• Marder (SECOND EDITION), 1.2, 1.3, 2.2, 2.3, 3.2, 3.5

• Extra Problem: Three parts

Show that the reciprocal lattice primitive vectors satisfy the relation

\[ b_1 \cdot (b_2 \times b_3) = \frac{(2\pi)^3}{a_1 \cdot (a_2 \times a_3)} \]  \hspace{1cm} (1)

You might want to approach this problem by writing \( b \) in terms of \( a \) and then using the orthogonality relationship between them.

• Show that the volume of a Bravais lattice primitive cell is

\[ V_{\text{cell}} = |a_1 \cdot (a_2 \times a_3)| \]  \hspace{1cm} (2)

• Show that the reciprocal lattice of the reciprocal lattice is the original real-space lattice—that is,

\[ 2\pi \frac{b_2 \times b_3}{b_1 \cdot (b_2 \times b_3)} = a_1 \]  \hspace{1cm} (3)

In doing this, you will find the following vector identities useful:

\[ A \times (B \times C) = B(A \cdot C) - C(A \cdot B) \]  \hspace{1cm} (4)

\[ (C \times A) \times (A \times B) = (C \cdot A \times B) \cdot A \]  \hspace{1cm} (5)

\[ (A \times B) \cdot (C \times D) = (A \cdot C)(B \cdot D) - (A \cdot D)(B \cdot C) \]  \hspace{1cm} (6)