Predictive Seismology

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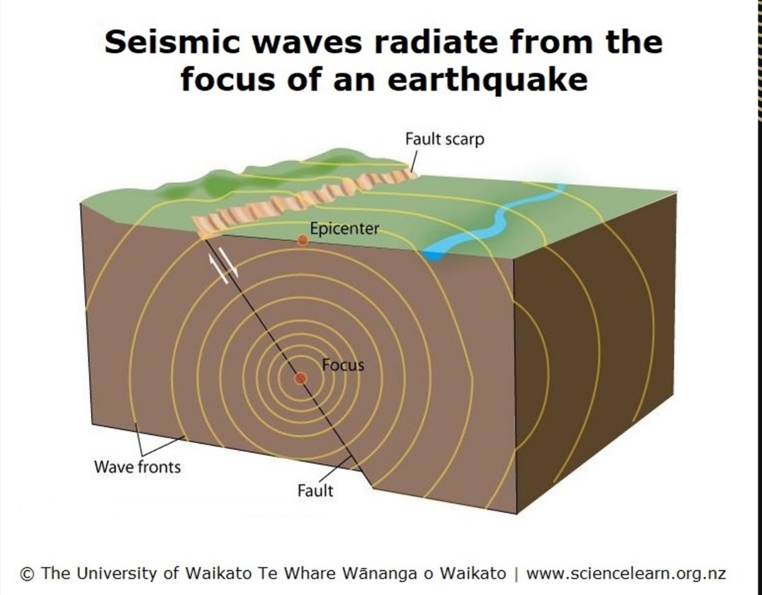
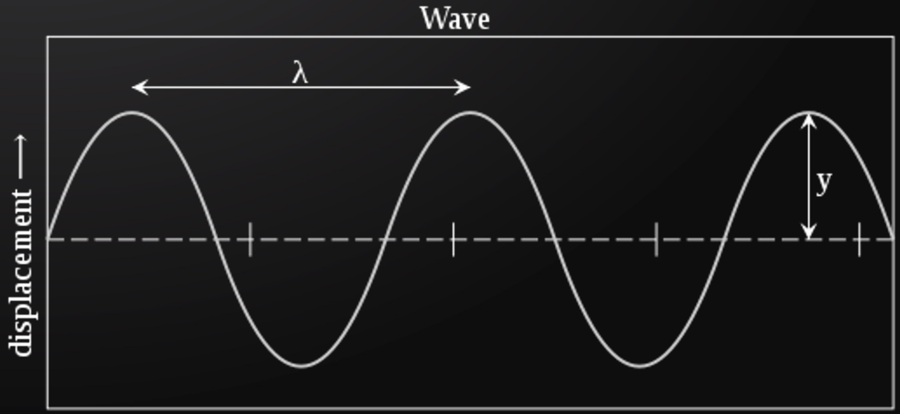
Earthquakes

- Tectonic Plates collide
- Send out waves through Earth and water



Wave oscillation

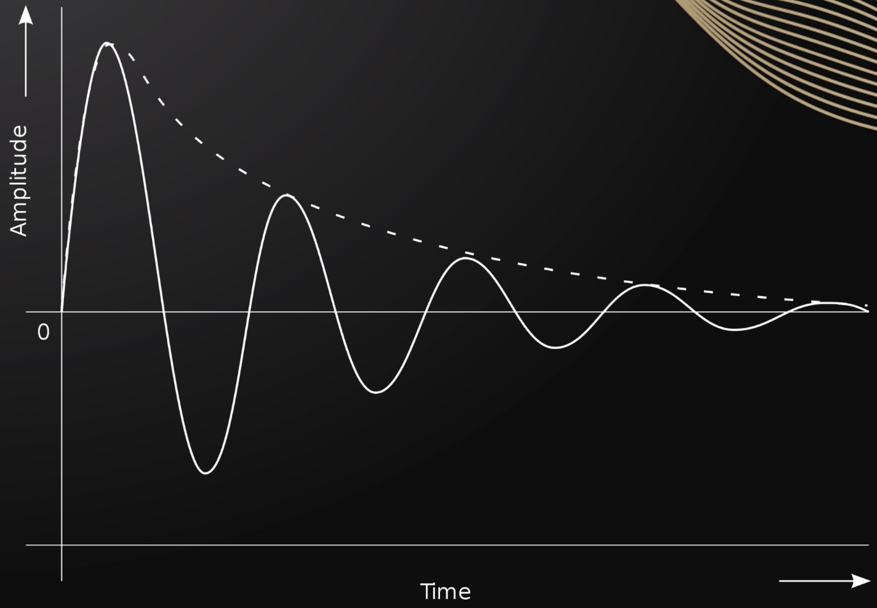
- Displacement along axes
- Propagation direction
- Single Point Origin



λ = wavelength
y = amplitude

Wave oscillation

- Following wave pattern/decay should allow prediction
- If predicted, can be counteracted



Uses

- Protecting sensitive device
- Minimizing damage to structures
- Stabilizing objects acted on by random force

Can we Predict Seismic Activity at a Point Based on Readings from Other Points?

Physics Tools

Large Punching Balloon
- Rubber maintains elasticity

Water
- High Inertia

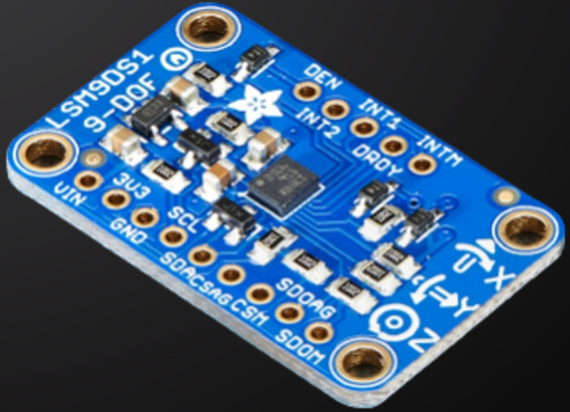
Plastic Bin
- Maintains boundary conditions
- Stabilizes Test Balloon

Balloon Popped

Modifications

- Plastic Tub
- Water OR Air filled
- Stretched rubber topper
- Accelerometers on flat rubber surface

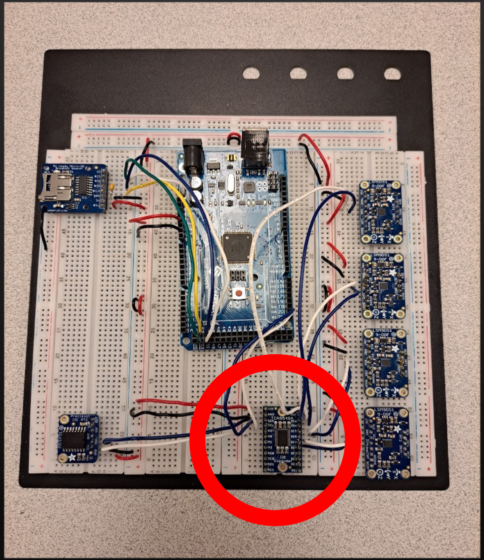
Data Taking Tools

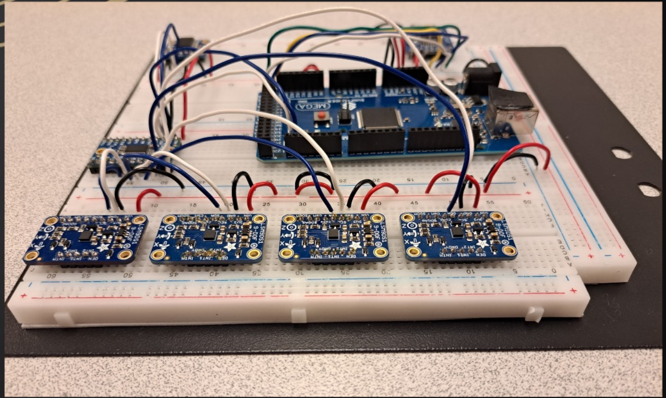


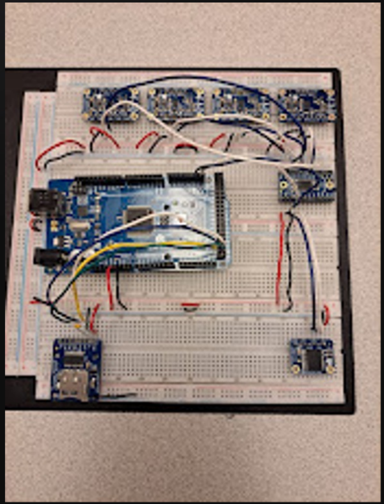
Arduino with…

- 4 Test Accelerometers
- 1 Control Accelerometer
- SD Card
- RTC - data timing
- Multiplexer - allows for unique readings from duplicate sensors

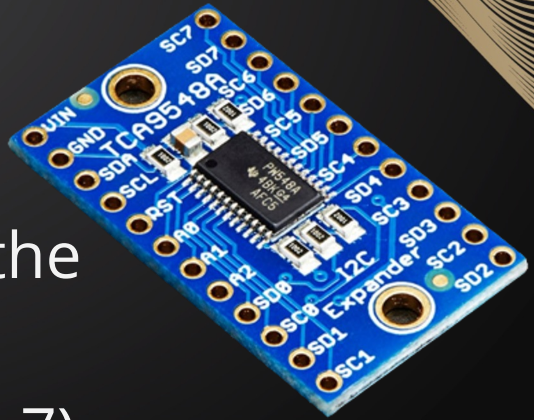
Diagram





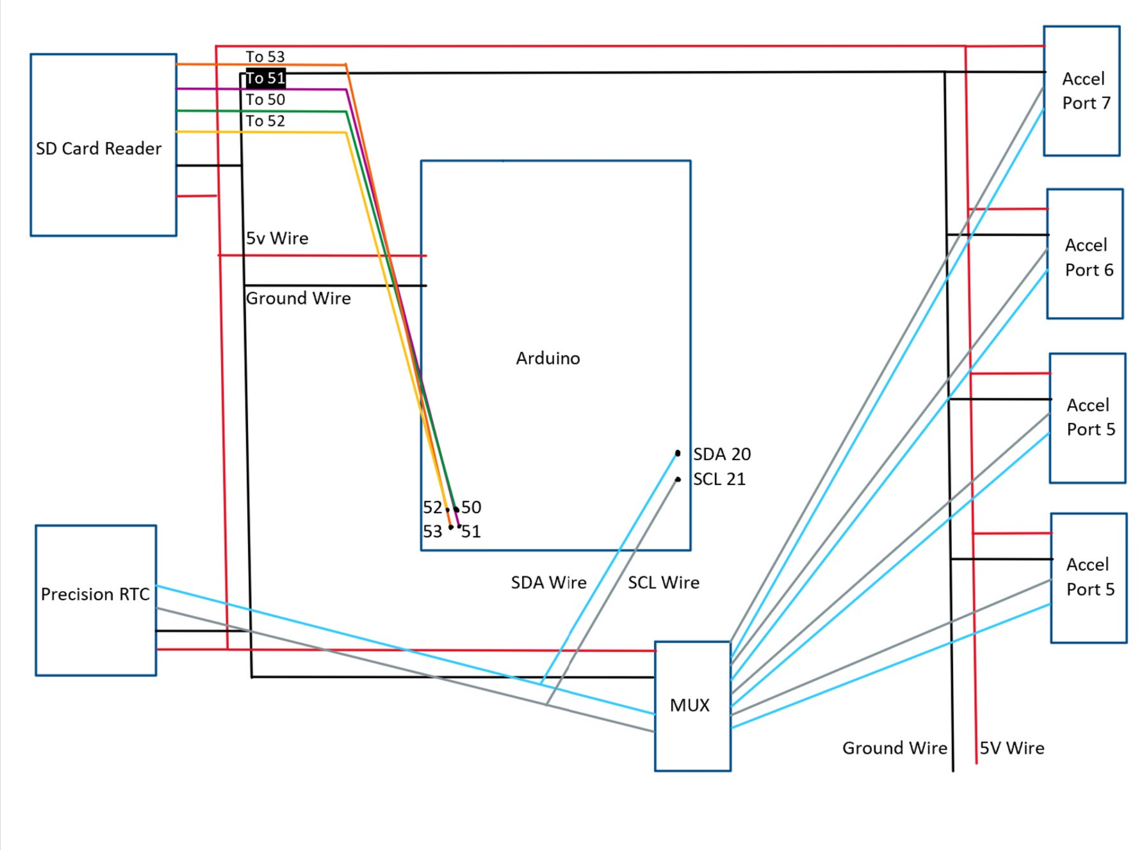


Multiplexer



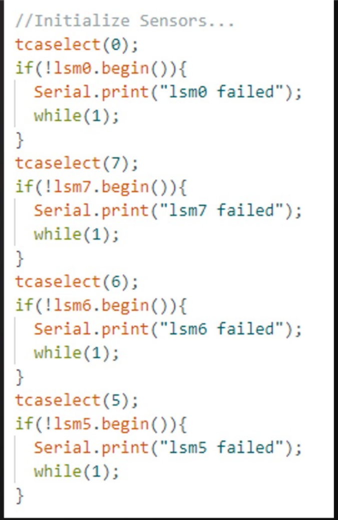
- Distinguishes between unique sensors that are of the same type
- Can support up to 8 sensors (0-7)
- Labels each sensor to differentiate data collected

Schematic

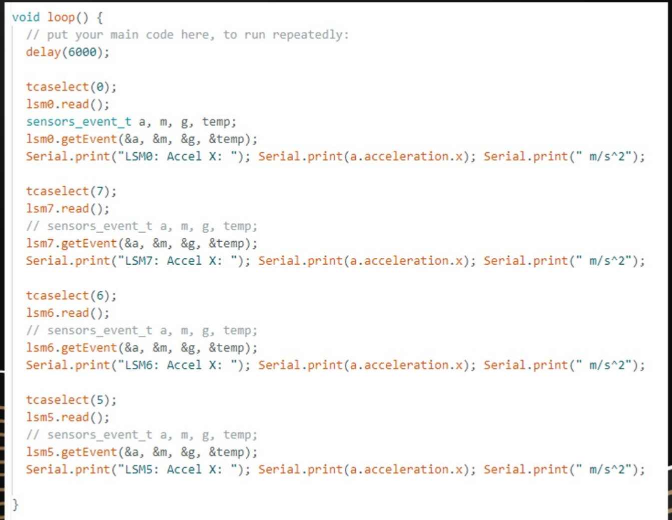

SD Card Reader
To 53
To 51
To 50
To 52
5v Wire
Ground Wire
Arduino
SDA 20
SCL 21
52
50
53
51
Precision RTC
SDA Wire
SCL Wire
MUX
Accel Port 7
Accel Port 6
Accel Port 5
Accel Port 5
Ground Wire
5V Wire

Code Example

```
#define TCAADDR 0x70

// assign names to each sensor
Adafruit_LSM9DS1 lsm0 = Adafruit_LSM9DS1(0);
Adafruit_LSM9DS1 lsm7 = Adafruit_LSM9DS1(7);
Adafruit_LSM9DS1 lsm6 = Adafruit_LSM9DS1(6);
Adafruit_LSM9DS1 lsm5 = Adafruit_LSM9DS1(5);

void setupSensor()
{
  lsm0.setupAccel(lsm0.LSM9DS1_ACCELRANGE_2G);
  lsm0.setupMag(lsm0.LSM9DS1_MAGGAIN_4GAUSS);
  lsm0.setupGyro(lsm0.LSM9DS1_GYROSCALE_245DPS);

  lsm7.setupAccel(lsm7.LSM9DS1_ACCELRANGE_2G);
  lsm7.setupMag(lsm7.LSM9DS1_MAGGAIN_4GAUSS);
  lsm7.setupGyro(lsm7.LSM9DS1_GYROSCALE_245DPS);

  lsm6.setupAccel(lsm6.LSM9DS1_ACCELRANGE_2G);
  lsm6.setupMag(lsm6.LSM9DS1_MAGGAIN_4GAUSS);
  lsm6.setupGyro(lsm6.LSM9DS1_GYROSCALE_245DPS);

  lsm5.setupAccel(lsm5.LSM9DS1_ACCELRANGE_2G);
  lsm5.setupMag(lsm5.LSM9DS1_MAGGAIN_4GAUSS);
  lsm5.setupGyro(lsm5.LSM9DS1_GYROSCALE_245DPS);
}
```

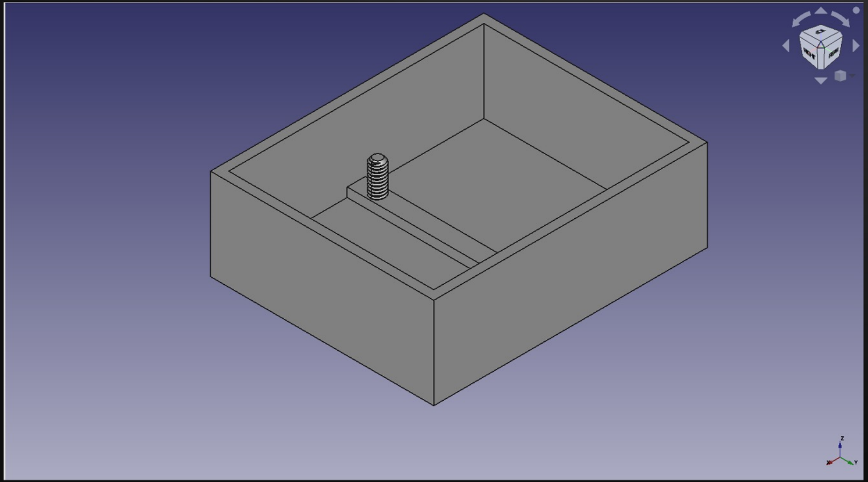


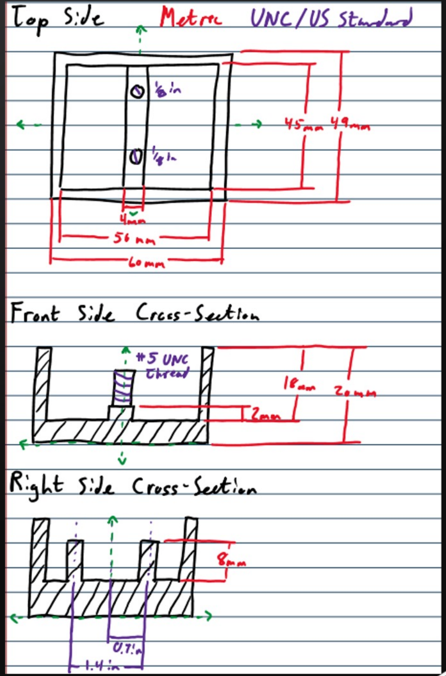
```
//Initialize Sensors...
tcaselect(0);
if(!lsm0.begin()){
  Serial.print("lsm0 failed");
  while(1);
}
tcaselect(7);
if(!lsm7.begin()){
  Serial.print("lsm7 failed");
  while(1);
}
tcaselect(6);
if(!lsm6.begin()){
  Serial.print("lsm6 failed");
  while(1);
}
tcaselect(5);
if(!lsm5.begin()){
  Serial.print("lsm5 failed");
  while(1);
}
```



```
void loop() {
  // put your main code here, to run repeatedly:
  delay(6000);

  tcaselect(0);
  lsm0.read();
  sensors_event_t a, m, g, temp;
  lsm0.getEvent(&a, &m, &g, &temp);
  Serial.print("LSM0: Accel X: "); Serial.print(a.acceleration.x); Serial.print(" m/s^2");

  tcaselect(7);
  lsm7.read();
  // sensors_event_t a, m, g, temp;
  lsm7.getEvent(&a, &m, &g, &temp);
  Serial.print("LSM7: Accel X: "); Serial.print(a.acceleration.x); Serial.print(" m/s^2");

  tcaselect(6);
  lsm6.read();
  // sensors_event_t a, m, g, temp;
  lsm6.getEvent(&a, &m, &g, &temp);
  Serial.print("LSM6: Accel X: "); Serial.print(a.acceleration.x); Serial.print(" m/s^2");

  tcaselect(5);
  lsm5.read();
  // sensors_event_t a, m, g, temp;
  lsm5.getEvent(&a, &m, &g, &temp);
  Serial.print("LSM5: Accel X: "); Serial.print(a.acceleration.x); Serial.print(" m/s^2");

}
```

Accelerometer Case





Calibrating the Accelerometer Data

Why Calibrate?

- Allows us to test the accuracy and precision of our accelerometers
- Allows us to test how feasible our current setup is
  - Will the accelerometers work our medium?

How will we Calibrate?

- 4 methods
  - Each Axis
  - Static
  - Constant Motion
  - On the Medium

Calibration Tests

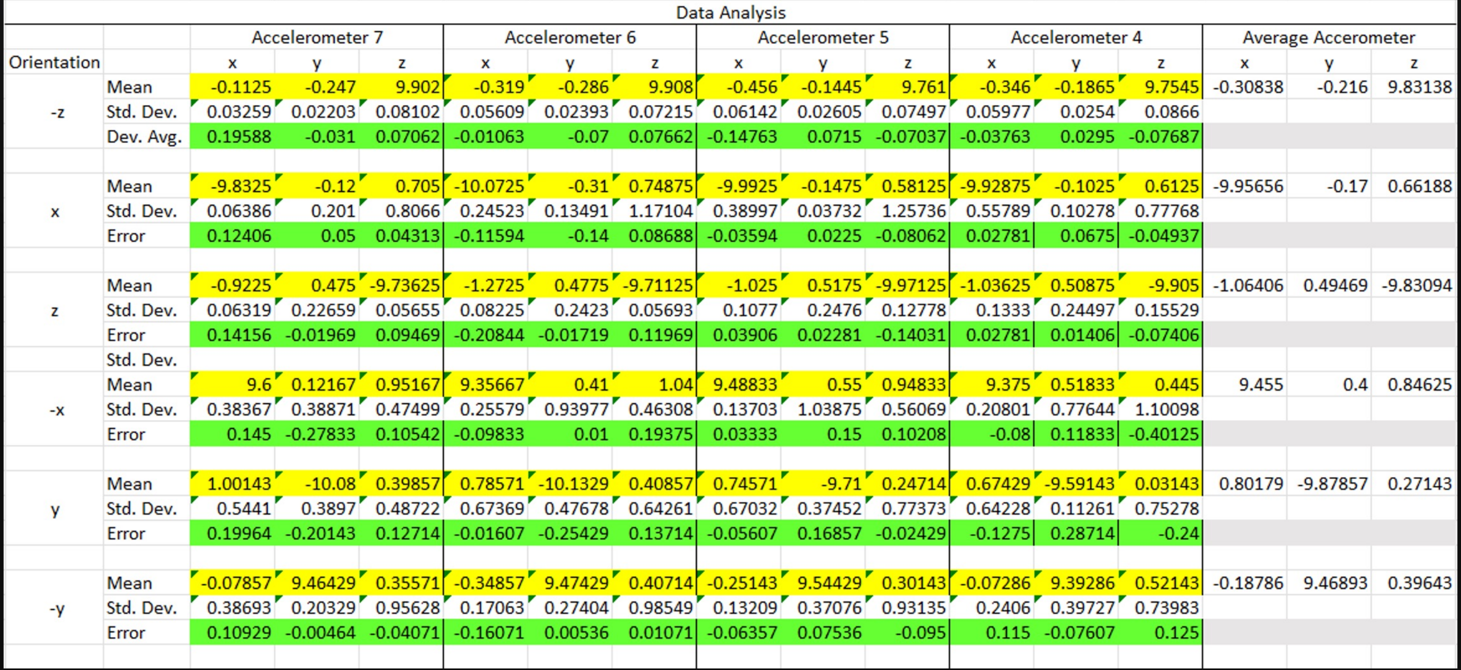
2 Methods have been completed

- Along Each Axis & Static
    - Could not do other two because of the popping of the balloon

Types of Data Collected & Analyzed

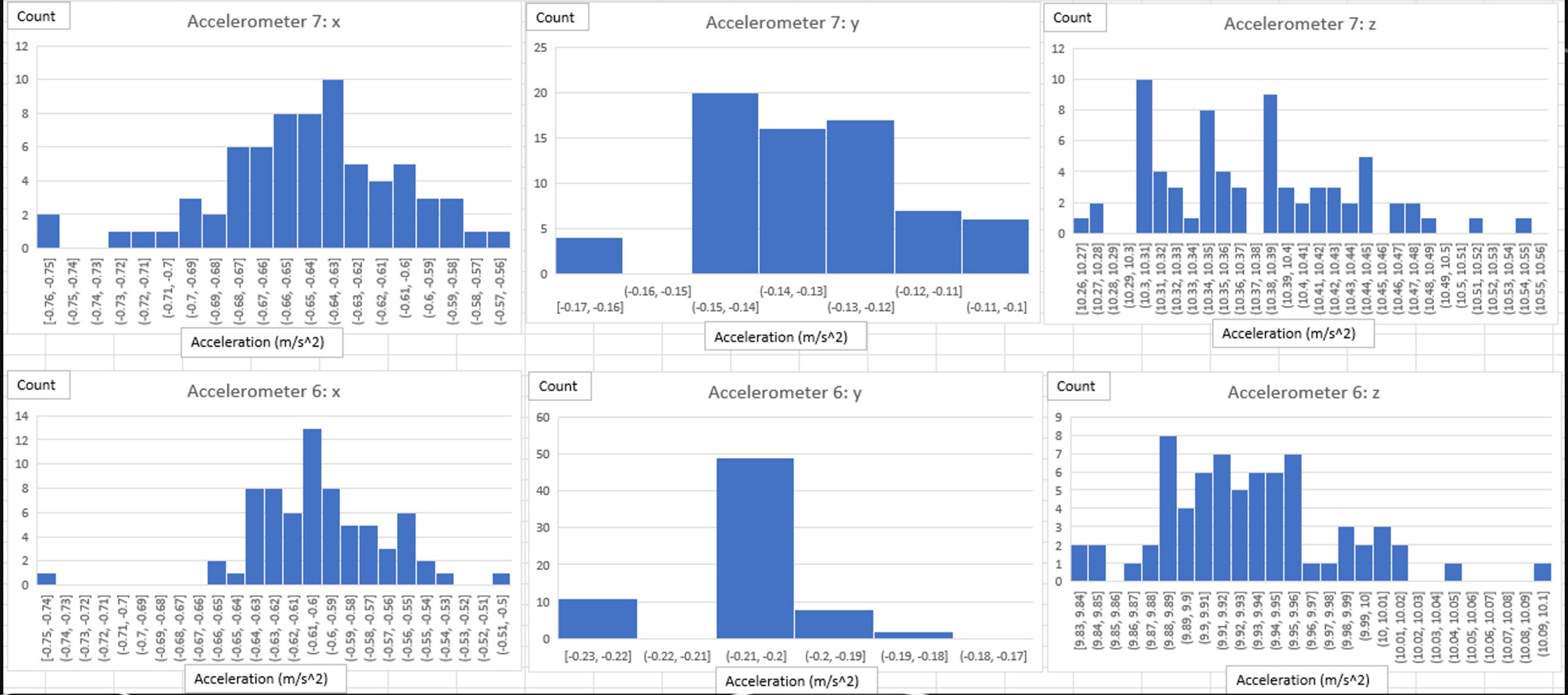
- Rotated the board about each axis & direction
- Let the board run without disturbing it
- From this, we found:
    - Mean readings of each axis
    - Standard deviations of each axis
    - Difference from total mean of each axis

Axial Calibration Analysis

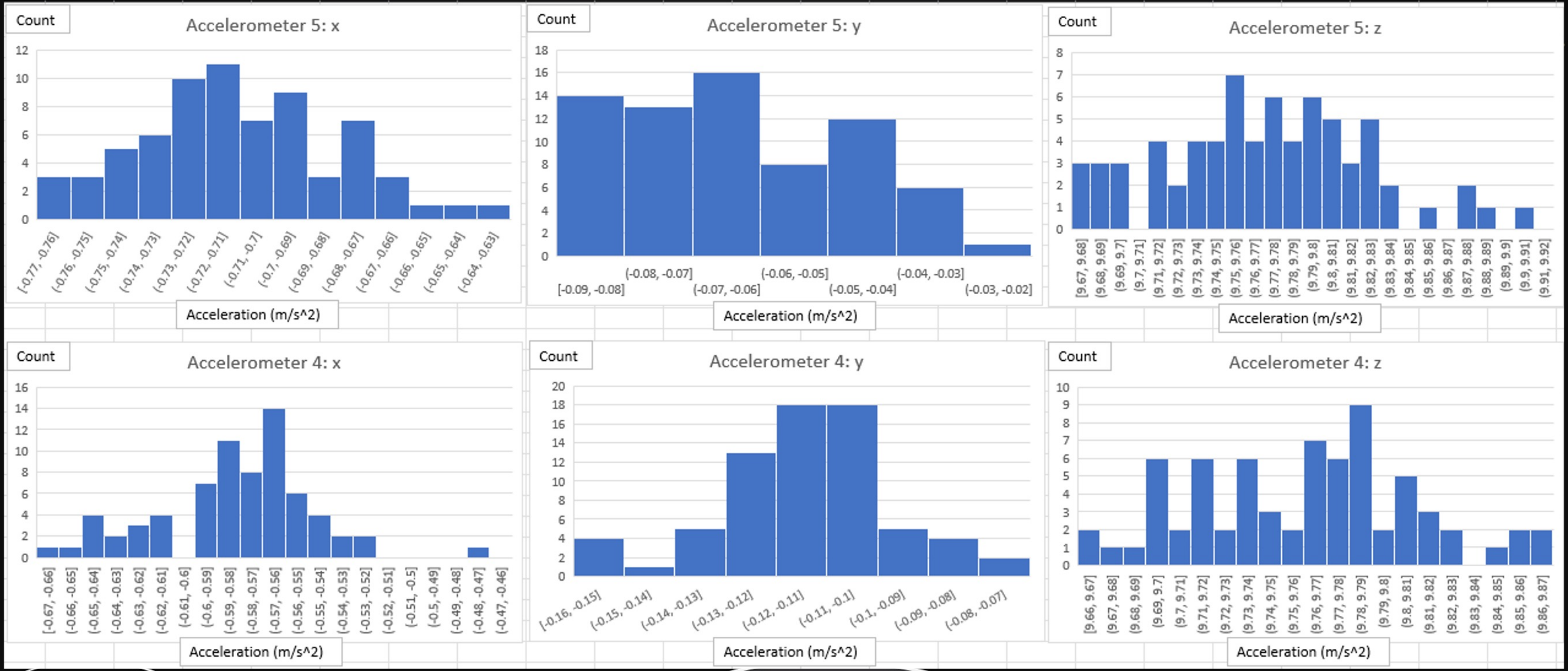


Data Analysis																
		Accelerometer 7			Accelerometer 6			Accelerometer 5			Accelerometer 4			Average Accerometer		
Orientation		x	y	z	x	y	z	x	y	z	x	y	z	x	y	z
-z	Mean	-0.1125	-0.247	9.902	-0.319	-0.286	9.908	-0.456	-0.1445	9.761	-0.346	-0.1865	9.7545	-0.30838	-0.216	9.83138
	Std. Dev.	0.03259	0.02203	0.08102	0.05609	0.02393	0.07215	0.06142	0.02605	0.07497	0.05977	0.0254	0.0866			
	Dev. Avg.	0.19588	-0.031	0.07062	-0.01063	-0.07	0.07662	-0.14763	0.0715	-0.07037	-0.03763	0.0295	-0.07687			
x	Mean	-9.8325	-0.12	0.705	-10.0725	-0.31	0.74875	-9.9925	-0.1475	0.58125	-9.92875	-0.1025	0.6125	-9.95656	-0.17	0.66188
	Std. Dev.	0.06386	0.201	0.8066	0.24523	0.13491	1.17104	0.38997	0.03732	1.25736	0.55789	0.10278	0.77768			
	Error	0.12406	0.05	0.04313	-0.11594	-0.14	0.08688	-0.03594	0.0225	-0.08062	0.02781	0.0675	-0.04937			
z	Mean	-0.9225	0.475	-9.73625	-1.2725	0.4775	-9.71125	-1.025	0.5175	-9.97125	-1.03625	0.50875	-9.905	-1.06406	0.49469	-9.83094
	Std. Dev.	0.06319	0.22659	0.05655	0.08225	0.2423	0.05693	0.1077	0.2476	0.12778	0.1333	0.24497	0.15529			
	Error	0.14156	-0.01969	0.09469	-0.20844	-0.01719	0.11969	0.03906	0.02281	-0.14031	0.02781	0.01406	-0.07406			
	Std. Dev.															
-x	Mean	9.6	0.12167	0.95167	9.35667	0.41	1.04	9.48833	0.55	0.94833	9.375	0.51833	0.445	9.455	0.4	0.84625
	Std. Dev.	0.38367	0.38871	0.47499	0.25579	0.93977	0.46308	0.13703	1.03875	0.56069	0.20801	0.77644	1.10098			
	Error	0.145	-0.27833	0.10542	-0.09833	0.01	0.19375	0.03333	0.15	0.10208	-0.08	0.11833	-0.40125			
y	Mean	1.00143	-10.08	0.39857	0.78571	-10.1329	0.40857	0.74571	-9.71	0.24714	0.67429	-9.59143	0.03143	0.80179	-9.87857	0.27143
	Std. Dev.	0.5441	0.3897	0.48722	0.67369	0.47678	0.64261	0.67032	0.37452	0.77373	0.64228	0.11261	0.75278			
	Error	0.19964	-0.20143	0.12714	-0.01607	-0.25429	0.13714	-0.05607	0.16857	-0.02429	-0.1275	0.28714	-0.24			
-y	Mean	-0.07857	9.46429	0.35571	-0.34857	9.47429	0.40714	-0.25143	9.54429	0.30143	-0.07286	9.39286	0.52143	-0.18786	9.46893	0.39643
	Std. Dev.	0.38693	0.20329	0.95628	0.17063	0.27404	0.98549	0.13209	0.37076	0.93135	0.2406	0.39727	0.73983			
	Error	0.10929	-0.00464	-0.04071	-0.16071	0.00536	0.01071	-0.06357	0.07536	-0.095	0.115	-0.07607	0.125			

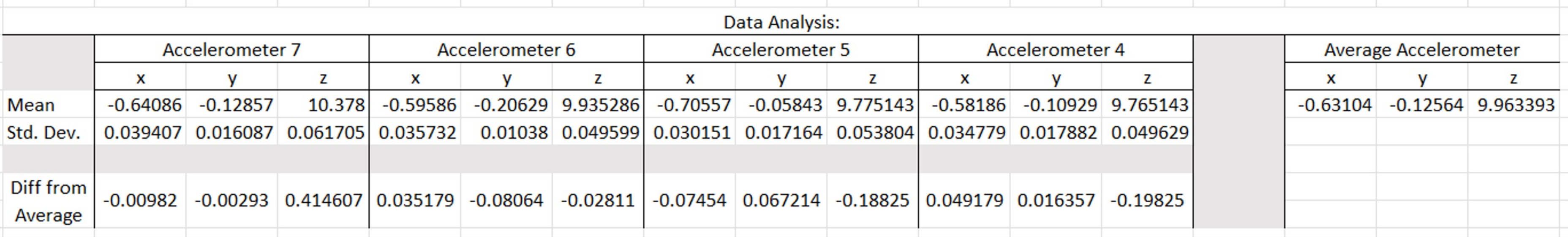
Static Calibration Data - 1



Static Calibration Data - 2



Static Calibration Analysis



Data Analysis:

	Accelerometer 7			Accelerometer 6			Accelerometer 5			Accelerometer 4				Average Accelerometer		
	x	y	z	x	y	z	x	y	z	x	y	z		x	y	z
Mean	-0.64086	-0.12857	10.378	-0.59586	-0.20629	9.935286	-0.70557	-0.05843	9.775143	-0.58186	-0.10929	9.765143		-0.63104	-0.12564	9.963393
Std. Dev.	0.039407	0.016087	0.061705	0.035732	0.01038	0.049599	0.030151	0.017164	0.053804	0.034779	0.017882	0.049629				
Diff from Average	-0.00982	-0.00293	0.414607	0.035179	-0.08064	-0.02811	-0.07454	0.067214	-0.18825	0.049179	0.016357	-0.19825				

Calibration Error & Conclusion

Possible Sources of Error

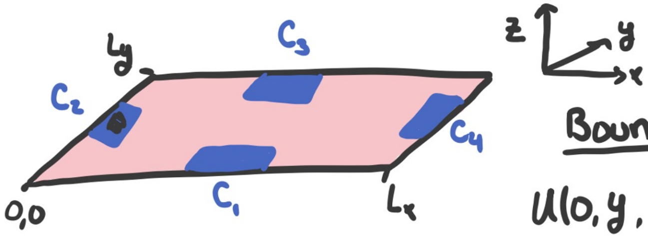
- Holding the breadboard
- Subtle vibrations around accelerometers
- Inconsistent calibration during manufacturing

Conclusions

- Replace Accelerometer 5
- Find more accelerometers and test them
- Conduct the other two calibration tests

Readings → Results
The Math Section

Developing a Mathematical Model



Boundary conditions

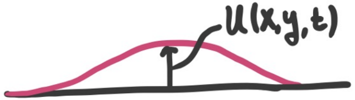
$U(0, y, t) = U(L_x, y, t) = 0$

$U(x, 0, t) = U(x, L_y, t) = 0$

$\partial_{xx} U(x, \frac{L_y}{2}, t) = C_{1,xx}\,\delta(x-\Delta)$

$\partial_{yy} U(x, \frac{L_y}{2}, t) = C_{1,yy}\,\delta(y-\frac{L_y}{2})$

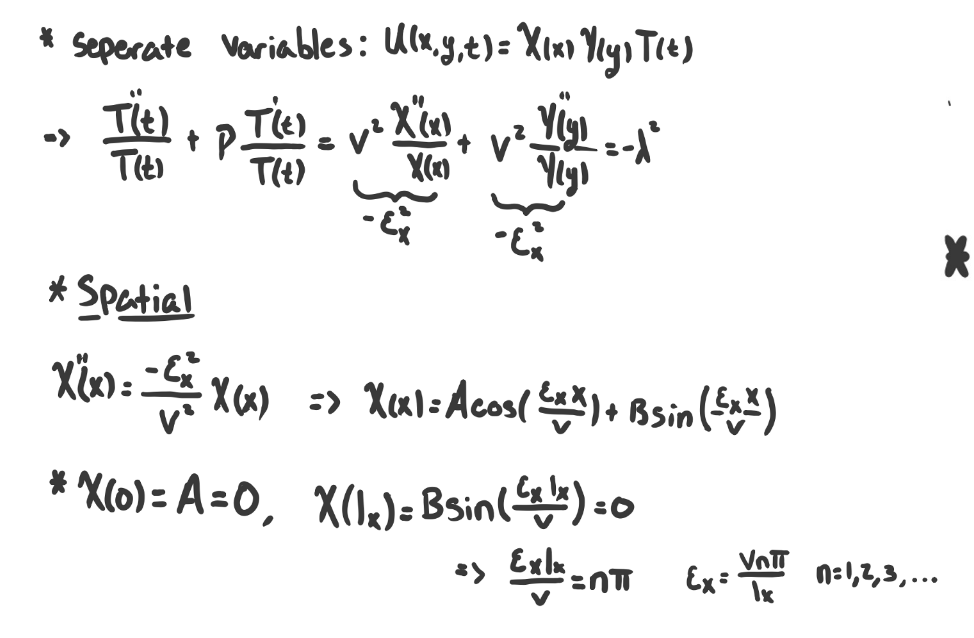
$\partial_{tt} U(x, \frac{L_y}{2}, t) = C_{1,zz}$ ↓ Similar for other Sensors



Wave eqn. w/ dampening.

$$\Rightarrow \partial_{tt} U + p\,\partial_t U = v^2(\partial_{xx} + \partial_{yy})U$$

Solving for an equation: Separation of Variables



* Seperate Variables: $U(x,y,t) = X(x)\,Y(y)\,T(t)$

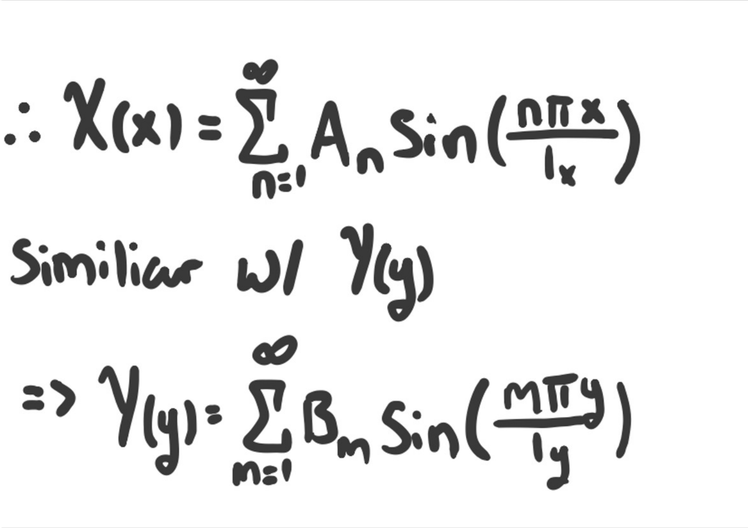
$$\Rightarrow \frac{\ddot{T}(t)}{T(t)} + p\frac{\dot{T}(t)}{T(t)} = \underbrace{v^2\frac{X''(x)}{X(x)}}_{-\varepsilon_x^2} + \underbrace{v^2\frac{Y''(y)}{Y(y)}}_{-\varepsilon_x^2} = -\lambda^2$$

* Spatial

$$X''(x) = \frac{-\varepsilon_x^2}{v^2}X(x) \Rightarrow X(x) = A\cos\left(\frac{\varepsilon_x x}{v}\right) + B\sin\left(\frac{\varepsilon_x x}{v}\right)$$

* $X(0) = A = 0,\quad X(l_x) = B\sin\left(\frac{\varepsilon_x l_x}{v}\right) = 0$

$$\Rightarrow \frac{\varepsilon_x l_x}{v} = n\pi \qquad \varepsilon_x = \frac{vn\pi}{l_x} \qquad n = 1, 2, 3, \ldots$$

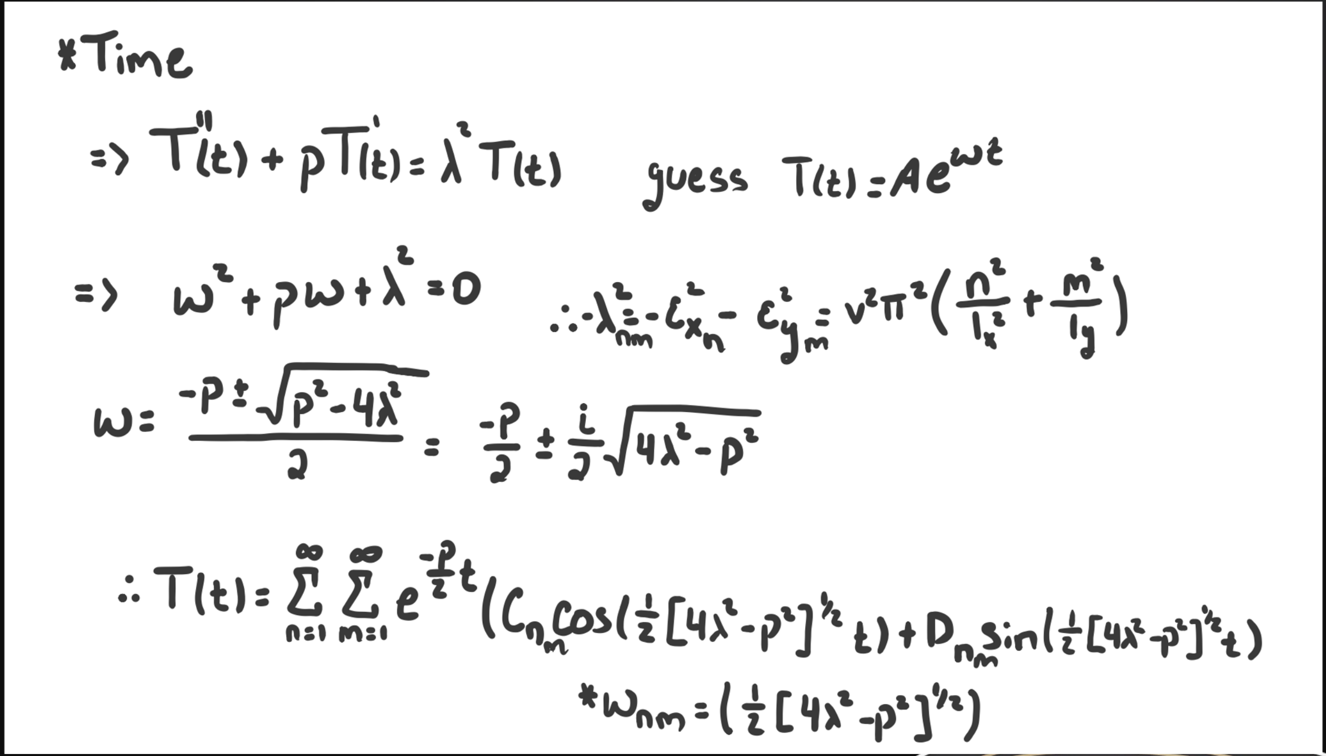


$$\therefore X(x) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi x}{l_x}\right)$$

* Similiar w/ $Y(y)$

$$\Rightarrow Y(y) = \sum_{m=1}^{\infty} B_m \sin\left(\frac{m\pi y}{l_y}\right)$$

Sperараtion cont.



*Time

$$\Rightarrow T''(t) + pT'(t) = \lambda^2 T(t) \qquad \text{guess } T(t) = Ae^{\omega t}$$

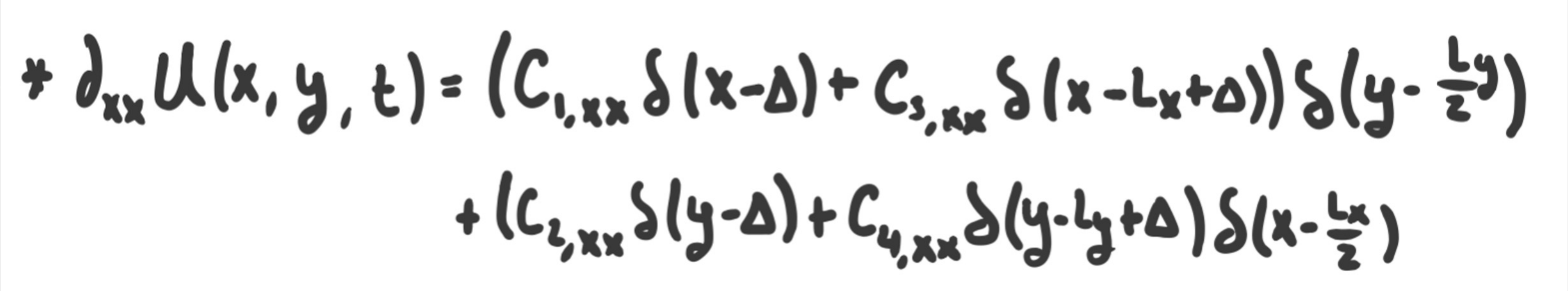
$$\Rightarrow \omega^2 + p\omega + \lambda^2 = 0 \qquad \therefore -\lambda_{nm}^2 = -\varepsilon_{x_n}^2 - \varepsilon_{y_m}^2 = v^2\pi^2\left(\frac{n^2}{l_x^2} + \frac{m^2}{l_y}\right)$$

$$\omega = \frac{-p \pm \sqrt{p^2 - 4\lambda^2}}{2} = \frac{-p}{2} \pm \frac{i}{2}\sqrt{4\lambda^2 - p^2}$$

$$\therefore T(t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} e^{-\frac{p}{2}t}\left(C_{nm}\cos\left(\tfrac{1}{2}[4\lambda^2 - p^2]^{1/2} t\right) + D_{nm}\sin\left(\tfrac{1}{2}[4\lambda^2 - p^2]^{1/2} t\right)\right.$$

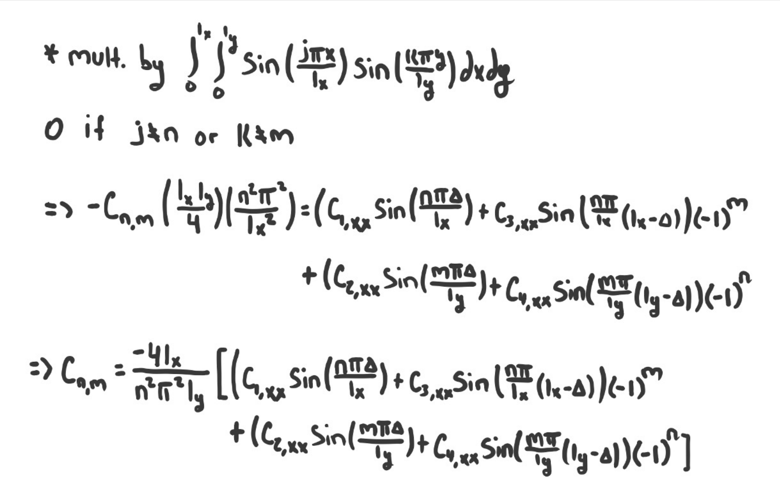
$$*\omega_{nm} = \left(\tfrac{1}{2}[4\lambda^2 - p^2]^{1/2}\right)$$

Boundary Conditions



$$* \; \partial_{xx} U(x, y, t) = (C_{1,xx}\,\delta(x-\Delta) + C_{3,xx}\,\delta(x - L_x + \Delta))\,\delta(y - \frac{L_y}{2})$$

$$+ (C_{2,xx}\,\delta(y-\Delta) + C_{4,xx}\,\delta(y - L_y + \Delta)\,\delta(x - \frac{L_x}{2})$$



$*$ mult. by $\int_0^{l_x}\int_0^{l_y} \sin(\frac{j\pi x}{l_x}) \sin(\frac{k\pi y}{l_y})\, dx\, dy$

0 if $j \neq n$ or $k \neq m$

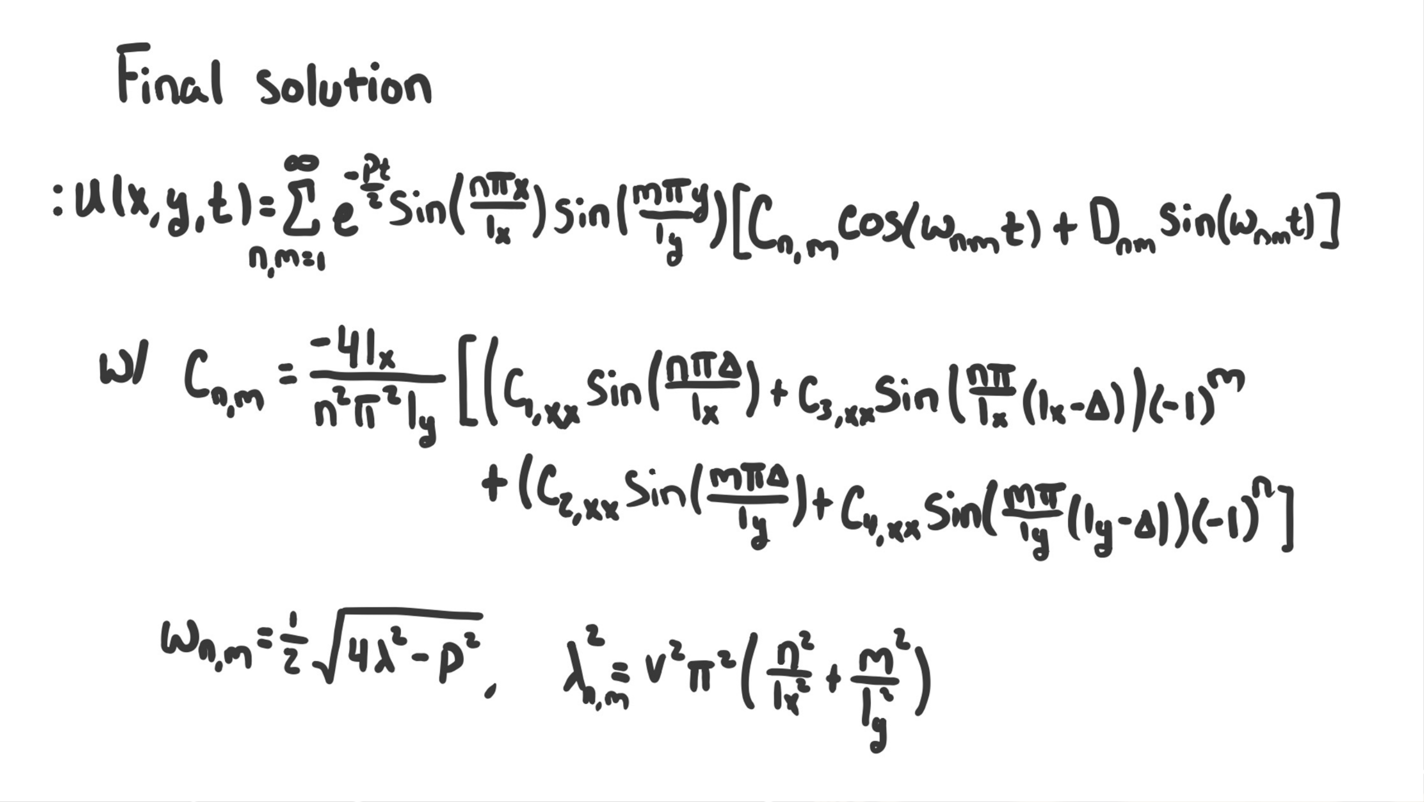
$$\Rightarrow -C_{n,m}(\frac{l_x l_y}{4})(\frac{n^2\pi^2}{l_x^2}) = (C_{1,xx} \sin(\frac{n\pi\Delta}{l_x}) + C_{3,xx} \sin(\frac{n\pi}{l_x}(l_x - \Delta))(-1)^m$$

$$+ (C_{2,xx} \sin(\frac{m\pi\Delta}{l_y}) + C_{4,xx} \sin(\frac{m\pi}{l_y}(l_y - \Delta))(-1)^n$$

$$\Rightarrow C_{n,m} = \frac{-4 l_x}{n^2\pi^2 l_y}[(C_{1,xx} \sin(\frac{n\pi\Delta}{l_x}) + C_{3,xx} \sin(\frac{n\pi}{l_x}(l_x - \Delta))(-1)^m$$

$$+ (C_{2,xx} \sin(\frac{m\pi\Delta}{l_y}) + C_{4,xx} \sin(\frac{m\pi}{l_y}(l_y - \Delta))(-1)^n]$$

Finally a Final Solution



Final solution

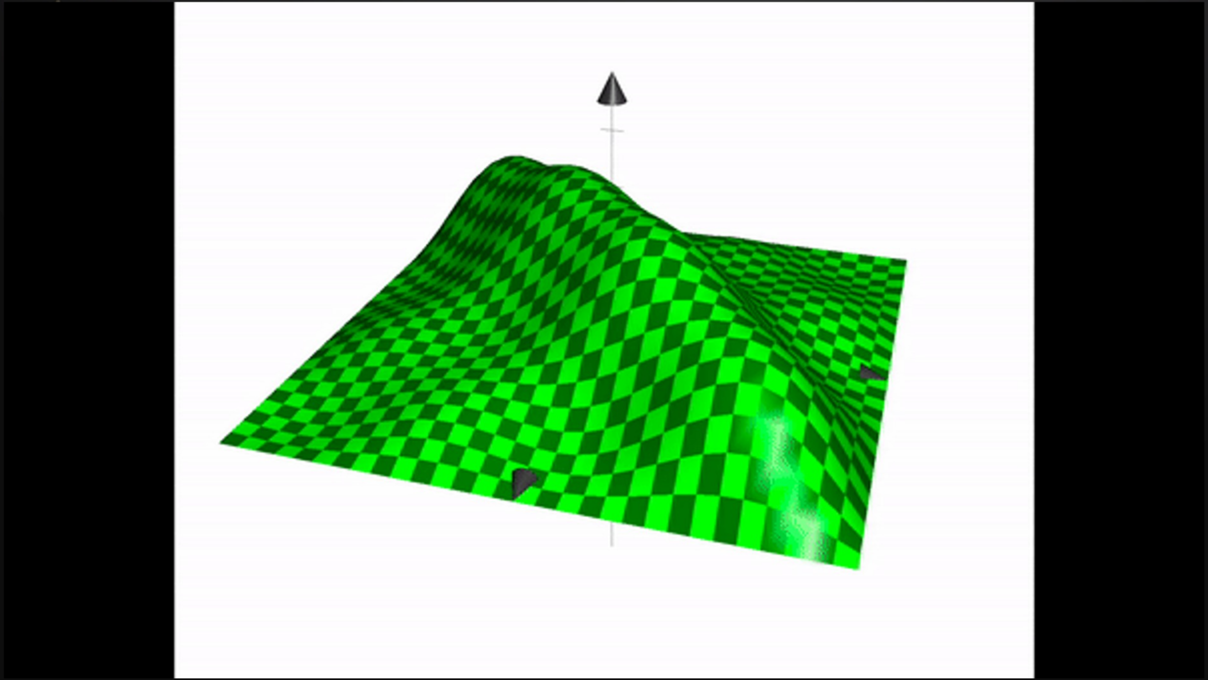
$$: u(x,y,t)=\sum_{n,m=1}^{\infty} e^{-\frac{pt}{2}}\sin\left(\frac{n\pi x}{l_x}\right)\sin\left(\frac{m\pi y}{l_y}\right)\left[C_{n,m}\cos(\omega_{nm}t)+D_{nm}\sin(\omega_{nm}t)\right]$$

$$\text{w/ } C_{n,m}=\frac{-4l_x}{n^2\pi^2 l_y}\left[\left(C_{1,xx}\sin\left(\frac{n\pi\Delta}{l_x}\right)+C_{3,xx}\sin\left(\frac{n\pi}{l_x}(l_x-\Delta)\right)\right)(-1)^m\right.$$

$$\left.+\left(C_{2,xx}\sin\left(\frac{m\pi\Delta}{l_y}\right)+C_{4,xx}\sin\left(\frac{m\pi}{l_y}(l_y-\Delta)\right)\right)(-1)^n\right]$$

$$\omega_{n,m}=\frac{1}{2}\sqrt{4\lambda^2-p^2}, \quad \lambda_{n,m}^2=v^2\pi^2\left(\frac{n^2}{l_x^2}+\frac{m^2}{l_y^2}\right)$$

Proof of Concept



What’s Next

-With Balloon popped, we move onto stretched rubber

- Shifting from 3-D wave equations to 2-D wave equations

-We could possibly try balloon again, but this is a hazardous option