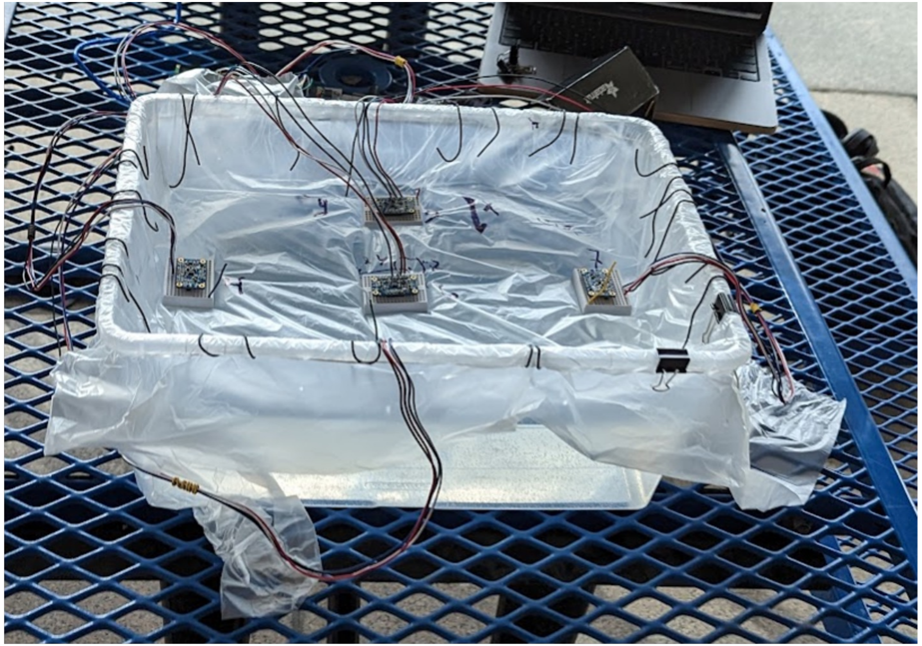
Analysis of Wave Propagation Data

Group 5 (The best one)

The Setup

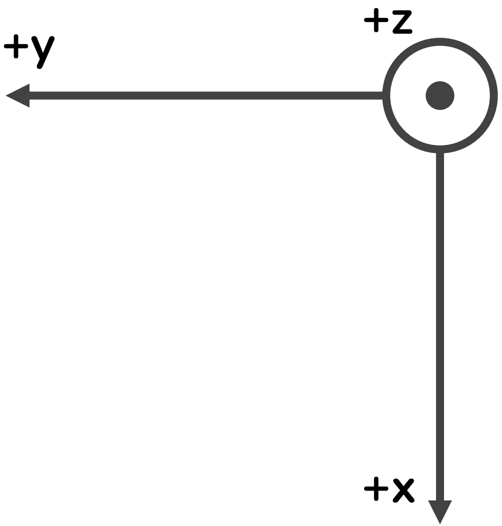


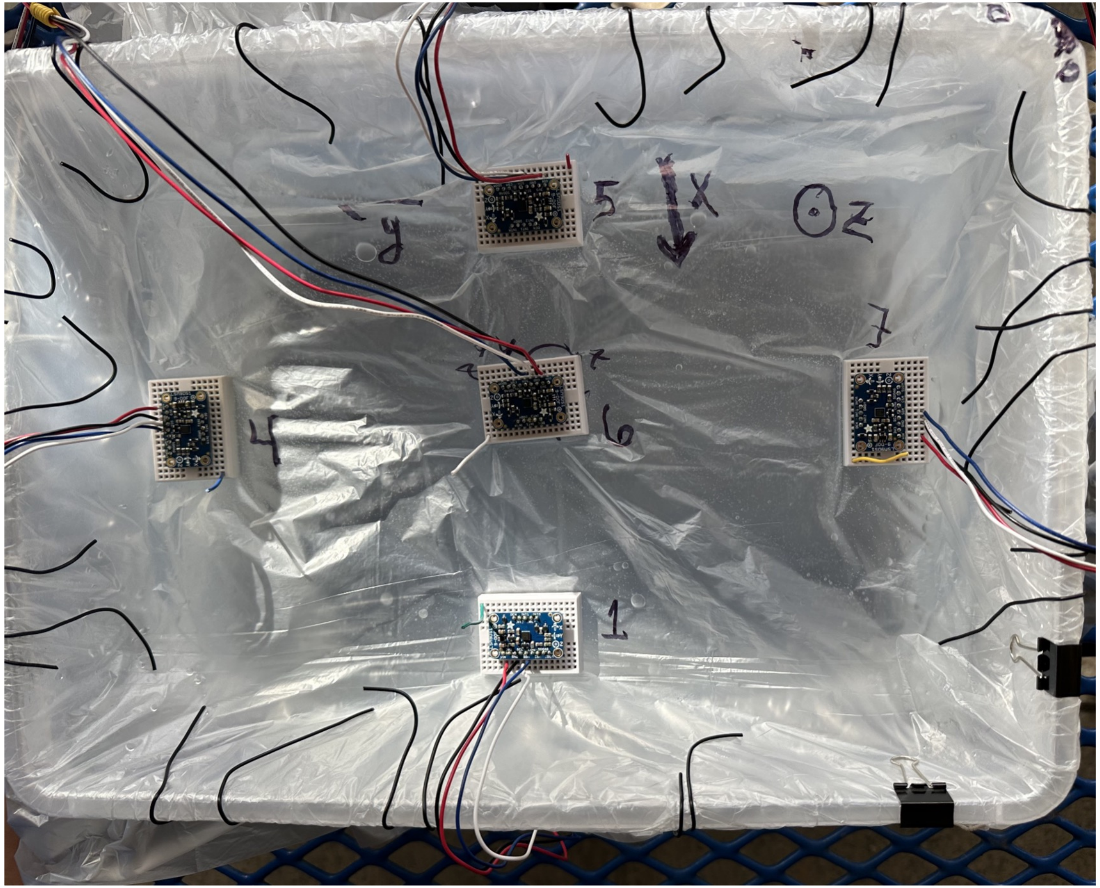
Medium Mayhem

Tested 5+ mediums, most relevant notes below

Balloon	Not flat, not easily replicable, popped, great inertia
Plastic Wrap	Low inertia, hard to keep a control state, did NOT pop
Bath Liner	Good inertia, flat surface, didn’t pop, replicable

The Setup

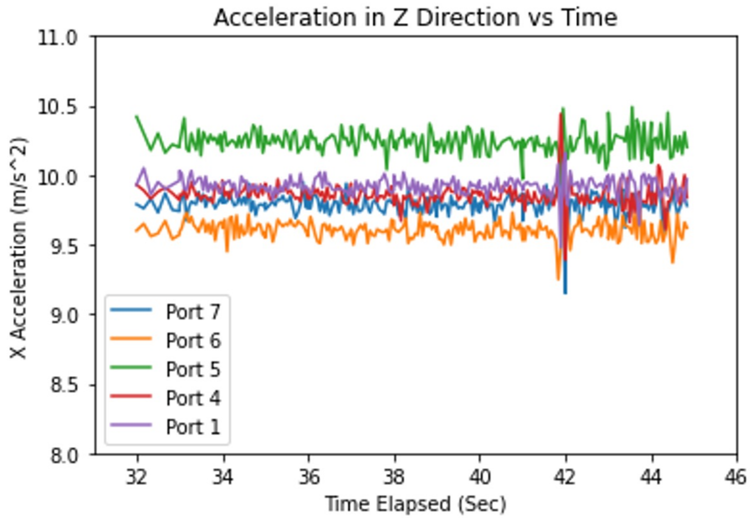


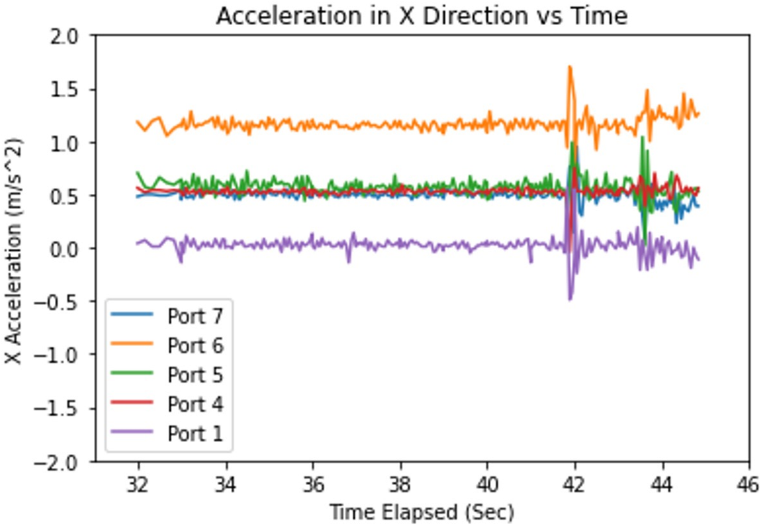


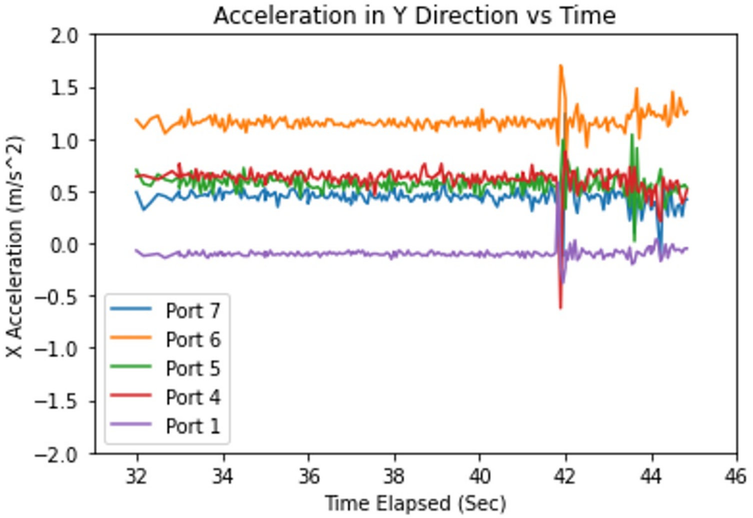
Acceleration Data Table (First 10 rows out of 250)

Accel 7 x m/s^2	Accel 7 y m/s^2	Accel 7 z m/s^2	Accel 6 x m/s^2	Accel 6 y m/s^2	Accel 6 z m/s^2	Accel 5 x m/s^2	Accel 5 y m/s^2	Accel 5 z m/s^2	Accel 4 x m/s^2	Accel 4 y m/s^2	Accel 4 z m/s^2	Accel 1 x m/s^2	Accel 1 y m/s^2	Accel 1 z m/s^2	Time s
-0.49	-0.48	9.79	1.18	-0.63	9.6	0.7	-1.08	10.42	-0.64	0.56	9.93	-0.04	0.07	9.93	32.16667
-0.32	-0.5	9.76	1.1	-0.67	9.65	0.57	-1.14	10.3	-0.65	0.52	9.89	-0.07	0.12	10.05	32.33333
-0.39	-0.5	9.83	1.19	-0.66	9.56	0.55	-1.18	10.18	-0.63	0.54	9.83	-0.01	0.11	9.86	32.5
-0.47	-0.49	9.73	1.22	-0.64	9.58	0.66	-1.13	10.3	-0.61	0.54	9.88	-0.01	0.1	9.95	32.66667
-0.45	-0.49	9.87	1.05	-0.67	9.67	0.61	-1.14	10.16	-0.65	0.53	9.91	-0.09	0.14	9.9	32.83333
-0.41	-0.51	9.75	1.12	-0.7	9.54	0.59	-1.12	10.22	-0.69	0.54	9.87	-0.07	0.11	9.93	33
-0.5	-0.54	9.74	1.16	-0.7	9.57	0.64	-1.16	10.2	-0.63	0.53	9.82	0.14	0.08	9.96	33.16667
-0.44	-0.45	9.78	1.14	-0.69	9.59	0.53	-1.15	10.22	-0.76	0.51	9.86	-0.05	0.12	10.03	33.33333
-0.44	-0.56	9.84	1.21	-0.7	9.62	0.6	-1.17	10.32	-0.63	0.49	9.85	0.05	0.11	10.01	33.5
etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.

Graphs


Acceleration in Z Direction vs Time
X Acceleration (m/s^2)
Time Elapsed (Sec)
Port 7
Port 6
Port 5
Port 4
Port 1


Acceleration in X Direction vs Time
X Acceleration (m/s^2)
Time Elapsed (Sec)
Port 7
Port 6
Port 5
Port 4
Port 1


Acceleration in Y Direction vs Time
X Acceleration (m/s^2)
Time Elapsed (Sec)
Port 7
Port 6
Port 5
Port 4
Port 1

Data Analysis Technique: Our model

$$X\ direction\ waves:\ \sin\left(\frac{n\pi x}{l_x}\right)$$

$$Y\ direction\ waves:\ \sin\left(\frac{n\pi y}{l_y}\right)$$

These give quantization for the type of waves that can occur in the x and y direction

$$Time\ dependence:\ e^{\frac{-pt}{2}}\left(C_{nm}\sin\left(w_{nm}t\right)+D_{nm}\cos\left(w_{nm}t\right)\right)$$

The time dependence gives us damping and the movement of waves through time and space

Data Analysis Technique: The issues

- The damping coefficient **p** and wave speed **v** are unknowns
- The speed of water waves can be calculated using:

$$v = \sqrt{\frac{g\lambda}{2\pi}\tanh\left(2\pi\frac{d}{\lambda}\right)}$$

λ = wavelength
d = depth
g = acceleration of gravity

From: http://hyperphysics.phy-astr.gsu.edu/hbase/watwav.html

- Will estimate the wavelength by using average wavelength of the water

Data Analysis Technique: Estimating p

- Treat our sensor data as single point perturbations
- Use one time point right after the initial perturbation as our baseline
  - set that as our $t = 0$
- These initial data points will determine our C_nm and D_nm
- Take the spatial derivative twice (either x or y) and pick one of the censors readings
  - With this equation and plug in the location of the sensor, the time difference between this point and our test point that we used to find C_nm and D_nm, and then equate this to the censors reading

Data Analysis Technique: Estimating p

```
# define function for estimating p (Example below is for Censor 1 readings for x acceleration)
def Estimation_p(p):
    result = 0
    for n in range(1, 25):
        for m in range(1, 25):
            term1 = (((-1)**n)*(n**2) * (np.pi**2)) / (lx**2)  #Comes from derivative and x = lx/2
            term2 = np.sin((m*np.pi*d) / ly)                   #Comes from y = d
            term3 = np.exp((-p*t) / 2)                         #t will be about 10ms - 100ms
            term4 = (-1)**m * np.sin(n*np.pi*d / lx)
            term5 = (-1)**m * np.sin(n*np.pi*(lx-d) / lx)
            term6 = (-1)**n * np.sin(m*np.pi*d / ly)
            term7 = (-1)**n * np.sin(m*np.pi*(ly-d) / ly)
            C = ((-4*lx)/((n**2)*(np.pi**2)*ly))*(-C_7yy*term4 - C_1xx*term5 + C_5xx*term6 + C_4yy*term7)
            term9 = (-1)**n * np.sin(m*np.pi*d / ly)
            term10 = (-1)**n * np.sin(m*np.pi*(ly-d) / ly)
            term11 = (-1)**m * np.sin(n*np.pi*d / lx)
            term12 = (-1)**m * np.sin(n*np.pi*(lx-d) / lx)
            l = (v**2)*(np.pi**2)*((n**2)/(lx**2)+(m**2)/(ly**2))
            w = (np.sqrt(np.abs(4*l - p**2)))/2
            exp = np.exp((p*np.pi)/(4*w))
            D = (exp*ly)/(lx*(np.pi**2)*(m**2))*(C_5yy*term9 - C_1yy*term10 + C_7xx*term11 -C_4xx*term12)
            result += term1 * term2 * term3 * (C * np.cos( w * t) + D * np.sin (w * t ))
    return result

# loop through values of p and print results
p = 0
while p < 100:
    result = (-1) * Estimation_p(p)   #Will make this into an array later and have code that will be able
    print(f"p={p}: {result:.6f}")     #to sort through the array and tell us which p gives the closest
    p += .001                         #value to the censors 1 reading after the time interval
```

Brute Force: Find p by running through different values and see which value of p yields the new reading.

Repeat this with all sensors for both x and y readings and average the p from there

Data Analysis Technique: Making the predictions

- Once **p** is estimated we can then start testing our model to see how accurate our predictions are
  - Plug in the location of the middle censor
  - Develop a time domain in which our model works
  - Find out how often we need to update our coefficients

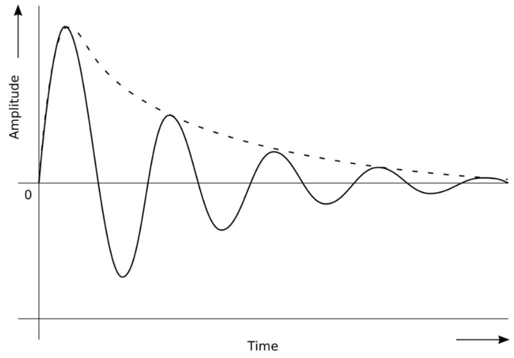
Further Testing

Test with single perturbations

- Look for decay rate of the damped oscillation
- Determine how the medium dampens the oscillation
- Insight will lead to better predictions of oscillations

Types of perturbations

- Different locations of perturbations
  - Inside vs outside of the box
  - Drive one accelerometer, wave pass through box

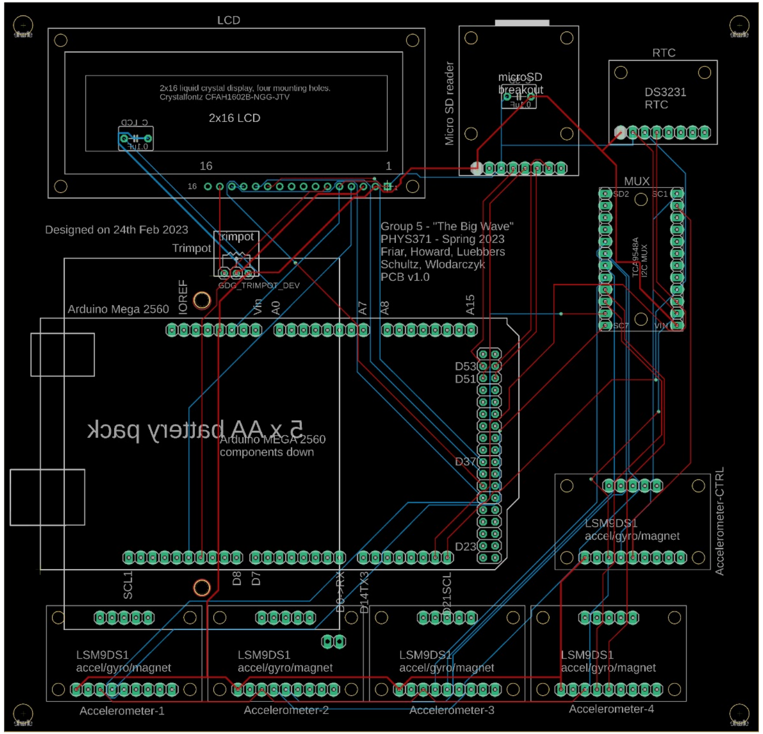


Example of Damped Oscillation (Not from our data)
Credit: Damped Simple Harmonic Motion: Definition, Expression, Example, Video (toppr.com)

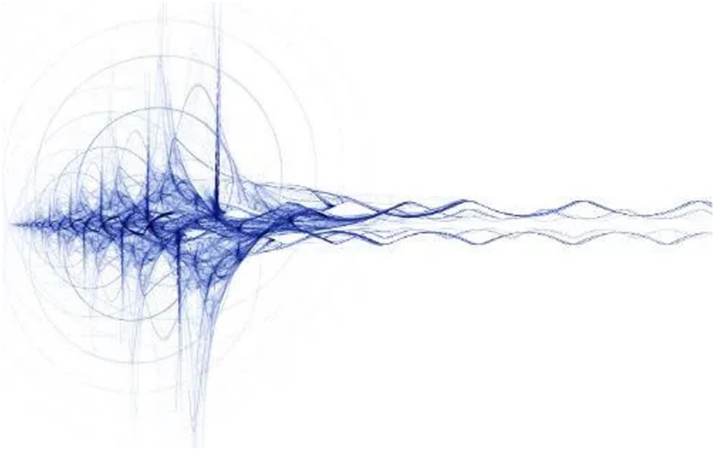
PCB

Once we have the PCB

- Start designing 3d-printed case
- Work on ribbon cable attachment
- Get final lengths for ribbon cables



Understanding Data



- Wave equation Analysis
- Values of acceleration allow for magnitude
- Magnitude over time allows for wave tracing
- Damping coefficients found by impulse -> end calculation
- External accelerometer data affects central

Experimental Validation

- Accurate Predictivity
- Time accuracy
- Testing: single impact, double impact
- Z coord agrees with gravity
- Repeatable results