Phys. 561

Problem Set 1: Due Sept. 11, 2025

1. 2.1, 2.2, 3.1, 3.2,3.3

2. Computation of $r_s$: For $d = 2$ and $d = 3$, compute $r_s$ for a general kinetic energy of the form $c_n k^n$. What physical system corresponds to $d = 2$ and $n = 1$?

3. This problem is on the Higgs mechanism for spontaneous symmetry breaking which we discussed in class. A mass term enters the Lagrangian in the form $m^2\phi^2$. Consider the Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + |D_\mu\phi|^2 - V(\phi) \tag{1}$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $D_\mu = \partial_\mu + ieA_\mu$. You are to a) show that this Lagrangian is invariant under the transformation $\phi \to e^{i\alpha(x)}\phi(x)$ and $A_\mu(x) \to A_\mu(x) - \frac{1}{e}\partial_\mu\alpha(x)$. b) Let $V(\phi) = -\mu^2|\phi|^2 + \frac{\lambda}{2}(\phi^*\phi)^2$. Show that the minimum of the potential is non-zero. What does this mean? c) Expand the potential about the minimum found in (b) by letting

$$\phi(x) = \phi_0 + \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2), \tag{2}$$

and show that the potential can be written as

$$V(\phi) = -\frac{\mu^4}{2\lambda} + \frac{1}{2}\cdot 2\mu^2\phi_1^2 + O(\phi_i^3)\cdots. \tag{3}$$

In terms of what we discussed in class, what does it mean that there is no term of the form $m^2\phi_2^2$? d) Now consider the kinetic part and show that it can be written as

$$|D_\mu\phi|^2 = \frac{1}{2}(\partial_\mu\phi_1)^2 + \frac{1}{2}(\partial_\mu\phi_2)^2 + \sqrt{2}e\phi_0\cdot A_\mu\partial^\mu\phi_2 + e^2\phi_0 A_\mu A^\mu + \cdots \tag{4}$$

What does the last term mean physically? Does the new Lagrangian preserve the original symmetry? If not, why? Explain based on what we discussed in class.