Name   **Net ID**

TA Name   **Discussion Section**

PHYS 212 James Scholar Assignment #4

The problems are to be done on paper, **showing all work**. Again, *the presentation should be neat, legible, and easy to follow*.

Your assignments will all be submitted using Gradescope.

In this assignment you will learn all about magnetic resonance imaging (MRI), or as it used to be called, nuclear magnetic resonance (NMR) imaging (the name was changed because the average Joe was afraid of anything with the word nuclear, or 'nucular', as George W. would say!).

UIUC Chemistry professor Paul Lauterbur (who passed away in March 2007) won the 2003 Nobel Prize in Medicine for his development of MRI. Lauterbur shared the Prize with Sir Peter Mansfield of the University of Nottingham in England. Mansfield was a research associate in the department of physics at Illinois from 1962-1964. Adding to these two the Physics Nobel Prize awarded that year to UIUC Physicist Anthony Leggett (http://www.news.uiuc.edu/NEWS/03/1007nobelphys.html) makes a total of three Nobel winners for UIUC in 2003. (Also, it should be not be overlooked that UIUC microbiologist Professor Carl Woese received the 2003 Crafoord Prize, awarded annually by the Royal Swedish Academy of Sciences to recognize accomplishments in scientific fields *not* covered by the Nobel Prizes; for details see http://www.news.uiuc.edu/news/03/0925woeseaward.html.)

There are many excellent MRI references and tutorials available.

> https://www.youtube.com/watch?v=OZjpaDMSXVM A-Level Physics HQ

> https://www.youtube.com/watch?v=Ok9ILIYzmaY   Introduction to MRI

Finally, for those of you that are really interested, the following is the best overall tutorial site I found (but it is longer than I expect most of you to go through):

- http://www.cis.rit.edu/htbooks/mri/

1. **Clinical MRI uses the magnetic properties of which nuclei?**
   - ☐ helium
   - ☐ nitrogen
   - ☐ hydrogen
   - ☐ helium *and* nitrogen

The ratio between the magnetic moment and the spin angular momentum and is called the 'gyromagnetic ratio', γ. For hydrogen, the nuclear spin angular momentum is entirely determined by the proton, since that's all there is in the nucleus. Since we know circulating currents produce a magnetic moment, it perhaps should not surprise us too much that a proton (with charge $+|e|$, and spin angular momentum $S = $ h-bar$/2$ [h-bar $=$ h$/2\pi$, and h is Planck's constant]) has a magnetic moment $\mu = \gamma S$. Both **S** and **μ** are vectors; because the sign of the charge is positive, they point in the same direction. [Note: Don't get too comfortable with this justification of the proton magnetic moment -- it turns out the neutron also has a magnetic moment, even though it has *no* net charge! This is due to the fact that it is composed of charged quarks.]

2. **If the experimentally measured value of the magnetic moment for hydrogen is 1.41×10^{-26} A m^2, what is the value of γ?**

3. **When placed in a large externally applied magnetic field B_0 (at room temperature), the nuclear magnetic moment will always align in the direction of the field.**
   - ☐ True
   - ☐ False

4. **What is the *difference* in the energy (in Joules) between the aligned and anti-aligned orientations of the magnetic moment, as given in Question 2, if the hydrogen is placed in a 1 Tesla magnetic field?**

5. **The magnitude of the net magnetization of the nuclei is proportional to the 'spin excess' -- the difference in the number of spins in the low-energy state and the number in the high-energy state.**
   - ☐ True
   - ☐ False

6. **According to quantum mechanics, the relation between energy E and frequency ν is $E = h\,\nu$. What is ν corresponding to the energy difference you calculated in Question 5? Give your answer in MHz.**

When an electromagnetic radio frequency pulse is directed onto the moment, it is possible to align the moment so that it is perpendicular to the main field B_0 = B_zz-hat. For example, after applying the RF pulse, the magnetic moment might point along x-hat. B_0 will then exert a torque τ on the magnetic moment. You may remember from Physics 211 that whenever a torque is applied at right angles to an angular momentum L, then L will start to precess (in the direction of τ). You learned then that the angular precession frequency $\Omega = \tau/L$.

7. For our tipped magnetic moment (value from Question 2), what is the number of revolutions/sec (i.e., what is $\Omega/2\pi$)? Give your answer in MHz.

8. Use the Larmor formula and the value of γ to calculate the resonant frequency of the hydrogen nuclear magnetic moment in a 1 Tesla magnetic field. To compare your value with your other calculations, divide it by 2π to make it a regular (and not an angular) frequency. Give your answer in MHz.

Hopefully from your answers to the previous two problems you understand somewhat where the Larmor formula comes from.

9. The FID
   ☐ is the signal induced in the receiver coil
   ☐ grows exponentially
   ☐ reduces the signal
   ☐ grows exponentially *and* reduces the signal

10. The loss of transverse magnetization during the FID occurs at a rate given by the
   ☐ gyromagnetic ratio
   ☐ lattice relaxation constant
   ☐ tissues T2* time
   ☐ T1 time

11. The regrowth of longitudinal magnetization (and consequently the reduction in the transverse magnetization that is detected by the induction coil) is given by the
   ☐ gyromagnetic ratio
   ☐ lattice relaxation constant
   ☐ tissues T2* time
   ☐ T1 time

12. **T1 and T2 relaxation processes are independent but occur at the same time.**

☐ True

☐ False

It is important to understand that the presence of other atoms can shift the resonant frequency of the nuclear moment. This is known as the Chemical shift. It can happen because the other atoms (their nuclei, and their electrons) also generate local magnetic fields, which contribute to the net field experienced by the hydrogen magnetic moment. In this way, it is possible to gain information about what molecules are nearby by looking at the precise resonance frequency.

Also, we see that the total signal which is detected will depend on the number of resonant nuclei being detected. In order to have position-resolved nuclear magnetic resonance, one uses a *non-uniform* B_0 field, i.e., a field with gradient dB_0/dx. Hydrogen molecules at different values of x will therefore experience a different strength magnetic field (see Lecture 13, p. 26). Through the resonance condition, $\Omega = \gamma B_0$, each position **x** corresponds to a different resonance frequency. Because the intensity of the NMR signal is a measure of the number of nuclei with a given resonance frequency, the field gradient leads to a unique corresponsence between the measured spectrum and the spatial arrangement of the nuclei.

13. **If the MRI gradient magnet can produce a maximum gradient of 0.1 Tesla/meter, and the device is intended to have a minimum spatial resolution (a slice size) of 1mm, the FID receiver must be able to distinguish frequencies separated by _____Hz.**

The idea to use a strong gradient in the magnetic field to localize the signal to a narrow slice of tissue was basically what Prof. Lauterbur got the Nobel prize for. There are two more experimental details. First, as the slice gets thinner and thinner, the number of nuclei contributing to the signal necessarily also becomes smaller, and their FID signal becomes harder and harder to see. One way to fight this is to use a bigger B_0, since this increases the spin excess.

Finally, the gradient technique determines how many nuclei are in a particular slice (corresponding to a particular resonant frequency). But this is not three dimensional information. In order to get that, the measurement is made by taking a slice in many orientations; the direction of the field gradient determines the orientation of the slice -- adjustable magnets inside the MRI device control the orientation of the magnetic field gradient (the switching of these on and off makes a rather loud knocking noise during an MRI scan). It is as though you were given an object and wanted to find out its shape, but the only measurements you could make were of various cross-sections. There is then a well-known mathematical algorithm (the 'inverse Radon Transform') for deducing the three-dimensional structure that produced the data obtained.

14. **To obtain the image shown,**
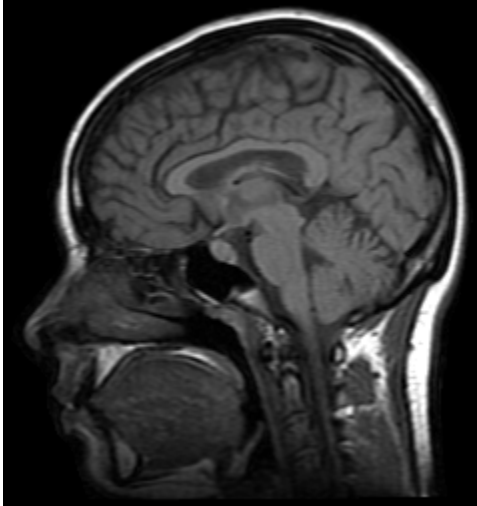
the gradient of the magnetic field pointed

- ☐ either to the left or the right
- ☐ either top to bottom, or bottom to top
- ☐ either into or out of the page

You may be interested/proud to know that *many* of the main nuclear magnetic resonance techniques were pioneered here. UIUC Physics Professor Charlie Slichter did much of the work using NMR to characterize condensed matter systems (he wrote the book that everyone in the field learns from!). And while he was here as a postdoctoral researcher, Erwin Hahn was the first person to experimentally discover spin echoes! And of course, now we have the Nobel Prize for the medical imaging applications of NMR. This is a perfect example of how fundamental science research can have an incredible impact on society. **Yea Physics!!**

15. **Magnetic resonance imaging is now used in a great variety of medical applications. List three for which MRIs are a particularly vital diagnostic tool.**