Unit 1: Waves

One of the main features of quantum mechanics is that particles can behave like waves, so we first start with a description classical waves, like sound. These waves are typically generated by a source, such as a speaker for sound waves, or a laser. The fundamental object is a function of space and time, which we will call $y(x,t)$ in this section*. y may represent the magnitude of the electric field for light, or the pressure in a sound wave, or the height of the water in a water wave.

After this unit, you should be able to

- Identify the basic properties of a harmonic wave.
- Compute the intensity and average intensity for harmonic waves.

Harmonic waves

To further simplify matters, we will consider harmonic waves of a single frequency. Such a wave propagating in the $+x$ direction may be written as

$$y(x,t) = A\cos(kx - \omega t + \phi), \tag{1}$$

where ω is the angular frequency and k is the wavenumber. For this class we will be considering waves either just in 1D or along a path in 3 dimensions, so you will not have to consider vector quantities. A summary is available in Table 1.

For concreteness, let's consider the classical (PHYS 212) description[1] of light waves polarized in the x direction. In this case, the wave equation describes the value of the electric field at a given position and time.

$$E_x(x,t) = E_{max}\cos(kx - \omega t + \phi) \tag{2}$$

Let's break this equation down. Since cos always returns a value between ± 1, the maximum electric field at any position or time is E_{max}. The electric field varies between $\pm E_{max}$. We will call this the **amplitude** of the wave.

Now let's consider the part inside the cos: $kx - \omega t + \phi$. Recall that if we have a function $f(x)$, then $f(x+a)$ shifts the entire function to the left by a. ϕ serves this role for Eqn 2. The easiest way to read off ϕ is to look at the value of the electric field at $t=0$ and $x=0$. Then

$$\phi = \cos^{-1}\left(E_x(0,0)/E_{max}\right). \tag{3}$$

Now let's consider $t=0$. The function is then $E_{max}\cos(kx+\phi)$. k measures the rate at which the **wave repeats in space;** the wavelength λ is equal to $2\pi/k$, since the value of the wave is the same for $x + n\lambda$.

[1] This description must be modified because light is actually *quantized*, which we will get to later in the course.

Symbol	Description	SI units	Light wave quantity
k	Wave number	m^{-1}	
λ	Wavelength	m	
ω	Angular frequency	s^{-1}	
ϕ	Phase offset	radians (unitless)	
T	Period	s	
f	Frequency	s^{-1}	
I	Intensity	W/m^2	
A	Amplitude	$\sqrt{W/m^2}$	

Table 1: Parameters that describe harmonic waves.

Now take $x = 0$; the function is then $E_{max}\cos(-\omega t + \phi) = E_{max}\cos(\omega t - \phi)$. ω is like k but for time; it's the rate at which the function oscillates in time. We call ω the angular frequency, because the function has the same value for $t + 2\pi/\omega$. We also sometimes use the frequency $f = \omega/2\pi$, and the period $T = 1/f$. The period is the equivalent of the wavelength λ for time; it's how often the function repeats itself.

We define the speed of a harmonic wave by tracking how quickly a maximum value moves through space. Eqn 2 is at a maximum where $kx_{max} - \omega t + \phi = 2\pi n$, where n is an integer. Let's consider $n = 0$ to track one maximum in particular:

$$x_{max} = \frac{\omega t - \phi}{k} \tag{4}$$

$$v = \frac{dx_{max}}{dt} = \frac{\omega}{k} \tag{5}$$

So the speed of such a wave is given by $v = \omega/k$, which may also be written as λf.

Amplitude and Intensity

For many waves of interest, we don't directly detect the wave amplitude y at a given position and time (e.g., the instantaneous pressure of a sound wave, or the electric field strength of an electromagnetic wave). Instead, a quantity of interest is the average intensity, which describes how loud the sound is or how bright the light is. For light, it tells us the power per square meter incident on a surface. The intensity of the wave at a given time and position is $I(x,t) = |y(x,t)|^2$.[2] We will average the intensity over a period T:

$$I_{\text{average}}(x) = \frac{1}{T}\int_0^T |A|^2 |\cos(kx - \omega t + \phi)|^2 dt = \frac{|A|^2}{2}, \tag{6}$$

for a harmonic wave. For a pure harmonic wave, the **average** intensity is the same at all places in space.

[2] For some variable x, $|x|^2$ is the absolute value squared. The absolute value is there because sometimes y is complex, which we will see later in the course.

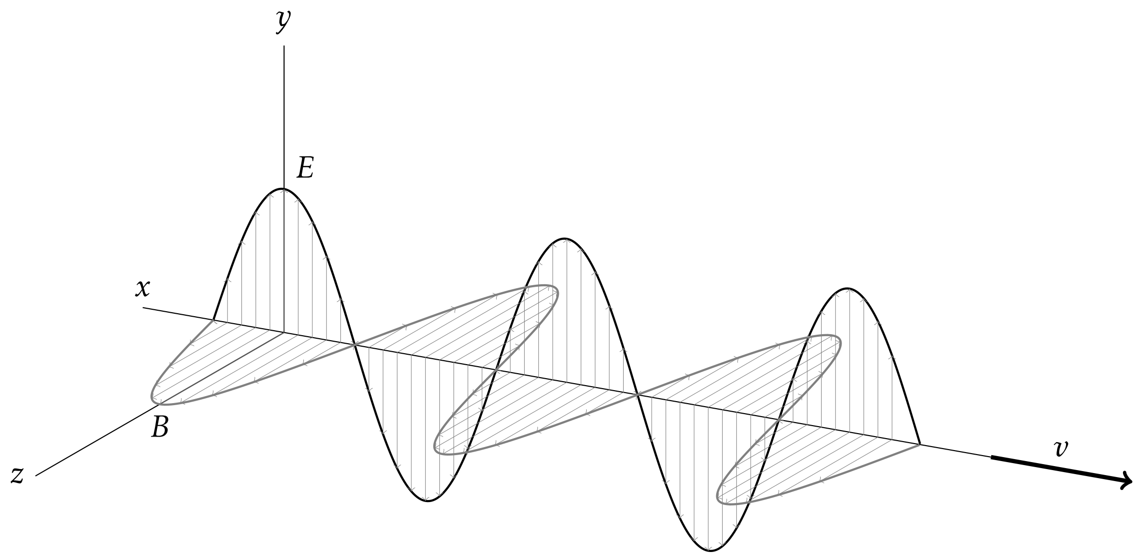

Figure 1: An electromagnetic wave. The fields oscillate in both space (wavelength λ) and time (frequency f).

In the next unit, we will consider what happens when we *superimpose* two harmonic waves. Using light waves as an example, depending on the location and time, the electric field may add or subtract, which will make the time-averaged intensity depend on position. This adding and subtracting of the field value is called interference.

Example: Light waves

In electromagnetism, light is described as a wave in the electric and magnetic field. The amplitude of the wave is proportional the maximum electric field[3], and the wavelength and frequency are related by $\lambda f = c$, where c is the speed of light. Note that the electric field is a vector, which means that it oscillates between pointing one direction and the opposite direction, as diagrammed in Fig 1.

The frequency/wavelength determine the properties of the light. For example, visible light has wavelengths between around 400 and 700 nm (1 nm is 10^{-9} m!), about 1/50 the width of a human hair. This is actually a very narrow proportion of all possible wave lengths! The classification of light waves depends on the reference you look at (it's really a continuum, so classifications are arbitrary), so be sure to check your references. We've provided a rough list in Table 2.

[3]The maximum magnetic field is required to be proportional to the maximum electric field, so we usually just track the electric field.

	Wavelength range	Frequency range	Example use
Radio	> 1m	0-300 MHz	Communication
Microwaves	1 mm - 1 m	300 MHz-300 GHz	Communication (i.e, WiFi 5.4 GHz)
Infrared	700 nm - 1 mm	300 GHz-430 THz	Thermography
Visible	400 nm - 700 nm	430 THz - 750 THz	Sight..
Ultraviolet	10 nm - 400 nm	750 THz - 30 PHz	CPU manufacturing
X-ray	0.1 nm - 10 nm	30 PHz - 30 EHz	Medical imaging
Gamma ray	< 0.1 nm	> 30 EHz	Cancer treatment

Table 2: Wavelength and frequency of various types of light.