# Your comments/notes/etc

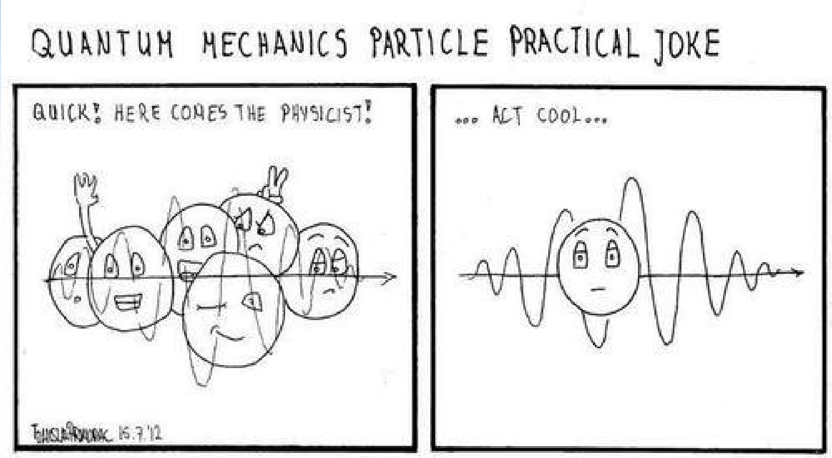
if a probability function is given, do we assume that the function is normalized?
Not necessarily. You may need to normalize it.

Its about time I get to learn why i is useful
You don't have to remember trig identities!

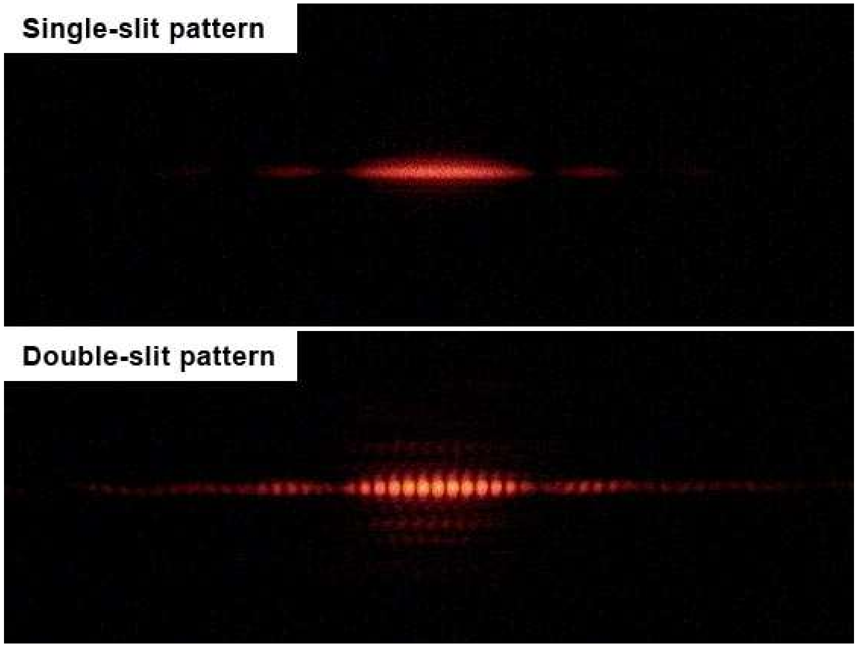
I had a really small problem with my car so I took it to a quantum mechanic

Why do we need probability density? Is it something like intensity that could describe the brightness? Yep! It determines the average number of photons we see -> intensity.

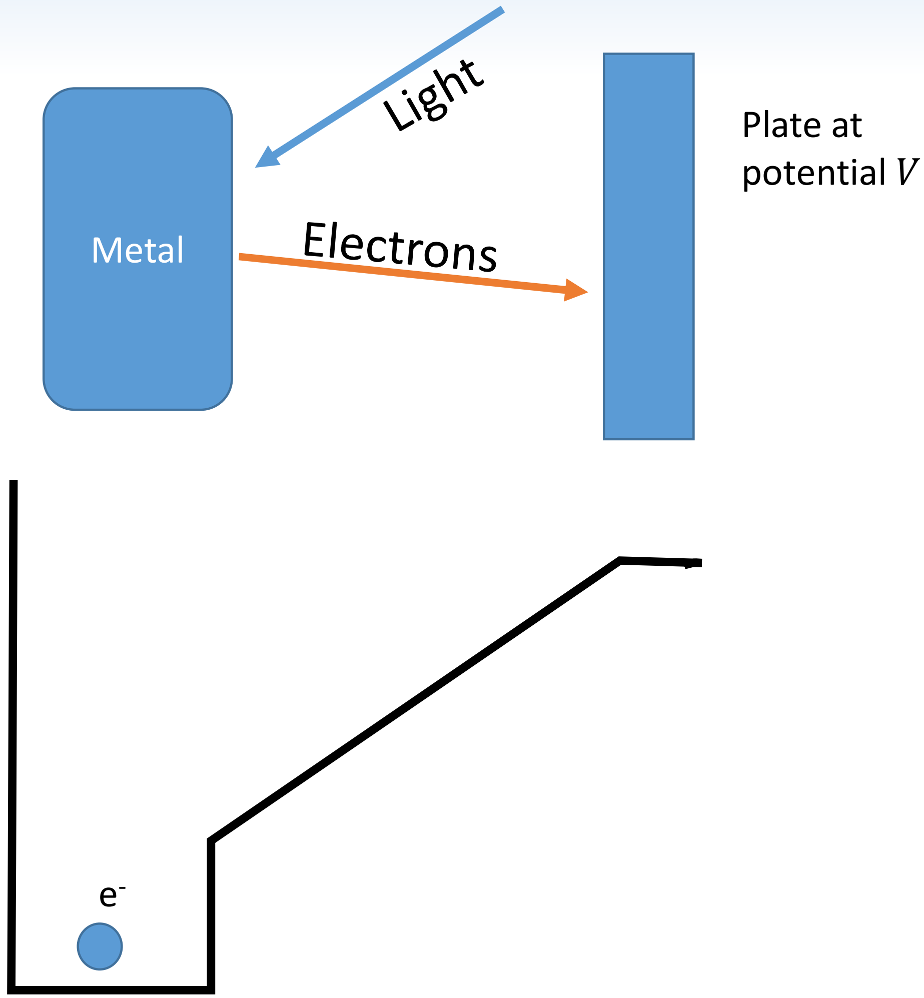
I know we did not just learn complex numbers in one unit...
We are not using *all* of complex numbers, mainly just i*i=-1 and $e^{i\theta} = \cos\theta + i\sin\theta$



# Lecture 5: Probability and Complex Numbers



# Last lecture: Photons




Light comes in quantized bits we call photons.

Energy can only be added and removed from the electromagnetic field amounts of $hf$.

$$E = hf$$

$$p = \frac{h}{\lambda}$$

$$\mathrm{K}E = \mathrm{hf} - \Phi$$

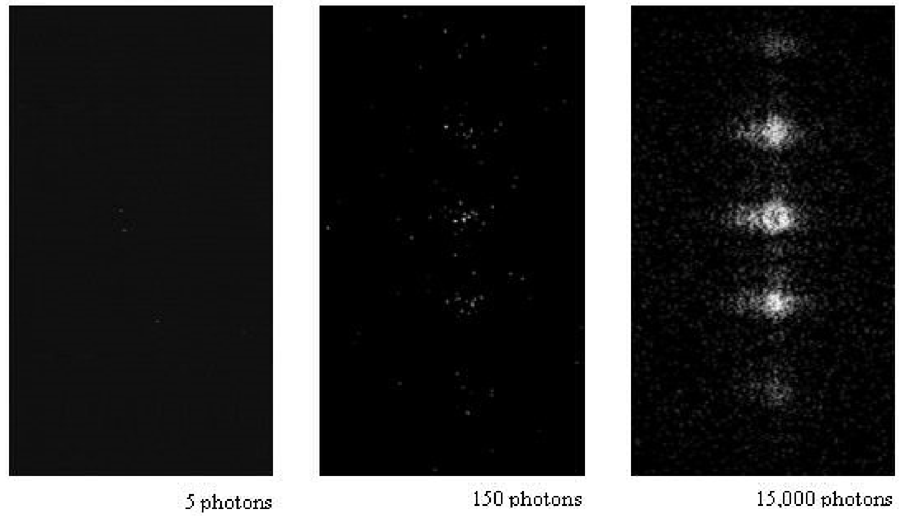
# Are photons billiard balls?

Let’s turn down the lights.

We send one photon at a time through a two-slit apparatus. Will we still observe interference fringes?

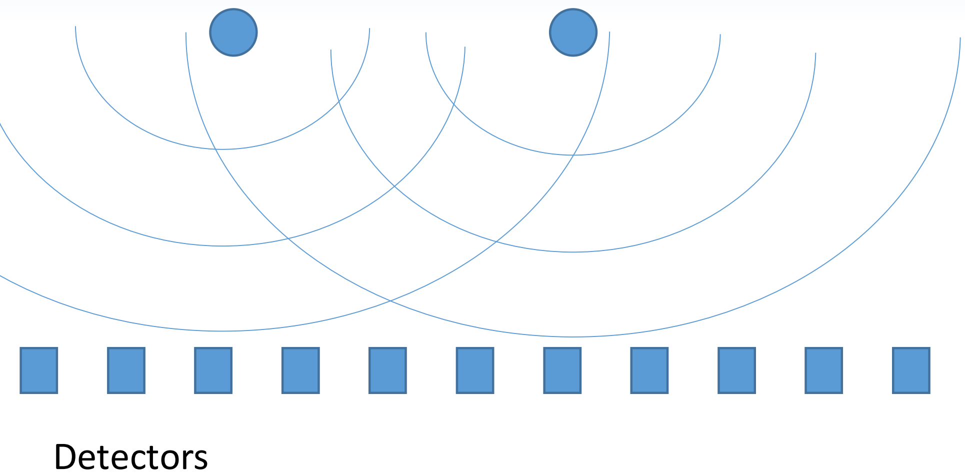
a) No, the fringes are due to photons interfering with one another.
b) No, interference is a wave property while photons are particles.
c) No, probability is positive, so it doesn’t interfere.
d) Yes, the photon is still an electromagnetic wave.
e) Yes, the photon’s probability of being observed will have interference.

# Are photons billiard balls?




*Low intensity interference experiment using a single photon counting camera. The photons first appear to arrive at random positions, but after many photons have arrived an interference pattern emerges.*

# We need to combine probability and interference




**Probability and intensity**

*Power on a detector:*
(absorbed number of photons/sec)*hf

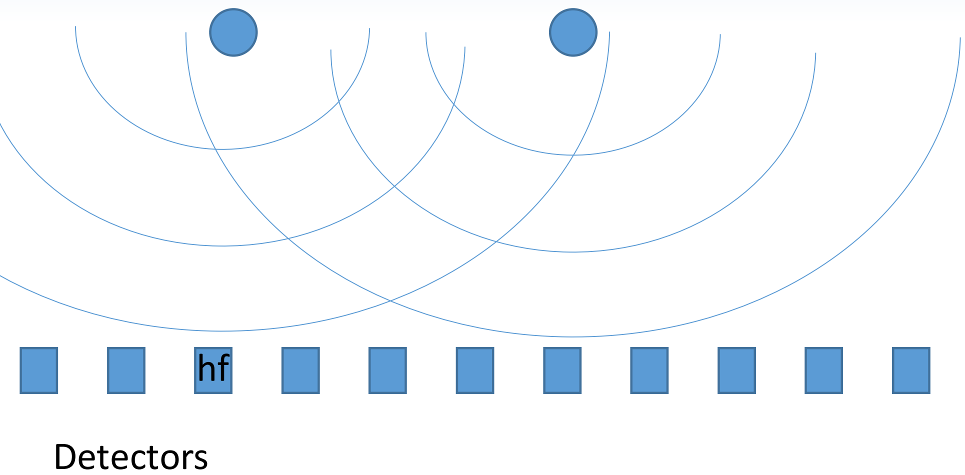
*Absorbed number of photons/sec:*
probability*(number emitted/sec)

1) Each time we detect a photon, we remove hf from the EM field.

2) The events come at random, even when the intensity is constant.

3) Even for a single photon, we see interference.

# We need to combine probability and interference

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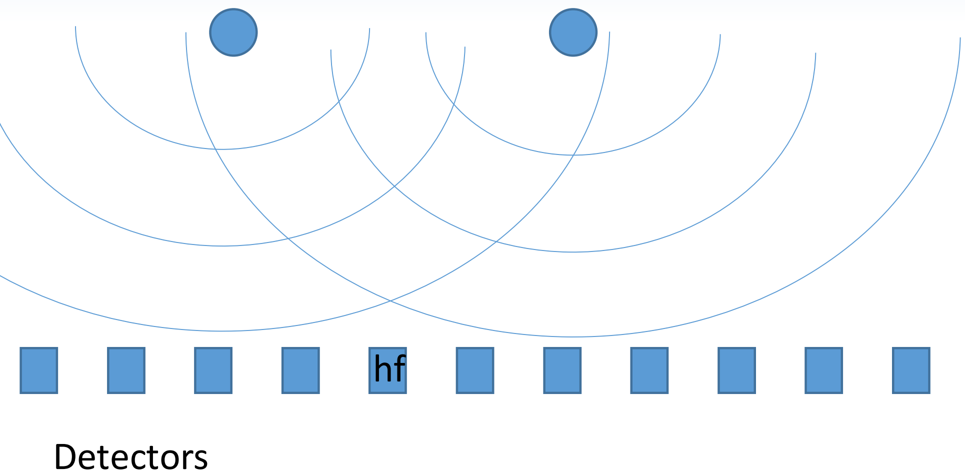


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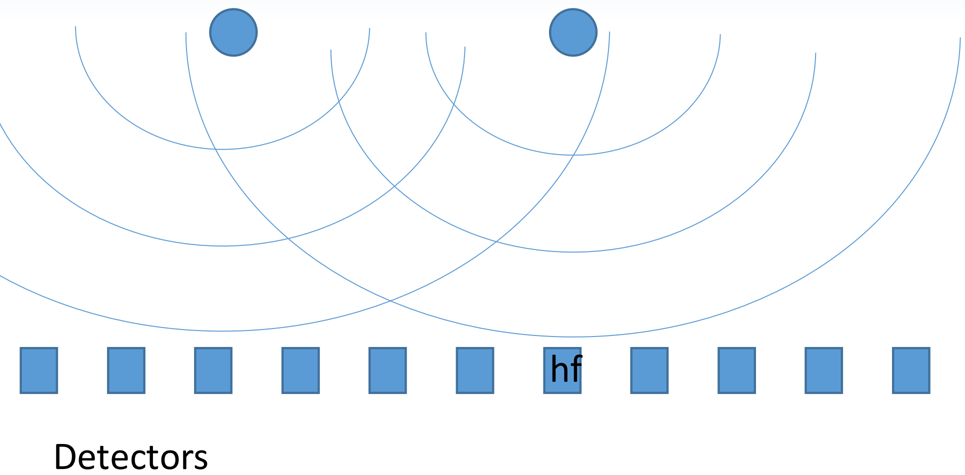


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# What we are learning today

Classical mechanics :
Position as a function of time $x(t)$

Electricity and magnetism:
vector fields $E(x,t), B(x,t)$

Quantum mechanics:
Complex probability amplitude $\psi(x,t)$
This applies to matter as well as light!

$\boxed{\text{Today: Review of probability and complex numbers}}$

# Probability, probability density

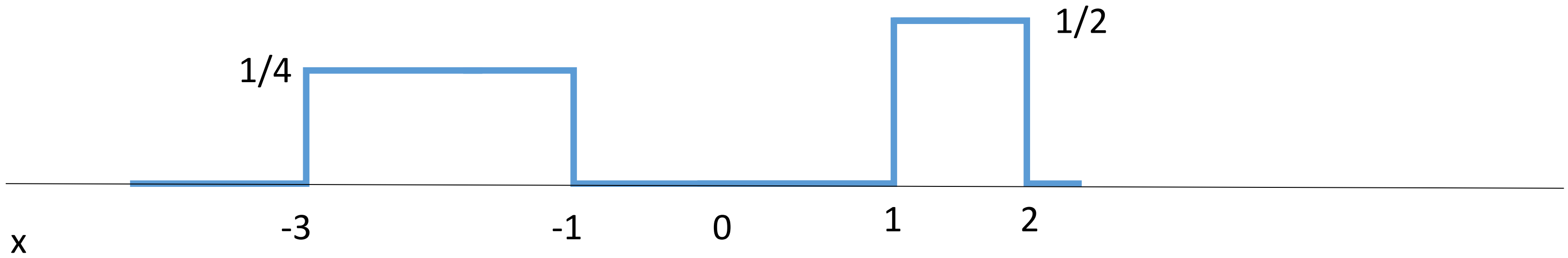
$$P(a < x < b) = \int_a^b \rho(x)dx = \int_a^b |\Psi(x)|^2 dx$$

$P(a < x < b)$: probability to find particle in the interval [a,b], unitless

$\rho(x)$: probability per length, always positive, units $m^{-1}$

$\Psi(x)$: Wave function, can be complex, units $m^{-1/2}$

# Probability density practice




Compare $P(x > 0)$ and $P(x < 0)$.

$a) P(x > 0) > P(x < 0)$
$b) P(x > 0) = P(x < 0)$
$c) P(x > 0) < P(x < 0)$

# Checkpoint

Suppose that we are told that the probability density for a photon is given by

$$\rho(x) = 1\ \mathrm{nm}^{-1}$$

between x=2 nm and x=2.5 nm, where there is a sensor that will register a 'click' if a photon is detected.


What is the probability that the sensor will click?

a) 0
b) ¼
c) ½
d) ¾
e) 1

$$P(a < x < b) = \int_a^b \rho(x)dx = \int_a^b |\Psi(x)|^2 dx$$

# Probability and intensity

Let’s suppose that N particles per second with kinetic energy E are impacting a screen with average (over time) probability density $\rho(x)$. What is the power per meter that impacts the screen?

a) $\rho(x)$
b) $N\rho(x)/E$
c) $N\rho(x)E$
d) $NE$

$$P(a < x < b) = \int_a^b \rho(x)dx = \int_a^b |\Psi(x)|^2 dx$$

# Complex numbers: $\Psi$ can be complex

$$z = x + iy$$

Euler's formula:

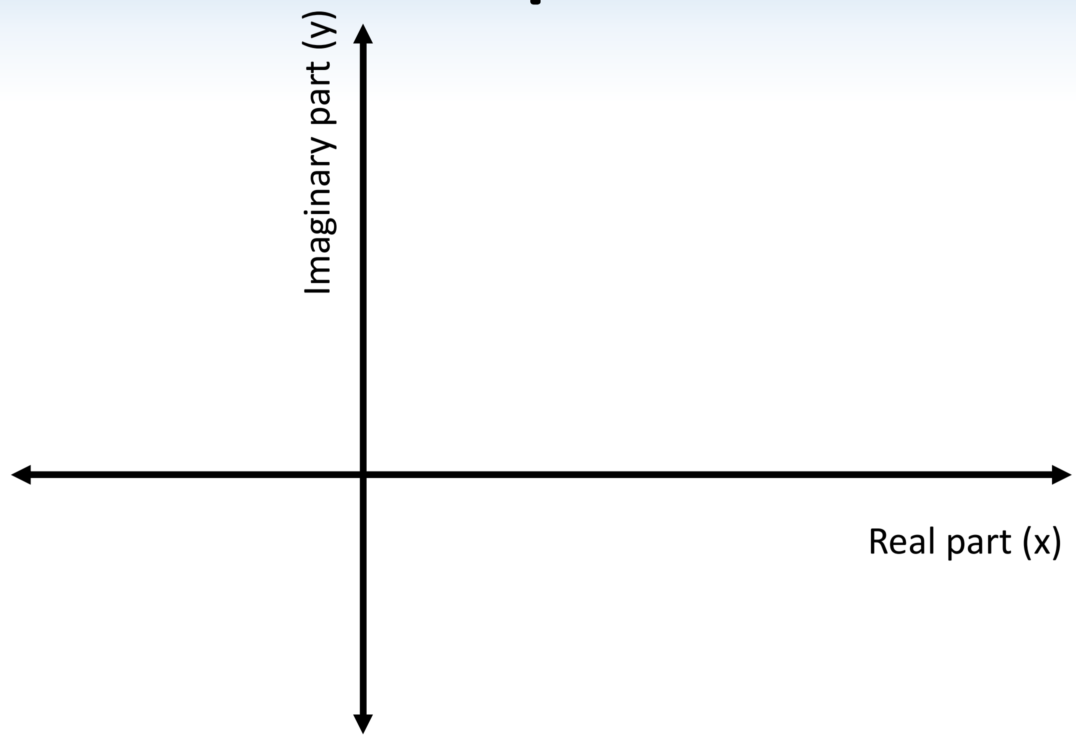
$$e^{i\phi} = \cos\phi + i\sin\phi$$

Complex conjugate

$$z^* = x - iy$$

Absolute value:

$$|z|^2 = z\, z^*$$




Complex numbers add just like phasors! The math is the same.

# Complex number practice

Suppose that $z = 3 + 4i$. What is $|z|^2$?

a) 7
b) -1
c) 25
d) 5

# Complex number practice

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Now let's try to write $z = Ae^{i\theta}$. How do we find θ?

*a)* $\tan(\theta) = 4/3$
*b)* $\tan(\theta) = 3/4$
*c)* $\theta = 0$
*d)* $\theta = \pi$

# Complex angle

Suppose that $Ae^{i\theta} = 2 + 2i$.

What is $\theta$?

$a)\ \pi$

$b)\ \frac{\pi}{2}$

$c)\ \frac{\pi}{4}$

What is A?

$a)\ 2\sqrt{2}$

b) 4

$c)\ \sqrt{2}$

d) 2

# Summary

Photons are NOT billiard balls

The probability to find the particle between $a$ and $b$ at time $t$ is

$$P(a < x < b, t) = \int_a^b \rho(x, t)\, dx$$

Intensity is given by (probability density)(energy)(number per second)

Next time: Complex numbers and Probability → Interference of Matter Waves