Superconducting magnetic field sensors: SQUID

SQUID – superconducting quantum interference device

SQUID helmet project at Los Alamos

https://www.newswise.com/articles/los-alamos-unveils-new-brain-imaging-system

Magnetic field scales:

Earth field: ~1G

Fields inside animals: ~0.01G-0.00001G

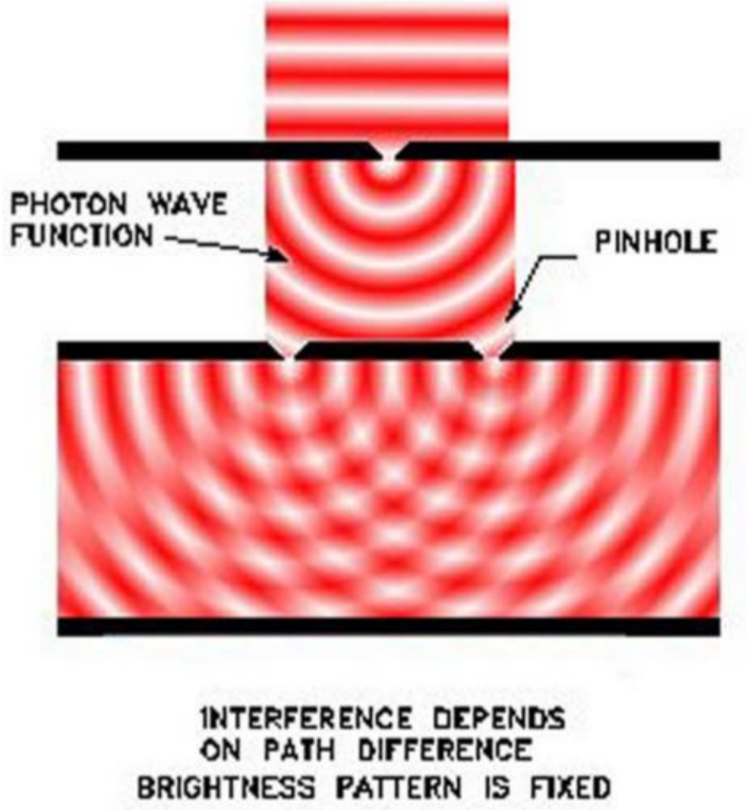
Fields of the **human brain**: ~0.3nG
This is less than a hundred-millionth of the Earth's magnetic field.

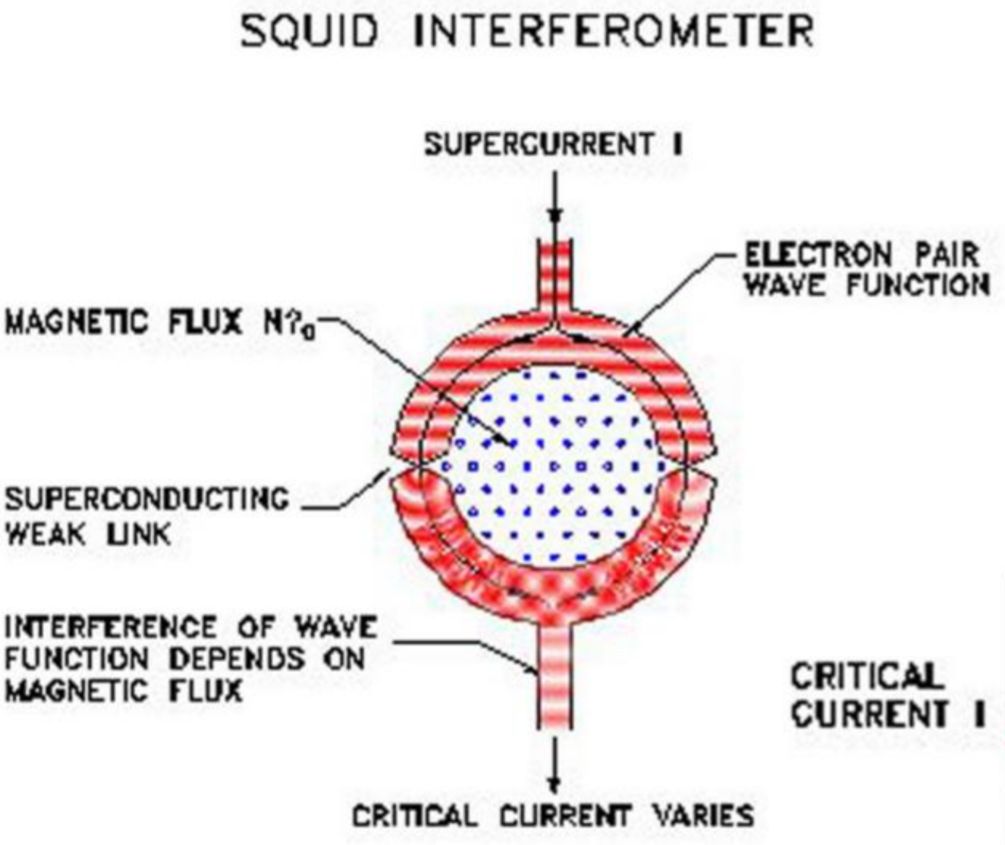
NanoGallery.info - Superconducting Helmet. SQUID used to measure brain magnetic fields

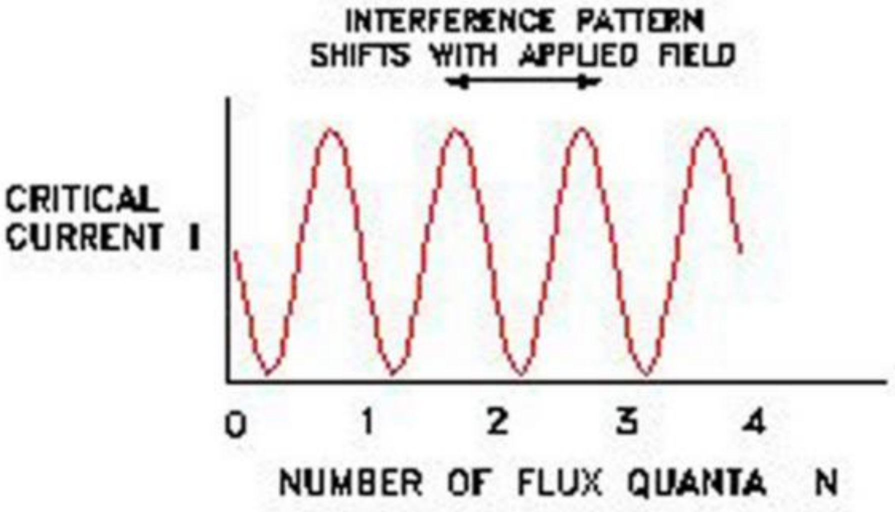
SQUIDs, or Superconducting Quantum Interference Devices, invented in 1964 by Robert Jaklevic, John Lambe, Arnold Silver, and James Mercereau of Ford Scientific Laboratories, are used to measure extremely small magnetic fields. They are currently the most sensitive magnetometers known, with the noise level as low as 3 $fT \cdot Hz^{-1/2}$. While, for example, the Earth magnet field is only about 0.0001 Tesla, some electrical processes in animals produce very small magnetic fields, typically between 0.000001 Tesla and 0.000000001 Tesla. SQUIDs are especially well suited for studying magnetic fields this small.
Measuring the brain's magnetic fields is even much more difficult because just above the skull the strength of the magnetic field is only about 0.3 picoTesla (0.0000000000003 Tesla). This is less than a hundred-millionth of Earth's magnetic field. In fact, brain fields can be measured only with the most sensitive magnetic-field sensor, i.e. with the superconducting quantum interference device, or SQUID.

Superconducting Quantum Interference Device (SQUID)

Optical analogue of SQUID







https://einstein.stanford.edu/STEP/information/data/equiv2.html

Superconducting Quantum Interference Device (SQUID)

The SQUID (**Superconducting Quantum Interference Device**) is a very sensitive *magnetometer*. The SQUID shown in the previous slide is a "DC" SQUIDs. This is the simplest design comprised of a superconducting ring broken by two very narrow (~10 Angstrom) insulating gaps or weak links. Current is injected into one half of the ring and removed from the other; the output is the voltage that develops across the gaps. The SQUID works *somewhat like a two-slit optical interferometer*, with the gaps playing the role of slits (“pinholes”). In the interferometer the slits are illuminated from one side by a coherent source of light waves. The waves pass through the slits and form an interference pattern on an imaging screen. The interference pattern depends on the path difference between the slits and on the wavelength used. In the DC SQUID electron pairs from the current source pass through the gaps, and their quantum mechanical wavefunctions interfere much like light waves. But the wavefunction's amplitude corresponds to the likelihood that an electron pair will cross the SQUID, rather than to brightness as in the optical case. Therefore, the *critical current of the SQUID depends on quantum interference, i.e., on the magnetic field applied*. This is because the phase of the electronic wave function depends on the vector potential. If the applied current is larger than the critical current, then a voltage occurs. This voltage oscillates as the critical current oscillates with the applied magnetic field.

https://einstein.stanford.edu/STEP/information/data/equiv2.html

Superconducting Quantum Interference Device (SQUID)

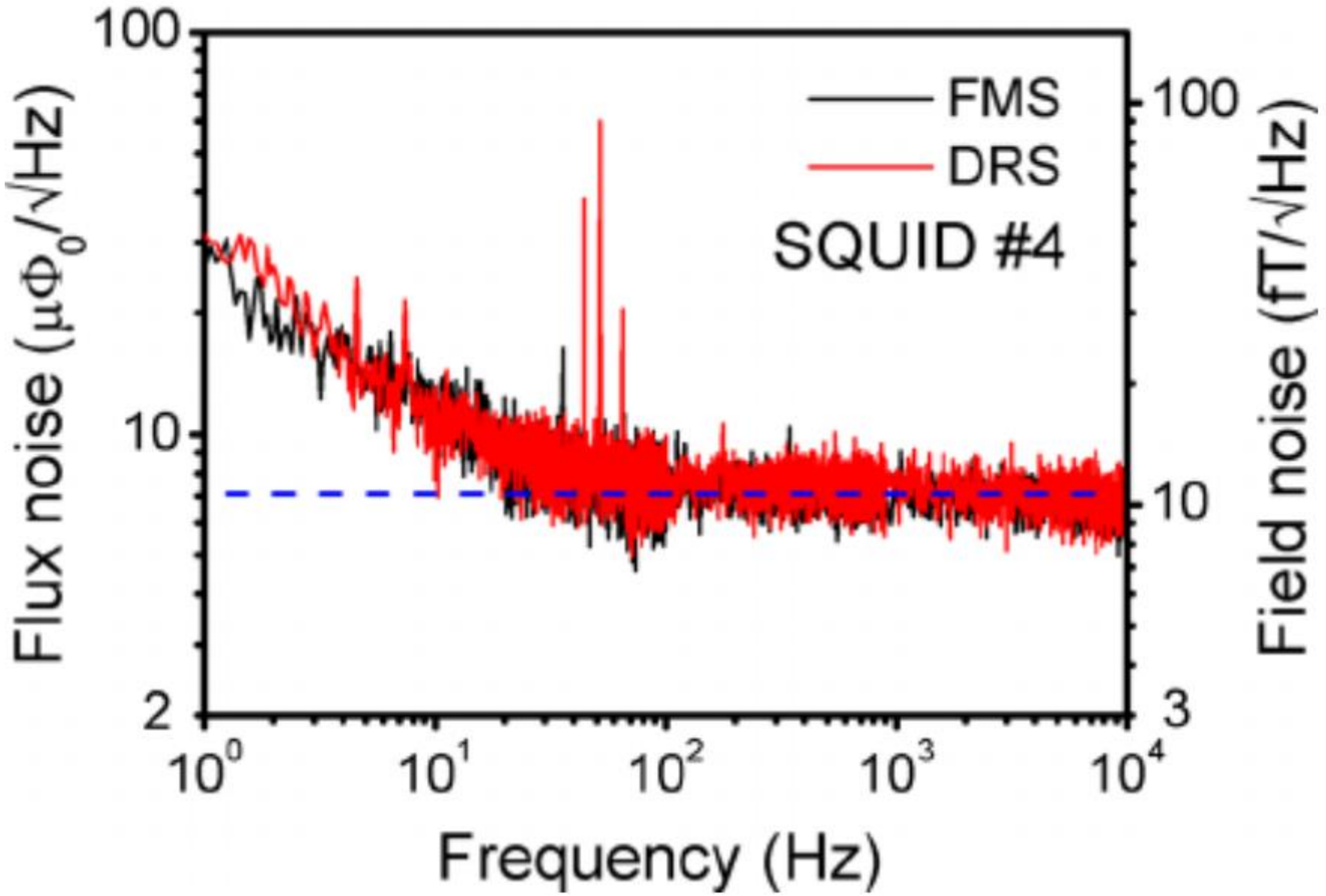
In the SQUID the current is carried by superconducting electrons which pair to one another. Such pairs effectively form a condensate analogous to the flux of coherent bosons, such as photons in laser light. Note that a bound pair of two fermions is a boson.
The electrons have an electric charge. Thus, they are deflected by a magnetic field. The effective path difference for electrons depends on the magnetic flux enclosed by the ring, which shifts the electron's interference pattern, and changes the voltage developed. There is no optical analog of this magnetic field dependence, since photons are not electrically charged.

The quantum interference pattern, like the optical pattern, is periodic and therefore the voltage developed across the SQUID repeats regularly with magnetic flux. The period of the variation is the flux quantum is about $h/2e = 2 \times 10^{-15}$ *weber.* In addition to intrinsically high sensitivity, SQUIDs can be very quiet as well. SQUIDS have been built with noise levels less than 10^{-7} of the quantum of flux.

https://einstein.stanford.edu/STEP/information/data/equiv2.html

Noise: Magnetic flux fluctuations in SQUIDs

$\Phi_0 = h/2e$



High intrinsic noise and absence of hysteresis in superconducting quantum interference devices with large Steward-McCumber parameter

 Jul 2013

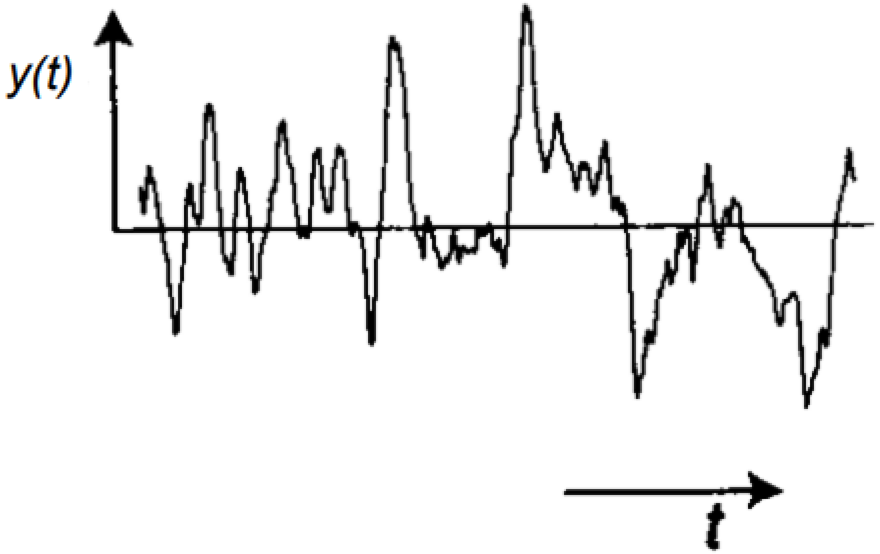
Jia Zeng · Yi Zhang · Michael Mück · [...] · Mianheng Jiang

We investigated niobium thin film superconducting quantum interference devices (SQUIDs) with large Steward-McCumber parameter (bc>1). No hysteresis was observed in the current-voltage (I-V) characteristics of the SQUIDs, even for bc � 17. We attribute the absence of hysteresis to an excess voltage noise of the junctions which increases the SQUID in...

Understanding "Noise"

Consider signal *y(t)*

time average of the signal fluctuations: $\langle y \rangle = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} y(t)dt$



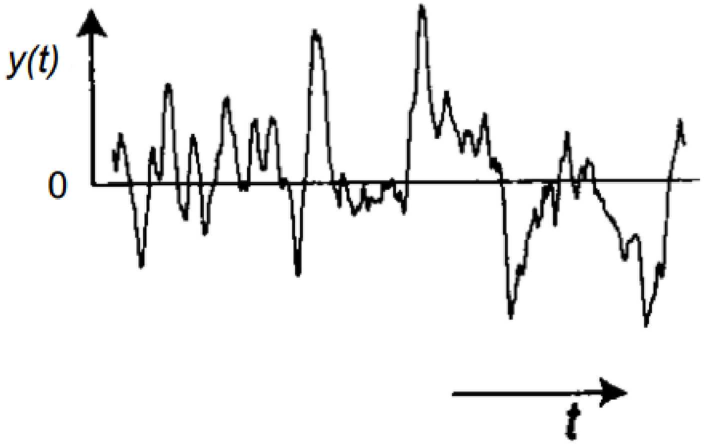
[Microsoft PowerPoint - Autocorr-Noise-combined.ppt [Compatibility Mode] (mcmaster.ca)]

Understanding "Noise"

Redefine the measured quantity as y<t> → y(t)-<y(t)>

So, from now on y(t) means the deviation of the measured quantity from its average or mean value. Therefore:

$$\langle y \rangle = 0$$

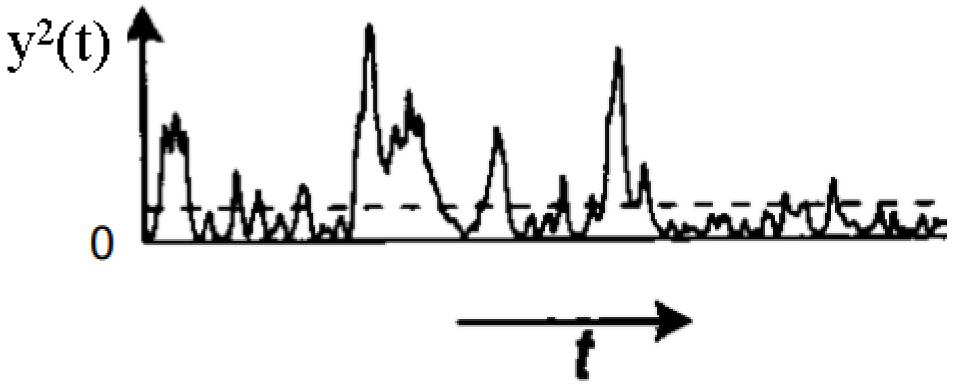


Understanding "Noise"

the mean of the square of the signal fluctuations is:

$$\langle y^2 \rangle = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} y^2(t)\,dt$$

since the square of the fluctuation is always positive, the mean square is not zero: $\langle y^2 \rangle \neq 0$



Understanding "Noise"

Autocorrelation function, $R(\tau)$

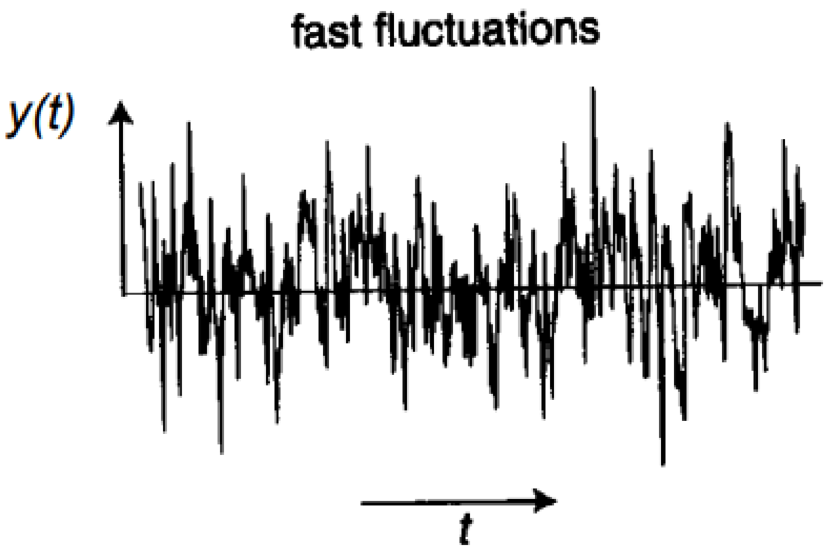
It defines how fluctuations from the mean value change in time.

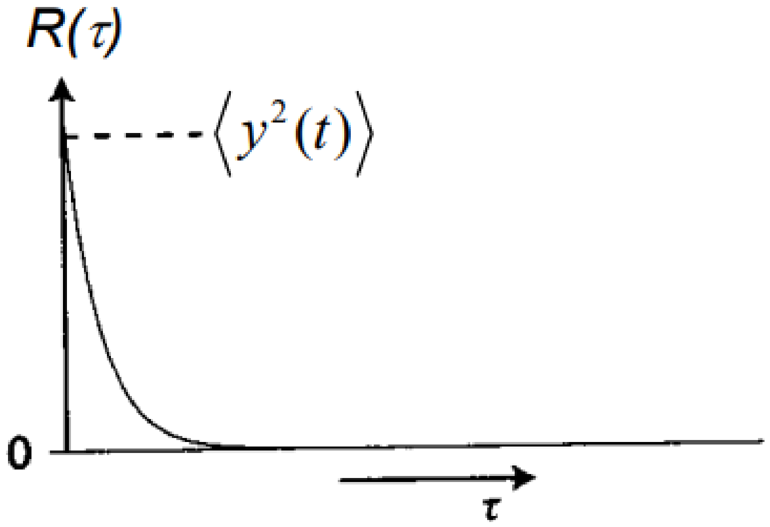
$$R(\tau) = \langle y(t)y(t+\tau)\rangle = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} y(t)y(t+\tau)dt \neq 0$$

t and the fluctuation at a later time t+tau.
If the fluctuations at different times would be completely independent from each other then the autocarrelation fucntion would be a delta-fucntion. In reality the fluctuations are correlated in time to some extent.

Understanding "Noise"

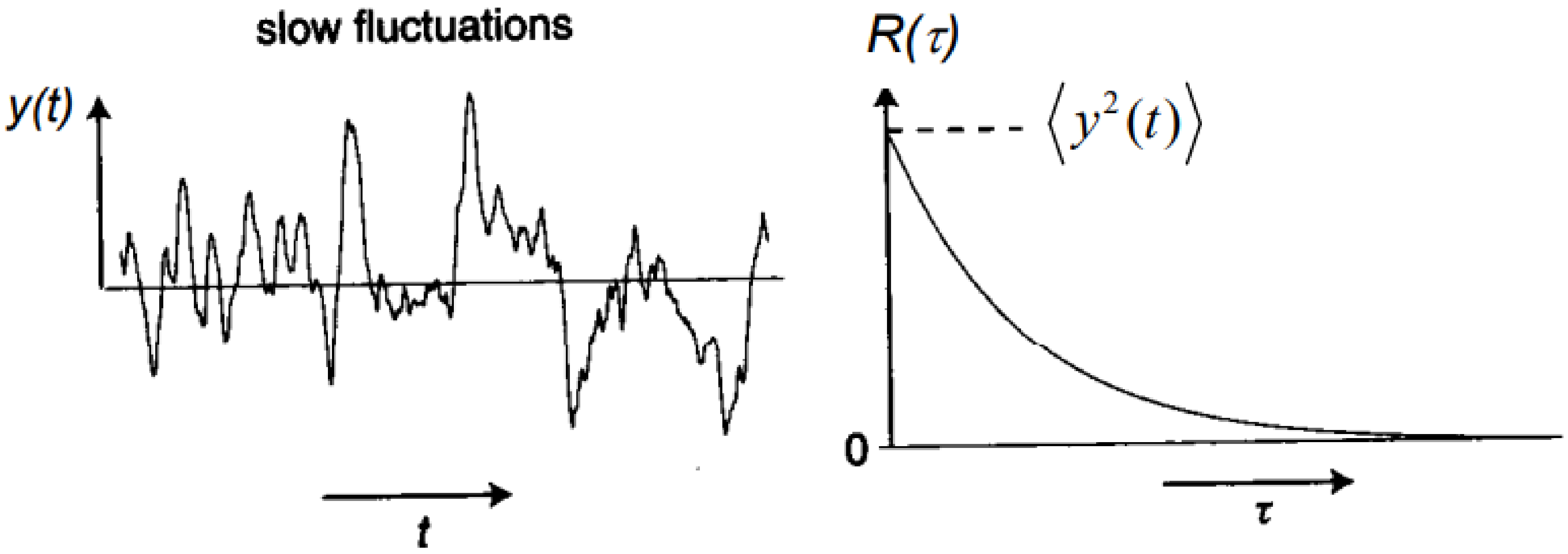
A rapidly fluctuating signal has an autocorrelation function that decays quickly with respect to τ:





Understanding "Noise"

A slowly fluctuating signal has an autocorrelation function that decays slowly with respect to τ:



Understanding "Noise"

- **By definition, the autocorrelation function at $\tau=0$ is equal to the mean square field:**

$$R(\tau = 0) = \langle y^2(t) \rangle$$

- **In general, the autocorrelation R(τ) tends to be large for small values of τ, and tends to zero for large values of τ.**

Understanding "Noise"

Power spectrum - is the Fourier transform of the autocorrelation function.

$$G(\omega) = \int_{-\infty}^{\infty} R(\tau) e^{-i\omega\tau} d\tau$$

F. T.

$$R(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) e^{i\omega\tau} d\omega$$

Parameter τ referred to as 'lag'.

Understanding "Noise"

- _**autocorrelation function is symmetric** $R(-\tau) = R(\tau)$

which means that:

$$R(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) e^{i\omega\tau} d\omega = \frac{1}{\pi} \int_{0}^{\infty} G(\omega) e^{i\omega\tau} d\omega$$

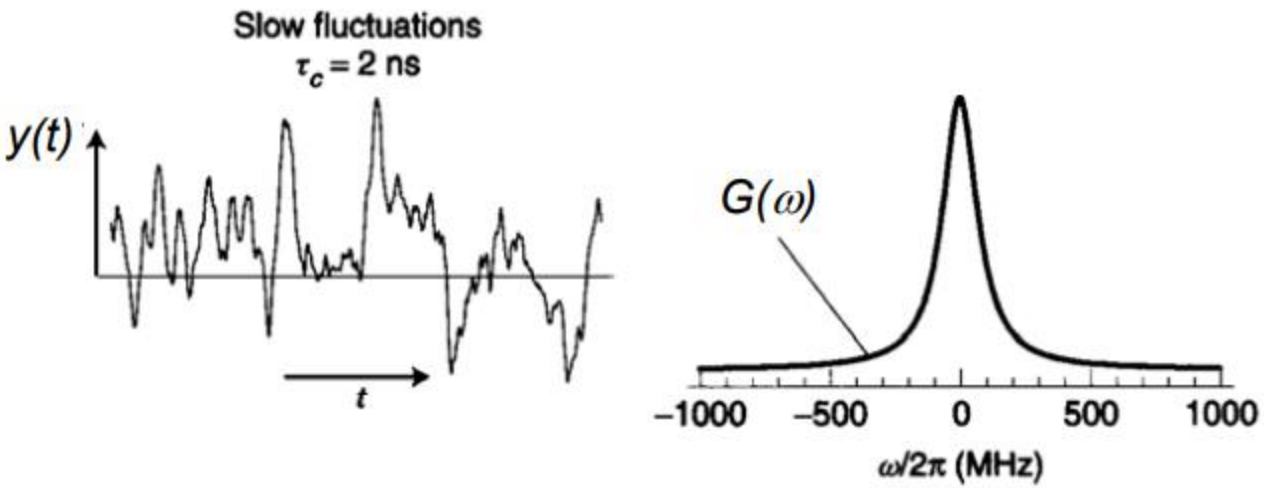
this condition leads to simplification:

$$R(\tau) = \frac{1}{\pi} \int_{0}^{\infty} G(\omega) \cos(\omega\tau) d\omega$$

consideration is then restricted to both $\omega \geq 0 \; and \; \tau \geq 0$

Understanding "Noise"

- if the signal fluctuates slowly, τ_{corr} is long and the power spectrum is narrow:



The area under the spectrum is independent of τ_{corr} and is given in the present model by $2\langle y^2(t)\rangle$ (twice the mean square amplitude of the signal *y(t)*).

Understanding “Noise”: A different view on the same question

start with the Fourier series representation of X(t) in real notation:

$$X(t) = \frac{A(0)}{2} + \sum_{n=1}^{\infty} [\, A(n)\cos(n\omega_0 t) + B(n)\sin(n\omega_0 t)]$$

As long as X(t) is continuous, the complete Fourier series obtained in this manner will reconstruct the original waveform without error.

Once A(n) and B(n) have been determined in this way, they can be plotted as functions of frequency ($n\omega_0$). These spectra are defined only for integer values of n.

The waveform of the observed segment T exactly determines the values of the Fourier coefficients:

$$A(n) = \frac{2}{T}\int_0^T X(t)\cos(n\omega_0 t)dt$$

and

$$B(n) = \frac{2}{T}\int_0^T X(t)\sin(n\omega_0 t)dt$$

https://ccs.fau.edu/~bressler/EDU/STSA/Modules/VI.pdf

Understanding “Noise”: A different view on the same question

A(n) and B(n) both refer to the same frequency; together they are equivalent to the complex spectrum:

$$Z(n) = \frac{A(n) - jB(n)}{2}$$

Power

Z(n)|^2 is referred to as power:

$$|Z(n)|^2 = \frac{A(n)^2 + B(n)^2}{4}$$

When displayed as a function of n or frequency, it becomes the power spectrum of X(t).

Z(n) is a complex number.

The absolute value or amplitude of Z(n) is defined as:

$$|Z(n)| = \sqrt{Z(n)Z(n)^*} = \frac{\sqrt{A(n)^2 + B(n)^2}}{2}$$

The amplitude of Z(n) displayed as a function of n (or frequency) is referred to as the amplitude spectrum of x(t).

https://ccs.fau.edu/~bressler/EDU/STSA/Modules/VI.pdf

Understanding "Noise": A different view on the same question

The total power of a time series can be determined either directly from the time series or from its spectral components. In the time domain, power is defined as the mean square (ms) value:

$$ms = \frac{1}{T}\int_0^T X(t)^2\,dt$$

or its root :

$$rms = \sqrt{\frac{1}{T}\int_0^T X(t)^2\,dt}$$

Parseval's Theorem

Parseval's theorem relates the mean square value and the sum of spectral powers as:

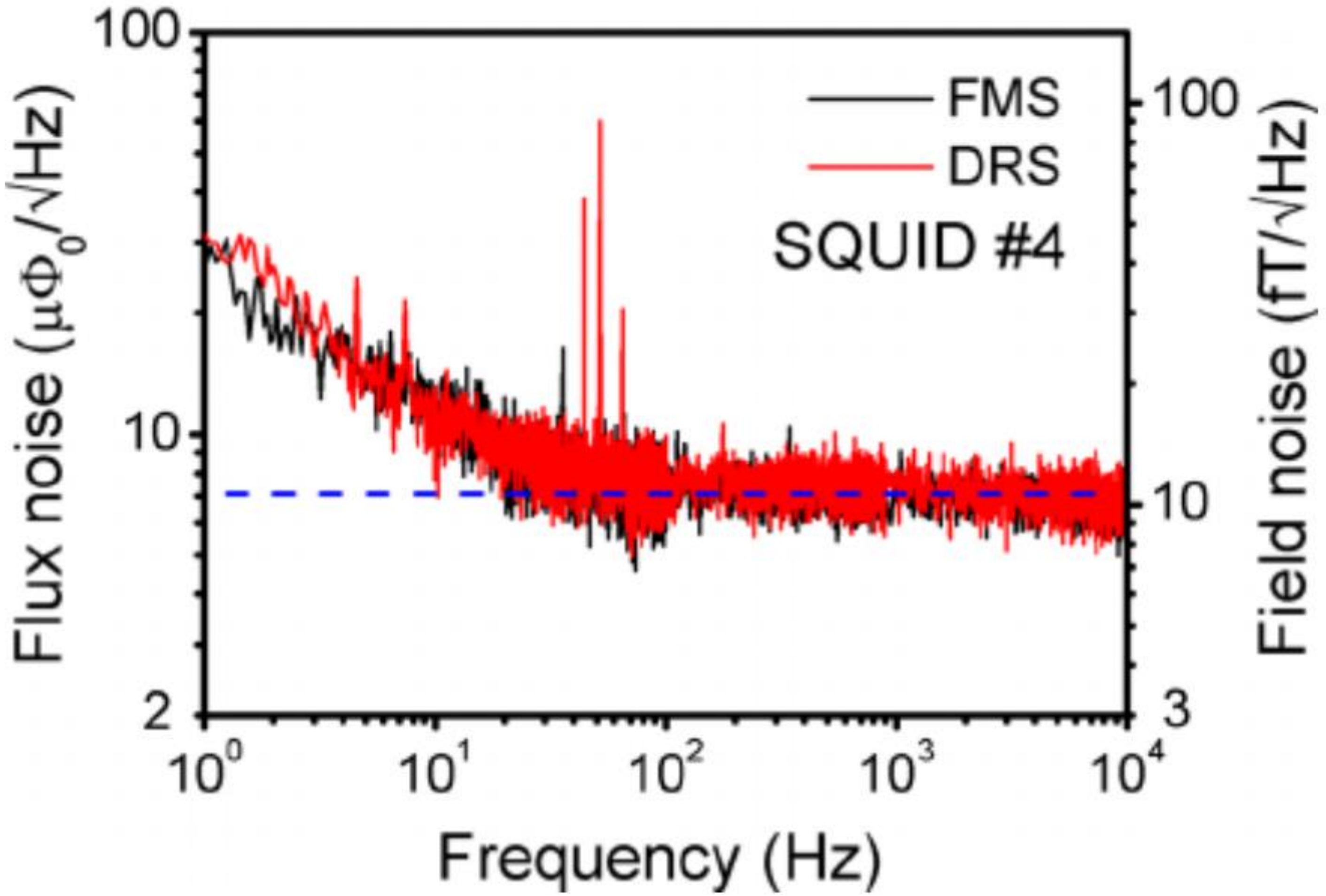
$$\frac{1}{T}\int_0^T X(t)^2\,dt = \sum_{n=-\infty}^{\infty} |Z(n)|^2 = \int Z^2\,df$$

Thus the total power of X(t) can be determined in the time domain as the mean square value, or in the frequency domain as the sum of spectral powers.

https://ccs.fau.edu/~bressler/EDU/STSA/Modules/VI.pdf

Noise power spectrum of the SQUID

$\Phi_0 = h/2e$



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