec{E} \cdot d\vec{A} = Q_{metal}/\varepsilon_0$

$$|E| = \frac{Q_{metal}}{4\pi\varepsilon_0 r^2} = \frac{6 \times 10^{-6}}{4\pi(8.854\times10^{-12})(7\times10^{-2})^2}$$

[✓ a.] $|E| = 1.1 \times 10^7$ N/C
b. $|E| = 1.8 \times 10^7$ N/C
c. $|E| = 7.7 \times 10^5$ N/C

9) Calculate the magnitude of the electric field E at a radius of 13 cm from the origin.

Note that 13cm is for $b < r < c$

$$|E| = \frac{Q_{enclosed}}{4\pi\varepsilon_0 r^2}$$ where $Q_{enclosed} = Q_{metal} + \rho V$

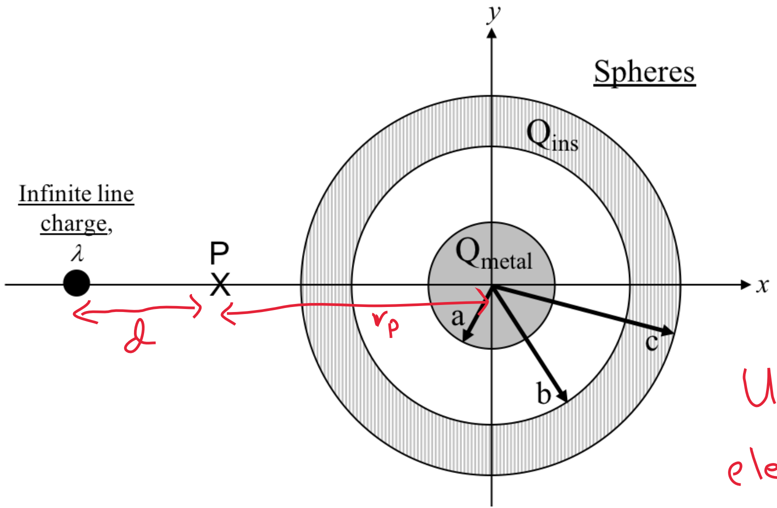
Note $V = \frac{4}{3}\pi r^3 - \frac{4}{3}\pi b^3 = 6.15 \times 10^{-3} m^3$

and $$\rho = \frac{Q_{ins}}{\frac{4}{3}\pi c^3 - \frac{4}{3}\pi b^3} = -228\,\mu C/m^3$$

a. $|E| = 8.2 \times 10^6$ N/C
b. $|E| = 7.4 \times 10^7$ N/C
[✓ c.] $|E| = 2.4 \times 10^6$ N/C
d. $|E| = 0$ N/C
e. $|E| = 5.3 \times 10^6$ N/C

$$|E| = \frac{(6 + (-228)(6.15\times10^{-3}))10^{-6}}{4\pi\varepsilon_0 (13/100)^2}$$

10) An infinite line charge having λ = 4 μC/m is now added parallel to the z-axis and centered at (x,y) = (-30 cm, 0), as shown. Find the magnitude of electric field at a point P located at (x,y) = (-21.5 cm,0). (The figure is not drawn to scale. Here we assume the line charge does not affect the surface charge distribution on the metal sphere.)



a = 5 cm
b = 9 cm
c = 17 cm
Q_{metal} = 6 μC
Q_{insul} = -4 μC

Use superposition of electric fields ⇒

E-field due to spheres at point P ⇒

$$\int \vec{E}\cdot d\vec{A} = \frac{Q_{total}}{\varepsilon_0} = \frac{Q_{metal}+Q_{ins}}{\varepsilon_0}$$

$$\vec{E}_s = -\left\{\frac{Q_{metal}+Q_{ins}}{4\pi r_p^2 \varepsilon_0}\right\}\hat{x}$$ → points in the -ve x direction

a. |E| = 7.2 × 10^6 N/C
b. |E| = 8.5 × 10^5 N/C
c. |E| = 1.9 × 10^6 N/C
d. |E| = 1.9 × 10^5 N/C
[✓ e. |E| = 4.6 × 10^5 N/C]

E-field due to line charge at P ⇒

$$\vec{E}_L = \frac{\lambda \cancel{L}\,\hat{x}}{2\pi d \cancel{L}\,\varepsilon_0} = \frac{\lambda}{2\pi\varepsilon_0 d}\hat{x}$$ → points in the +ve x-direction.

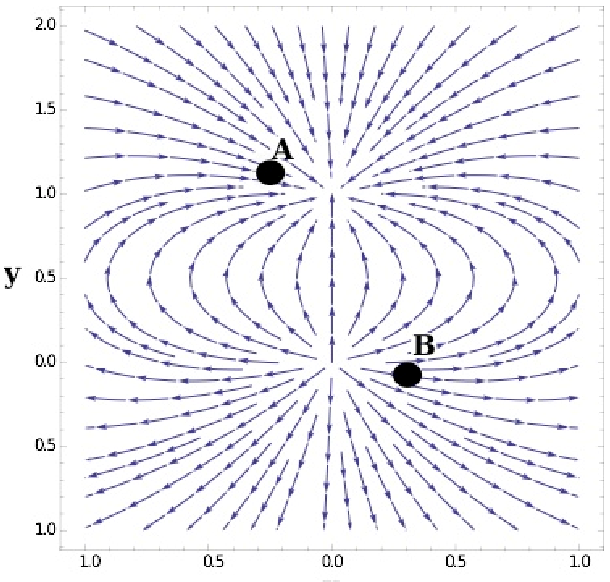
$$\text{so } |E| = -\left\{\frac{Q_{metal}+Q_{ins}}{4\pi\varepsilon_0 r_p^2}\right\} + \frac{\lambda}{2\pi\varepsilon_0 d}$$

$$= -\left\{\frac{6\times10^{-6} - 4\times10^{-6}}{4\pi\varepsilon_0 (21.5\times10^{-2})^2}\right\} + \frac{4\times10^{-6}}{2\pi\varepsilon_0 (8.5\times10^{-2})}$$

$$|E| = 4.6\times10^{5}\ \text{N/C}$$

The next three questions pertain to the situation described below.

The electric field due to two point charges is given as a cross sectional view in the x-y plane.There are two points labeled A and B on the graph.



11) What is the position of the point charges?

a. A positive charge at (1,0) and a negative charge at (0,0)
[✓ b.] A positive charge at (0,0) and a negative charge at (0,1)
c. A positive charge at (0,1) and a negative charge at (0,0)

Note that field lines point away from (0,0) → must be +ve and field lines point towards (0,1) → must be -ve

12) Which of these lines is an equipotential?

[✓ a.] y=0.5
b. y=0
c. x=0.5
d. x=1
e. x=0

Equipotential lines are always perpendicular to the electric field lines.

13) What is the relationship between the potential at A and B?

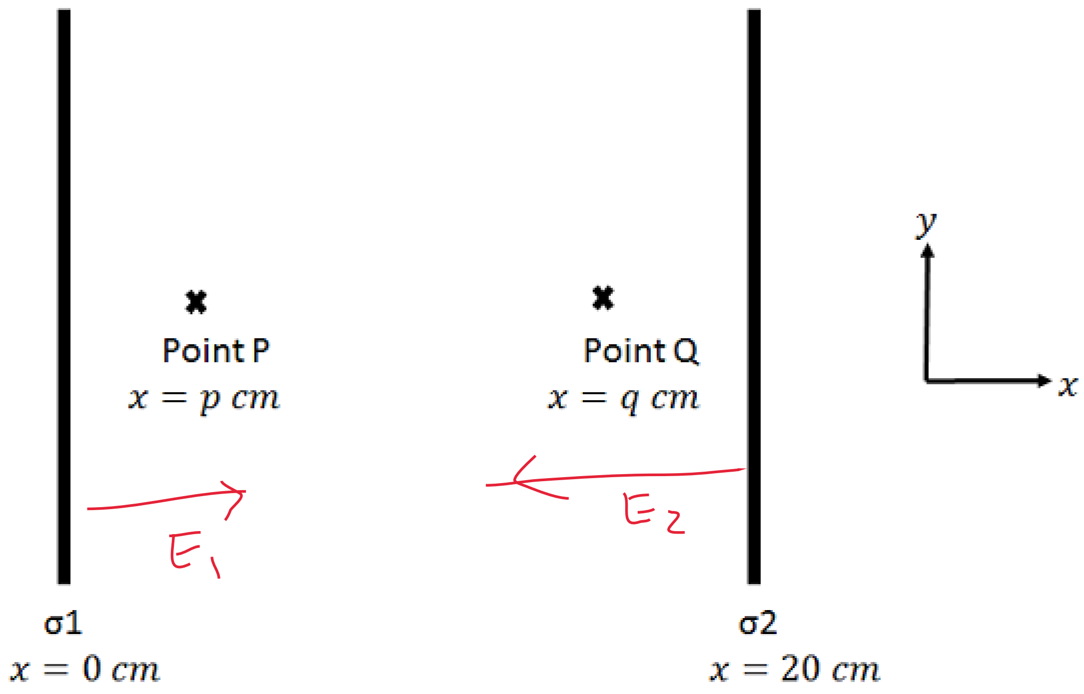
a. $V_A > V_B$
[✓ b.] $V_A < V_B$
c. $V_A = V_B$

Note that potential at B is positive while potential at A is negative

The next three questions pertain to the situation described below.

As seen in Figure 1, two charged, infinite plates have charge densities of $\sigma_1 = 5\ \mu C/m^2$ and $\sigma_2 = 9\ \mu C/m^2$ and are placed at x = 0 cm and x = 20 cm respectively.

Figure 1



14) What is the potential difference between point Q ,(q cm , 0 cm) and point P, (p cm , 0 cm)? Let q = 17 cm and p = 2 cm.

a. $\Delta V = 84$ kV
b. $\Delta V = 120$ kV
[✓ c. $\Delta V = 34$ kV]
d. $\Delta V = 280$ kV
e. $\Delta V = 230$ kV

$$\Delta V = -\int E \cdot dl$$

for an infinite charged plate $E = \sigma / 2\varepsilon_0$

$$E_{total} = E_1 - E_2 = \frac{\sigma_1}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0}$$

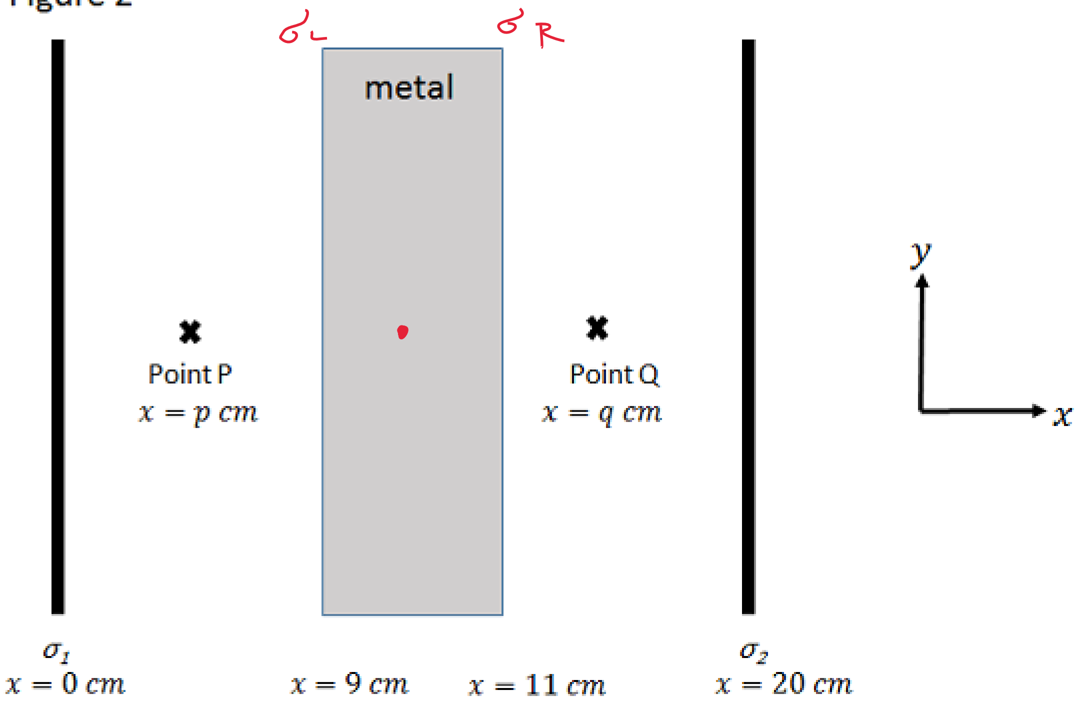
$$\Delta V = -\left(\frac{\sigma_1 - \sigma_2}{2\varepsilon_0}\right)(q - p)$$

$$= -\left(\frac{5 - 9}{2\varepsilon_0}\right)(15\ cm)\ \frac{mC}{m^2}$$

$$\Delta V = 34\ kV$$

15) For the next two problems, consider Figure 2.

Figure 2



A slab of metal with no net charge(shaded region; left edge at x =9 cm, right edge at x = 11cm) is now placed between the two charged, infinite plates.

What are the surface charge densities for the left side of the slab (given by σ_L) and the right side of the slab (given by σ_R)?

Efield in metal $= \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_L}{2\varepsilon_0} - \frac{\sigma_R}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0} = 0$ — ①

a. $\sigma_L = -7\ \mu C/m^2, \sigma_R = 7\ \mu C/m^2$
b. $\sigma_L = -2\ \mu C/m^2, \sigma_R = 2\ \mu C/m^2$
[✓ c.] $\sigma_L = 2\ \mu C/m^2, \sigma_R = -2\ \mu C/m^2$
d. $\sigma_L = 0\ \mu C/m^2, \sigma_R = 0\ \mu C/m^2$
e. $\sigma_L = 7\ \mu C/m^2, \sigma_R = -7\ \mu C/m^2$

also no net charge, $\sigma_L + \sigma_R = 0$

$\sigma_1 + \sigma_L + \sigma_L - \sigma_2 = 0$

$\sigma_L = \frac{\sigma_2 - \sigma_1}{2}$, $\underline{\sigma_R = -\sigma_L}$

16) With the addition of the metal slab, what is the potential difference between point Q and point P?

$\Delta V = E\ d$

a. ΔV = 100 kV
b. ΔV = 34 kV
c. ΔV = 0 kV
[✓ d.] ΔV = 29 kV
e. ΔV = 120 kV

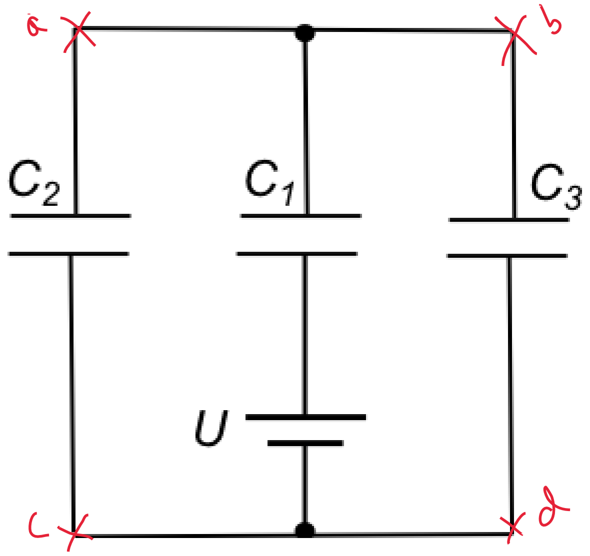
$E = \frac{\sigma_1}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0}$

$\Delta V = E(q - p - 2cm)$

$\Delta V = 29KV$

The next two questions pertain to the situation described below.

Three capacitors and an ideal battery are connected as shown in the figure below. The voltage of the battery is U= 9 V. The capacitances of the capacitors are $C_1 = 4\ \mu F$, $C_2 = 4\ \mu F$, and $C_3 = 6\ \mu F$.



17) Capacitors C_2 and C_3 are connected _______

Potential at 'a' = potential at 'b'

potential at 'c' = potential at 'd'

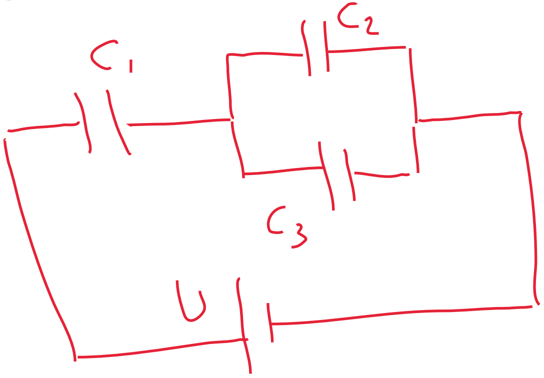
a. in series
[✓ b. in parallel]
c. neither in parallel nor in series

18) What is the total energy stored in all three capacitors?

[✓ a. 120 μJ]
b. 570 μJ
c. 26 μJ
d. 230 μJ
e. 1100 μJ

Total energy stored $= \frac{1}{2} C_{total} U^2$

Redraw circuit



$C_{23} = C_2 + C_3$

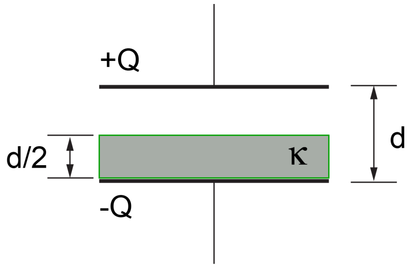
$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2 + C_3}$$

$$C_{total} = \frac{C_1 (C_2 + C_3)}{C_1 + C_2 + C_3}$$

So Total energy $= \frac{1}{2} \left\{ \frac{C_1 [C_2 + C_3]}{C_1 + C_2 + C_3} \right\} U^2$

The next two questions pertain to the situation described below.

A parallel plate capacitor is constructed with a dielectric slab with $\kappa = 1.7$ inserted between the plates. The area of each plate is 6 cm^2, and the distance between the two plates is 2 mm. Assume the infinite plane approximation.



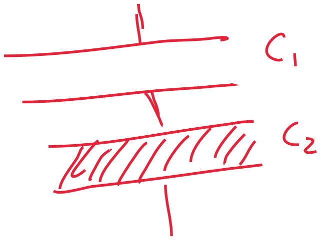
19) If we fix the charge on the capacitor to be $Q = 6\times10^{-11}$ C, what is the potential difference between the top and bottom plates? Note that in the region containing the dielectric medium, the electric field is $E = E_0 / \kappa$, where E_0 is the electric field in vacuum.

Consider this as two capacitors in series ⇒

[✓ a.] $\Delta V = 18$ V
b. $\Delta V = 6.6$ V
c. $\Delta V = 11$ V
d. $\Delta V = 23$ V
e. $\Delta V = 4.2$ V

$$C_1 = \frac{\varepsilon_0 A}{(d/2)} \quad , \quad C_2 = \frac{K\varepsilon_0 A}{d/2}$$

$$\frac{1}{C_{tot}} = \frac{d}{2\varepsilon_0 A} + \frac{d}{2K\varepsilon_0 A}$$



$$C_{tot} = \frac{2K\varepsilon_0 A}{d(K+1)}$$

$$\Delta V = \frac{Q\, d(K+1)}{2K\varepsilon_0 A}$$

20) If the dielectric slab is removed, the capacitance of the capacitor will

a. remain the same
[✓ b.] decrease
c. increase

as $C_{tot} = \frac{2K\varepsilon_0 A}{d(K+1)}$

if dielcetric is removed

C_{tot} will decrease