1.) Introduction:

Solid state physicists study the properties of solids in so far as they are influenced by the properties of the atoms and electrons. It turns out it's not free for all. There are rules governing how the atoms want and how such wave weaves influence the interactions between the electrons. E-Phonon interactions give rise to superconductivity. E- E interactions give rise to magnetism, the Kondo effect, and antiferromagnetism.

In this class we will make a first pass at studying this physics. For a more complete treatment you need to take 561.

2.) Crystal Structure:

a) Crystalline Order: the simplest way of
organizing atoms in a solid is a lattice. It is complicated to show that a regular lattice minimizes the energy of a collection of atoms. Except for the deform crystals at T=0. Crystallinity appears to be an emergent property. That there is no obvious physical principle here suggests that it might be an emergent property. Emergent properties need an organizing principle to be understood. It is worth thinking about: What is the organizing principle for crystallinity.

b) Bravais lattice

Def.: specific array in which repeated units of the crystal are arranged.

More technically, a collection of points in which the neighborhood around each point is the same under translation. First we do 2D lattices. All Bravais lattices in 2D can be written down explicitly from

\[ \mathbf{R} = n_1 \mathbf{a}_1 + n_2 \mathbf{a}_2 \]

the \( \mathbf{a}_i \)'s are primitive vectors.

1) \[ \mathbf{a}_1 = a(1,0) \] Square \[ \mathbf{a}_2 = a(0,1) \]
2.) Hexagonal
\[ \vec{a}_1 = a(1, 0) \]
\[ \vec{a}_2 = a(\frac{1}{2}, \sqrt{3}/2) \]

The neighborhood of each point is identical and the vectors are linearly independent.

\[ \alpha_1 \vec{a}_1 + \alpha_2 \vec{a}_2 \neq \alpha_3 \vec{a}_3 \]

3.) Rectangular:
\[ |a_1| = |a_2|; \phi = 90^\circ \]

4.) Centered Rectangular
\[ |a_1| = |a_2|; \phi = 90^\circ \]

5.) Oblique
\[ |a_1| = |a_2|; \phi \neq 90^\circ \]

There are other lattices in 2D but they do not have an underlying triangular lattice. Consider the Hexagonal lattice.
Honeycomb lattices are formed from a Bravais lattice but by adding a basis as an identical assembly of atoms. Start with a hexagonal lattice, then add particles at:

$$\vec{V}_1 = a(0, \frac{1}{2\sqrt{3}})$$

$$\vec{V}_2 = a(0, -\frac{1}{2\sqrt{3}})$$

(c) **Primitive Cells**

The smallest unit that is needed to repeat the lattice. All such cells must encompasses the same volume because a primitive cell in a Bravais lattice contains exactly one atom. Here are some choices for a square lattice. All...

But end to end, all such cells fill the crystal. This is a tiling or tesselation.

Wigner-Seitz Cell: A primitive cell that is invariant under all symmetry operations that leave the crystal invariant. Here are the rules for constructing a W.S. cell. Pick any lattice point. Then draw the bisector
of the line between that point and its neighbors.

Note there are 6 neighbors all equidistant, this answers the question in class.

d) Symmetries:
These are operations that leave the lattice invariant: reflections, rotations, translations. All such operations are generated by

\[ \mathbf{G} = \mathbf{a}^2 + R(\delta \theta). \]

(trans. \hspace{1cm} rot.)

The point group of a crystal denotes all such rotations and reflections.

Symmetry group:
all rigid operations which leave the lattice invariant. The symmetry group is not simply the point group \times translation group. This occurs because in some cases, there are combinations of translations and proper rotations which leave the system unchanged but neither of which is a symmetry itself.
Screw axes which are absent from 2D but present in 3D are a case in point. Two groups $(G, G')$ are equivalent up to an overall linear change of the coordinate system if $G$ a matrix $S$ such that

$$S^{-1} + SRS^{-1} = N' + \omega' = \omega'. \quad \text{(6)}$$

**3D Lattices**

3D Bravais lattices $\mathbf{R} = n\mathbf{a} + m\mathbf{b} + l\mathbf{c}$.

1. **Simple Cubic**
   
   $(0,0,0), (1,0,0), (0,1,0), (0,0,1)$

2.1. **FCC**

   $\frac{a}{2} (1,1,0), \frac{a}{2} (1,0,1), \frac{a}{2} (0,1,1)$.

3.1. **BCC**

   $\frac{a}{2} (1,1,-1), \frac{a}{2} (1,-1,1), \frac{a}{2} (-1,1,1)$.

4. **CaF$_2$**

   sc with basis to indicate the filling of space.
There are 7 distinct point symmetry groups and 14 Bravais lattices in 3D.

3.) Reciprocal Space

This is useful in the context of X-ray scattering. Consider a plane wave \( e^{-iK \cdot r} \).

\[ e^{-iK \cdot r} = e^{iK \cdot r} e^{iK \cdot r} \]

\[ e^{iK \cdot r} = 1 \quad \Rightarrow \quad K \cdot r = 2\pi n. \]

where \( n \) is an integer.

\[ K = m\hat{a}_1 + n\hat{a}_2 + p\hat{a}_3. \]

demand that \( n\hat{a}_1 - m\hat{a}_2 = 2\pi \mathbf{b}_2 \)

regardless of the \( \mathbf{b}_2 \) become as set of reciprocal lattice vector that work.

\[ \mathbf{b}_1 = \frac{2\pi}{\hat{a}_1 \cdot \hat{a}_2} \mathbf{a} \times \mathbf{a}_3. \]
\[ b_2 = 2\pi \frac{a_2^2 \times a_1}{a_2 \cdot a_1} \quad b_3 = 2\pi \frac{a_3^2 \times a_1}{a_3 \cdot a_1} \]

From these definitions, it is clear that 
\[ b_1 \cdot g = \text{integers}. \]

For the \( fcc \) lattice

\[ b_3 = \frac{4\pi}{a} (1, -1, -1) \Rightarrow \text{lattice spacing is } \frac{4\pi}{a} \]

in reciprocal space.

Miller indices: a set of 3D planes \( \perp \) to a family of planes in reciprocal space.

\( (h, k, l) \rightarrow \perp \) to reciprocal \((K)\) space

\[ [h,m,n] \rightarrow \perp \text{ real space} \]

(1, 0, 0) \( \perp \) to \( k = \frac{2\pi}{a} (1, 0, 0) \)

\[ [1, 0, 0] \perp \rightarrow a_1^* \]

S.C.