Optimally Scrambling Chiral Spin-Chain with Effective Black Hole Geometry

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Journal Club presentation
Physics 596
Team 12

Motivation for the study

- **Black Hole Information Paradox** :
  - Information “lost” upon crossing the *event horizon*.
  - Contradicts Quantum Mechanics (Unitarity).
- Possible resolution:
  - **Optical Scrambling of Information** inside horizon.
  - Causes **rapid thermalization** leading to **Hawking Radiation.**
  - Can be tested by examining analogous systems.
  - Example : **Chiral Spin Chain.**
  - **Phase Transition** analogous to event horizon.

Chiral Spin Chain

Mean Field Theory approach to derive the quasi-particle dispersion and chirality.

Chiral Spin Chain- the Hamiltonian

- Chiral Spin Chain

$$H = \frac{1}{2}\sum_{i=1}^{N}\left[-u\left(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y\right) + \frac{v}{2}\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1} \times \boldsymbol{S}_{i+2}\right]$$

- In terms of Pauli operators

$$H = \sum_n [-\frac{u}{8}\left(\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y\right) + \frac{v}{32}\epsilon_{abc}\sigma_n^a \sigma_{n+1}^b \sigma_{n+2}^c]$$

Breaks time-reversal symmetry and Reflection symmetry!

Chiral Spin Chain- Map to Fermion

- Jordan- Wigner Transformation: map to spinless fermion

$$\sigma_n^+ = \exp\left(-i\pi \sum_{m<n} c_m^\dagger c_m\right) c_n^\dagger, \sigma_n^- = \exp\left(i\pi \sum_{m<n} c_m^\dagger c_m\right) c_n, \sigma_n^z = 2c_n^\dagger c_n - 1$$

- Transformed Hamiltonian

$$H = \frac{1}{4}\sum_n [-uc_n^\dagger c_{n+1} - \frac{iv}{4} c_n^\dagger c_{n+2} + \frac{iv}{4}\left(c_n^\dagger c_{n+1}\sigma_{n+2}^z + c_{n+1}^\dagger c_{n+2}\sigma_n^z\right)] + h.c.$$

Has four fermion interaction term, not a free fermion model.

Chiral Spin Chain- Mean Field Theory

- Mean field theory can help us to convert interacting Hamiltonian into free one

$$AB = \langle A \rangle B + A \langle B \rangle - \langle A \rangle \langle B \rangle + \delta A \delta B \qquad \delta A = A - \langle A \rangle$$

Here <...> refers to ground state expectation value, after self- consistency regarding the particle-hole symmetry by the original Hamiltonian

$$H_{\mathrm{MF}} = \frac{1}{4} \sum_n \left(-u c_n^\dagger c_{n+1} - \frac{iv}{4} c_n^\dagger c_{n+2} \right) + \mathrm{H.c.}$$

nearest neighbor and next.N.N hopping

- Fourier transform:
quasi-particle in momentum space

$$H_{\mathrm{MF}} = \sum_k \left[-\frac{u}{2} \cos(k) + \frac{v}{8} \sin(2k) \right] c_k^\dagger c_k$$

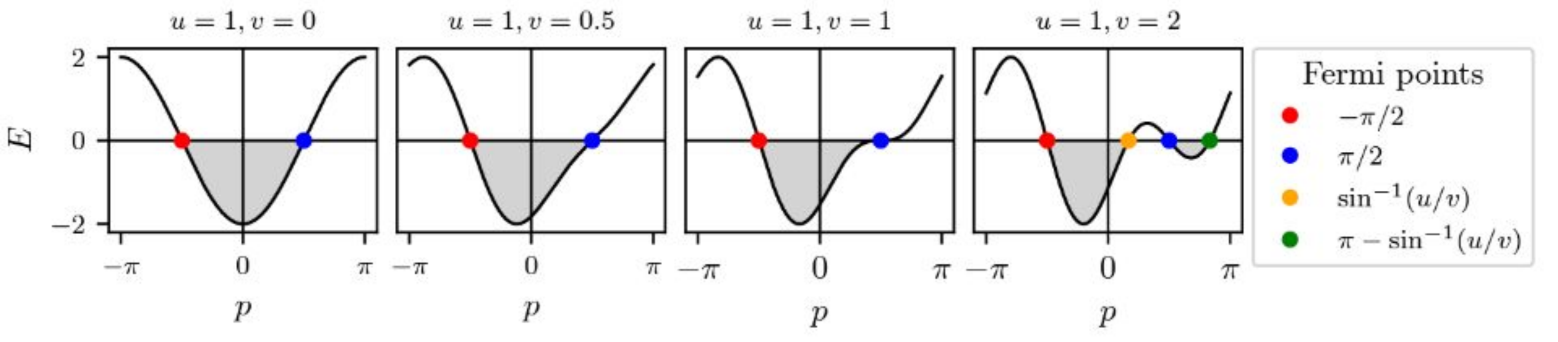
Chiral Spin Chain- Chirality

- Fermi velocity is defined by the group velocity $\frac{\partial \epsilon(k)}{\partial k}$ near Fermi points $\epsilon(k) = 0$

  $k_{\mathrm{R,L}} = \pm\frac{\pi}{2}$ if $|v|/2 > |u|$, we have two additional $k_1 = \sin^{-1}\left(\frac{u}{v/2}\right), k_2 = \pi - k_1$

- The diagram of the spectrum, with unequal left and right Fermi velocity- chirality

$$v_{\mathrm{R,L}} = \frac{\partial \epsilon(k)}{\partial k}|_{k_{\mathrm{R,L}}} = 2(\pm u - v/2)$$



Chiral Spin Chain- Validation of MFT I

- To validate MFT, we may compare the result of MFT and numerical approach, like MPS (matrix product state) .
- First, we may compare the phase transition point.

From the self-consistent MFT we have ground state energy (fully occupy negative energy states)

$$\rho_0 = \lim_{N\to\infty}\frac{1}{N}\sum_{p:E(p)<0} E(p) = \frac{1}{2\pi}\int_{p:E(p)<0} \mathrm{d}p E(p)$$

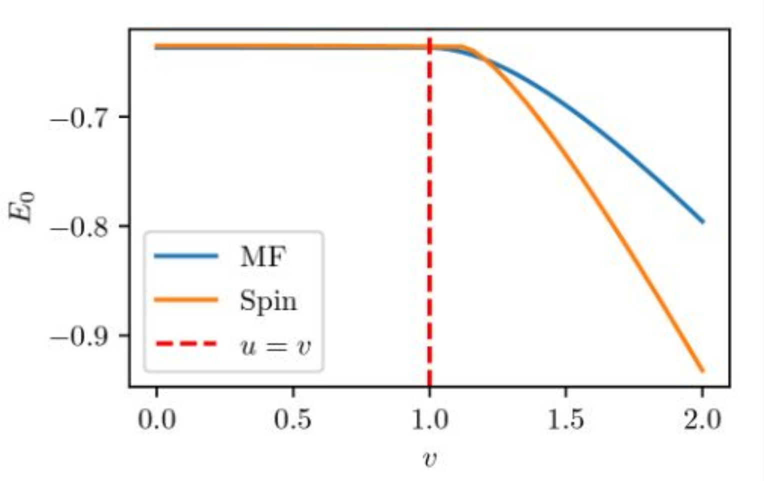
$$\rho_0 = \begin{cases} -\frac{2u}{\pi} & v/2 \leq u \\ -\frac{1}{\pi}\left(\frac{u^2}{v/2} + v/2\right) & v/2 > u \end{cases}$$

in which $\partial^2\rho_0/\partial v^2$ is discontinuous at $v/2 = u$

Chiral Spin Chain- Validation of MFT II

- Comparison with MPS result,

  The phase transition point shown by MFT is slightly smaller than that shown by MPS.



This test demonstrates that there exists a second-order phase transition about the point $v/2 = u$ and shows that for $v/2 < u$, the effect of the interactions on the model is negligible.

Chiral Spin Chain- Validation of MFT III

- Next we compare chirality. In MFT, the chirality $\chi_i = \boldsymbol{S}_i \cdot (\boldsymbol{S}_{i+1} \times \boldsymbol{S}_{i+2})$ can be written as $\chi_n = -2ic_n^\dagger c_{n+2} + \text{ H.c.}$ Thus, the result of MFT is

$$\langle \chi_n \rangle = 4\,\mathrm{Im}\,(C_{n,n+2}) = \begin{cases} 0 & v/2 \leq u \\ \frac{4}{\pi}\left(1 - \frac{u^2}{(v/2)^2}\right) & v/2 \geq u \end{cases}$$

This form indicates the scaling behavior $\chi_n(v) \approx \chi_n(u) + (v/2 - u)\chi_n'(u) \propto v/2 - u$

with critical exponent $\gamma = 1$

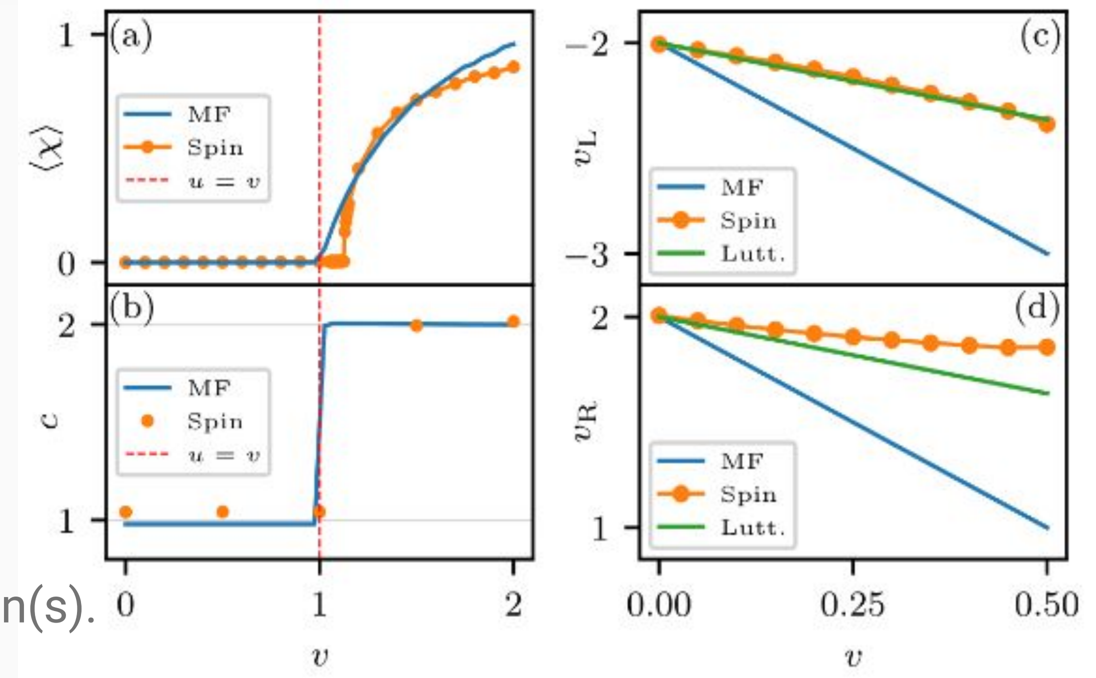
Chiral Spin Chain- Validation of MFT IV

- Comparison with DMRG result

MFT: $v/2 = u$ and $\gamma = 1$

DMRG: $v/2 \approx 1.12u$, $\gamma \approx 0.39$

The critical points are near, and the change of central charge indicates the change of number of Dirac fermion(s).

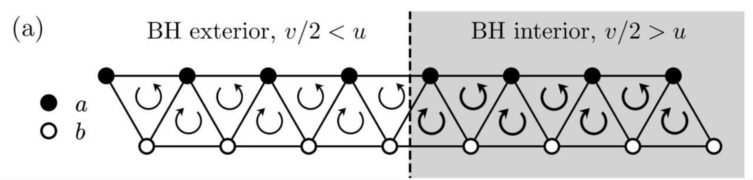


Black Hole Geometry

Emergent from low-energy effective theory of the chiral spin chain.

Connection to Black Holes - Unit Cell

- Introduce a unit cell with two sites, A and B.



- Mean Field Hamiltonian then takes the form -

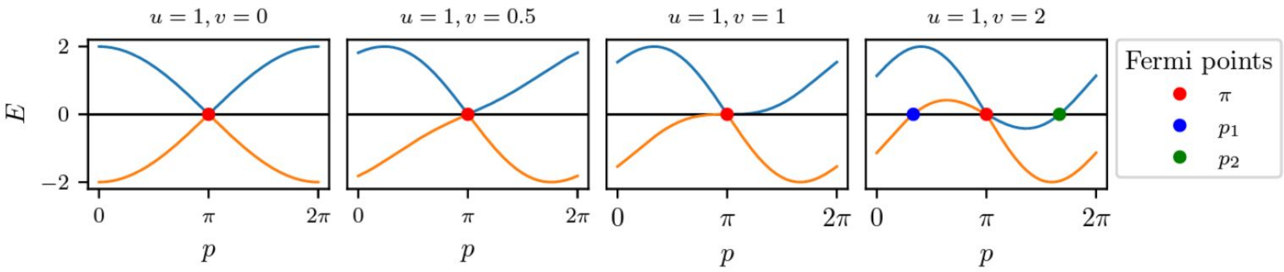
$$H_{\text{MF}} = \sum_n \left[-u a_n^\dagger \left(b_n + b_{n-1} \right) - \frac{iv}{4} \left(a_n^\dagger a_{n+1} + b_n^\dagger b_{n+1} \right) \right] + \text{H.c.}, \qquad u, v \in \mathbb{R}$$

$$\{a_n, a_m^\dagger\} = \{b_n, b_m^\dagger\} = \delta_{nm}$$

Connection to Black Holes - Dispersion Relation

- Dispersion relation obtained - depends on **u** and **v**.

$$E(p) = g(p) \pm |f(p)| = \frac{v}{2} \sin(a_c p) \pm u\sqrt{2 + 2\cos(a_c p)}$$



Note that $|\frac{v}{2}| > |u|$ for p_1 to exist.

$$p_0 = \frac{\pi}{a_c}, \quad p_1 = \frac{1}{a_c} \arccos\left(1 - \frac{2u^2}{(v/2)^2}\right)$$

Connection to Black Holes - Action

- Work backwards to obtain the corresponding **action integral** -

$$S = \int_M \mathrm{d}^{1+1} x \chi^\dagger(x) \left(i \overleftrightarrow{\partial_t} + i e^i_a \alpha a \overleftrightarrow{\partial_i} \right) \chi(x) = \int_M \mathrm{d}^{1+1} x \bar{\chi}(x) i e^\mu_a \gamma^a \overleftrightarrow{\partial_\mu} \chi(x)$$

- Amazingly, this corresponds to the **Dirac Fermion Action in Curved Spacetime** -

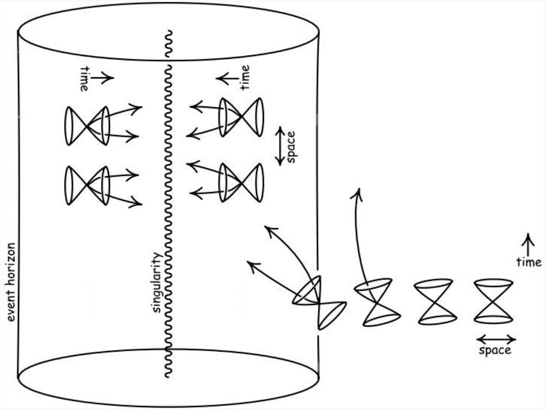
$$S_{Dirac} = \int_M d^{1+1} x |e| \left[\frac{i}{2} (\bar{\psi} \gamma^\mu D_\mu \psi) - D_\mu \psi \gamma^\mu \bar{\psi} - m \bar{\psi} \psi \right]$$

Connection to Black Holes - Metric Tensor

- Importantly, we read off the metric tensor as -

$$g_{\mu\nu} = \begin{pmatrix} 1 - v^2/u^2 & -v^2/u^2 \\ -v^2/u^2 & -1/u^2 \end{pmatrix}$$

This is the **Schwarzschild metric** in *Gullstrand-Painleve* coordinates!

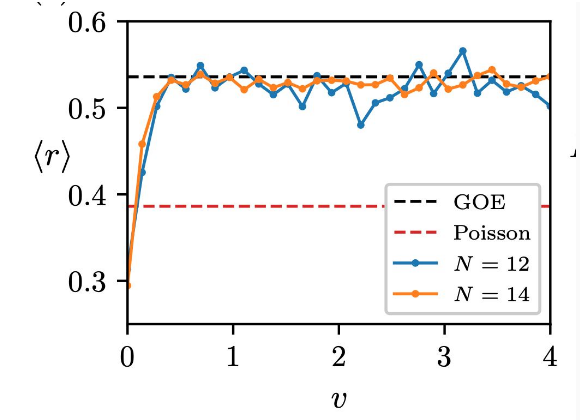
- 

"Light cone tipping" in this spacetime.

Connection to Black Holes - Quantum Chaos

- Study energy level statistics in **interior region**.
- Relevant Quantity -

$$r_n = min(s_n, s_{n-1})/max(s_n, s_{n-1}) \quad s_n = E_n - E_{n-1}$$



- Characteristic of **Quantum Chaos**! ⇒ Black Holes can potentially exhibit **maximum information scrambling!**

Lyapunov Exponent λ

Insights into quantum chaos of chiral spin-chain model

Basic Goal

- <r> is crude measure of chaotic behavior

  → Choose to use λ to quantify chaotic system rate of thermalization

- Seeking optimal scrambling

  → Need to determine if chiral spin-chain is capable of agreeing with universal bound for chaotic systems: $\lambda \leq 2\pi T$

- In QM framework, λ obtained using **out-of-time ordered correlators (OTOCs)**

Basic Framework

- **Regularized** OTOC:

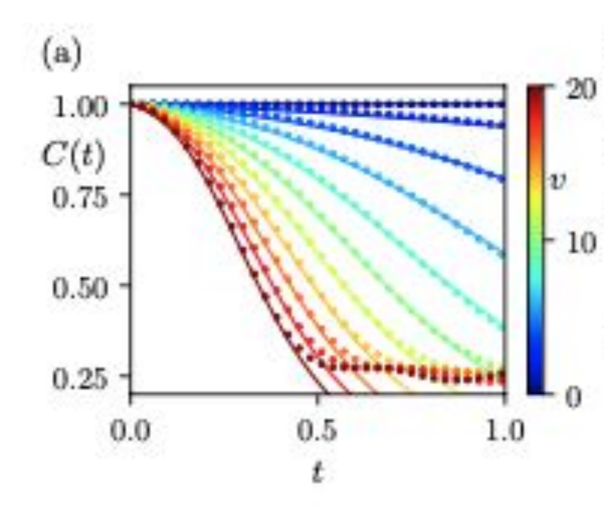
$$C(t) = \langle O_i(t)\rho^{1/4}O_j(0)\rho^{1/4}O_i(t)\rho^{1/4}O_j(0)\rho^{1/4}\rangle$$

- λ extracted by fitting numerical data to semi-classical functional form at low T:

$$C(t) = U\left(\frac{1}{2}, 1, Ne^{-\lambda t}\right)\sqrt{N}e^{-\lambda t/2}$$

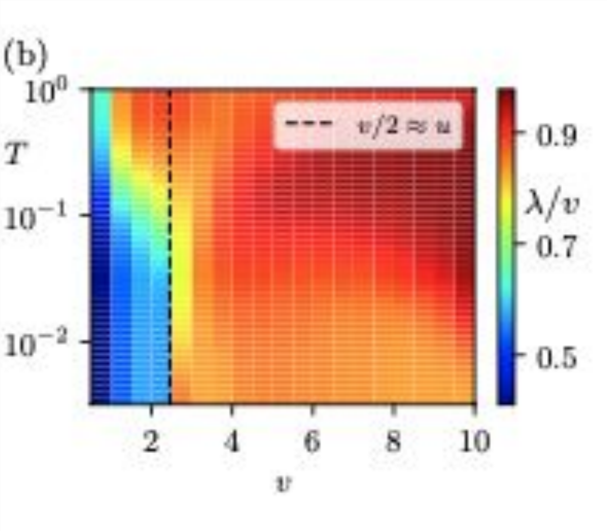
OTOC Behavior and λ Fitting

- Points show numerically evaluated OTOC
- Lines show fit to semi-classical functional form
- For large v, OTOC exhibits exponential decay → extract λ



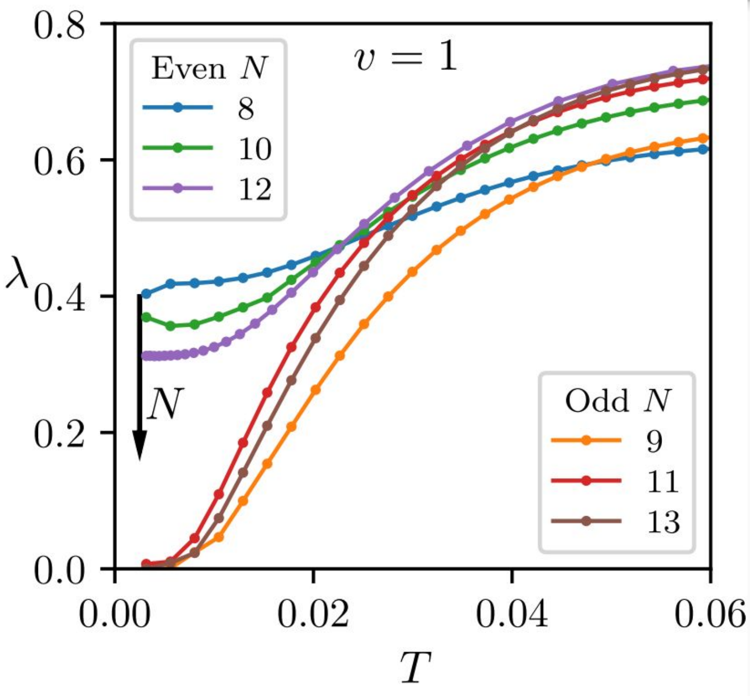
Temperature Dependence

- Using same framework, vary T in addition to v
- Different regimes split at phase transition v/2 ≈ u
- Large values of λ observed for large v, T → extract λ
- Universal bound condition: λ ≤ 2**π**T



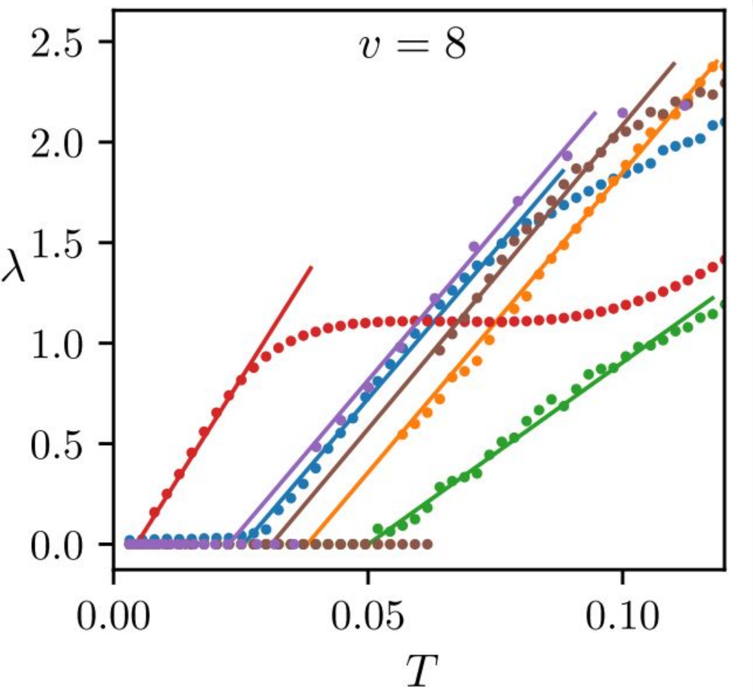
Lyapunov Experiment Results

Temperature Dependence of λ in Weakly Interacting Regime



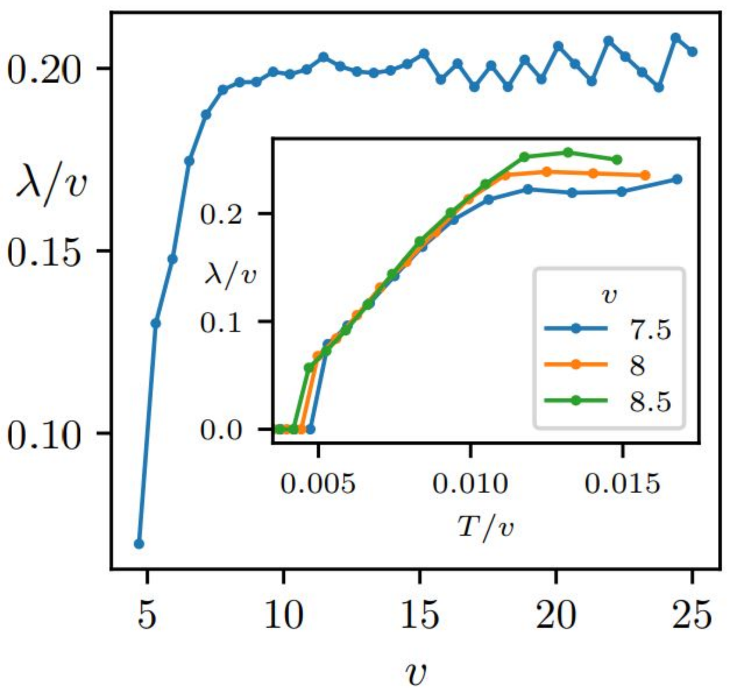
- Regime where interactions between different spins are weak ($v/2 < u$)
- **Quadratic** relationship between temperature and λ
- λ is non-zero for systems with even numbers of spin
  - tends towards zero as N goes to infinity

Temperature Dependence of λ in Strongly Interacting Regime



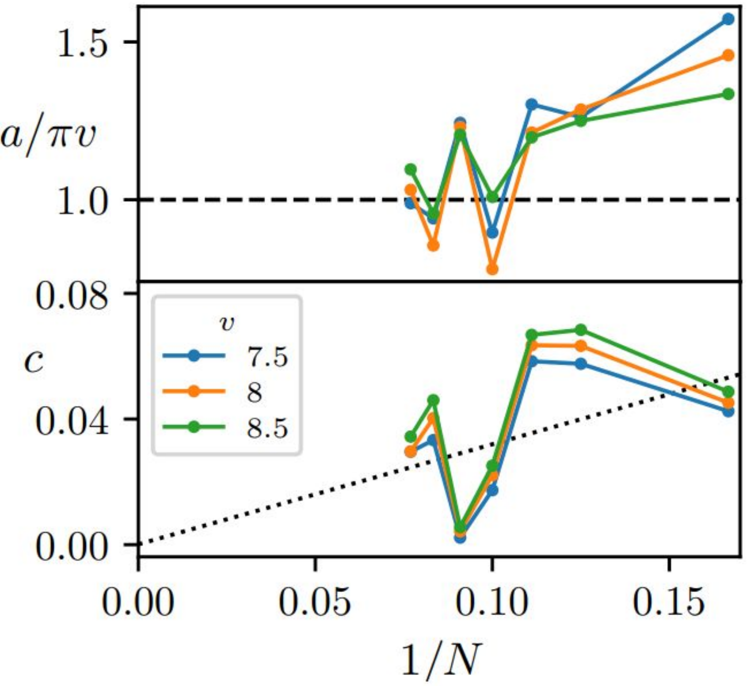
- Regime where interactions between different spins are strong (v/2 > u)
- **Linear** relationship between temperature and λ
- Scrambling is proportional to $e^{\lambda(T)}$
  - Since λ ~T, then scrambling in the strongly interacting regime is exponential (which we want)
- Agrees with prior SYK Model Studies[3]

Coupling Dependence of λ in Strongly Interacting Regime



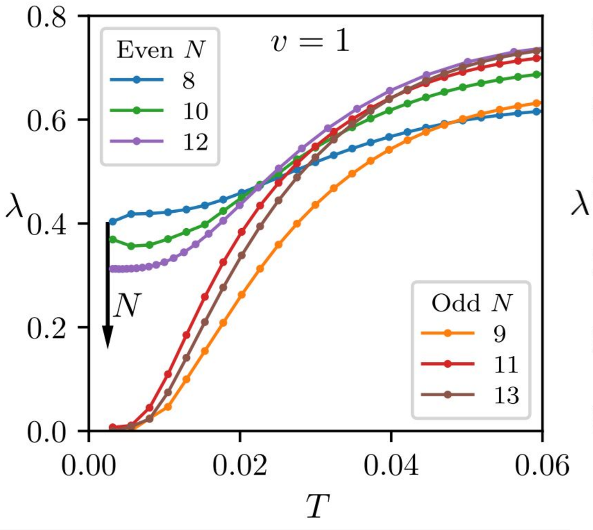
- We want to study the strongly interacting regime to understand optimal scrambling
- Question: Is coupling directly proportional to λ
- λ plateaus after reaching a sufficiently coupled spin chain
  - plateaus much faster than prior SYK model studies[4]

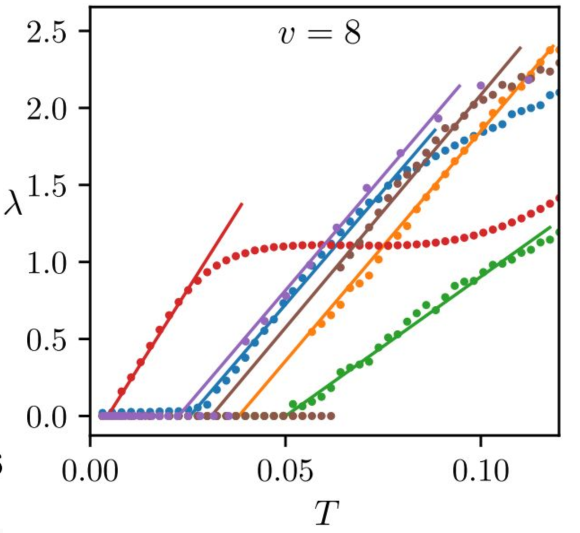
Further Analysis of λ in Strong Coupling Regime



- Linear fit of λ:
  - λ = a(T - c) where c is offset from zero
- Two things to note:
  - Oscillatory behavior shows boundary condition dependence given strong coupling
  - The rough linear fit for c demonstrates that offset increases as N decreases
- Motivates further studies of strongly coupled spin chain inside black holes

Coupling and Phase Transition





- The main takeaway is that there is a quantum phase transition going from the weakly to strongly interacting spin chain system
- This is directly seen through the distinctive change in thermalisation properties of the system

Summary and Conclusions

Summary - Methods

- Applied OTOCs to a model of a Chiral Spin-Chain
- Exponential decay of OTOCs yields Lyapunov exponents
- Exponents quantify rate of Chaos

$$H = \frac{1}{2}\sum_{i=1}^{N}\left[-u\left(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y\right) + \frac{v}{2}\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1} \times \boldsymbol{S}_{i+2}\right]$$

$$C(t) = U\left(\frac{1}{2}, 1, Ne^{-\lambda t}\right)\sqrt{N}e^{-\lambda t/2}$$

Summary - Conclusions

- When **v/2>u** → optimal scrambling → Behaves like a Black Hole!
- Experimentally Feasible → Can be a proxy to study BH properties
- Other future possibilities:
  - Theoretical derivation of λ
  - Phase transition **v/2≅u**
  - Higher dimensions

Citation Report

- Published in April 2024, but zero citations
  - Relatively recent publication
  - Not a big advancement compared to main references

Chiral Spin-Chain Interfaces Exhibiting Event-Horizon Physics

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Exploring interacting chiral spin chains in terms of black hole physics

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Thank you!

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