Entanglement Swapping using Sum-Frequency Generation

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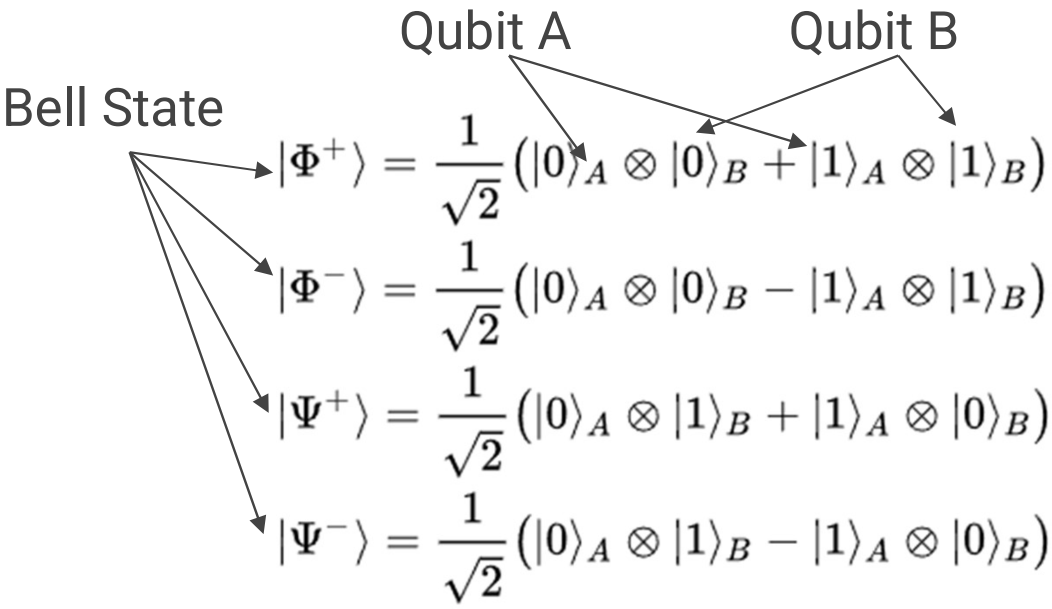
N. Sangouard, B. Sanguinetti, N. Curtz, N. Gisin, R. Thew, and H. Zbinden, Faithful entanglement swapping based on sum-frequency generation, Phys. Rev. Lett. 106, 120403 (2011).

What is a Bell state?

Maximally entangled states of two qubits

Four states form a basis



$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B\right)$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A \otimes |0\rangle_B - |1\rangle_A \otimes |1\rangle_B\right)$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A \otimes |1\rangle_B + |1\rangle_A \otimes |0\rangle_B\right)$$

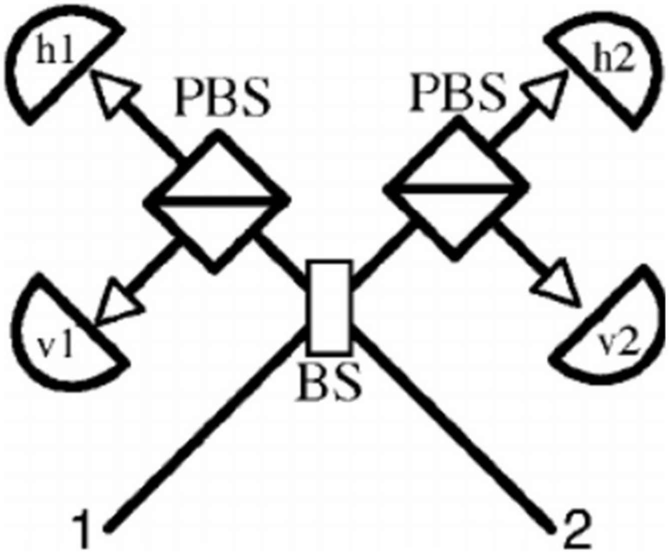
$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |0\rangle_B\right)$$

Bell State Measurements

Entangled photons measurement
PBS: Polarized Beam Splitter
BS: Beam Splitter

Determines which Bell state a system is in

Typically performed using beam splitters, phase shifters, and single-photon detectors.



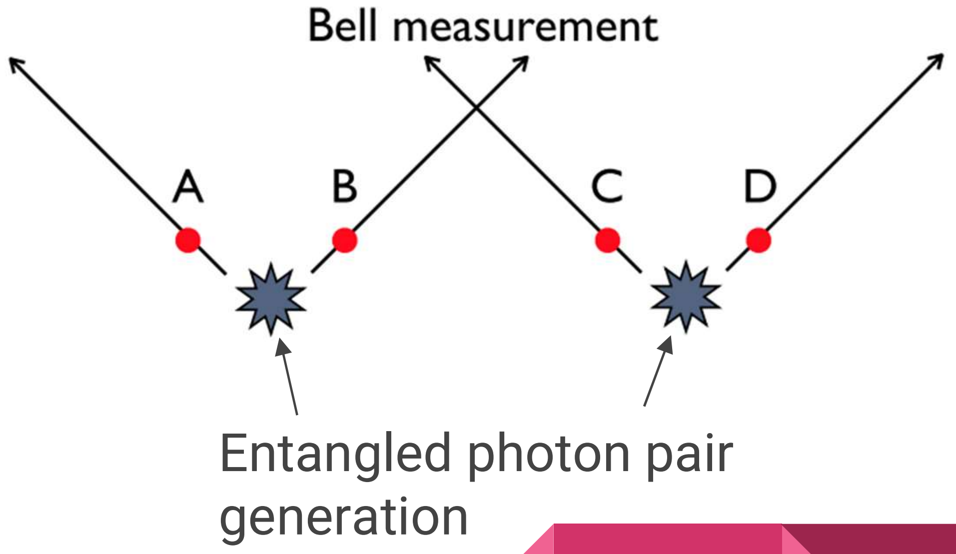
Humble, Travis & Grice, Warren. (2006). Spectral effects in quantum teleportation. Physical Review A. 75. 10.1103/PHYSREVA.75.022307.

Entanglement Swapping of Photons

Start: Two entangled photon pairs (A&B, C&D)

Apply Bell State Measurement on photons B & C

Outcome: Photons A & D become entangled



Time Bin Entanglement

Qubits represented by photon are based on detection time

Early or Late arrival

Pairs are time-bin entangled as they are generated simultaneously

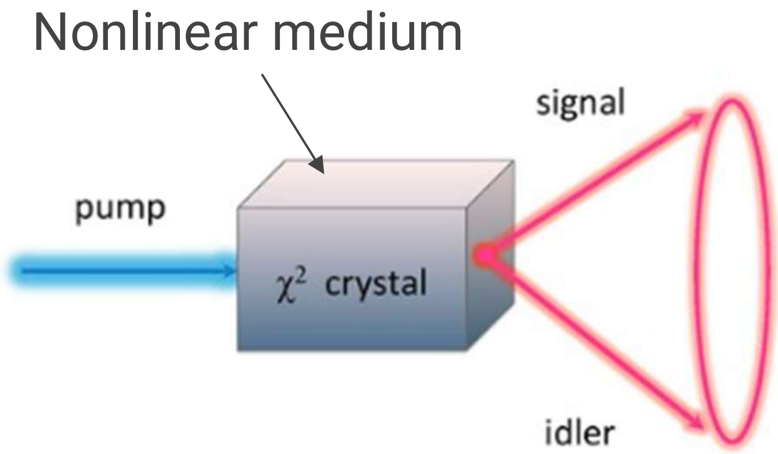
Bell state for time bin entangled photons

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|E\rangle_S\,|E\rangle_I \pm |L\rangle_S\,|L\rangle_I\right)$$

E: Early, L: Late

Bracht, Thomas K., et al. ‘Theory of Time-Bin Entangled Photons from Quantum Emitters’. arXiv [Quant-Ph], 2024, http://arxiv.org/abs/2404.08348. arXiv.

Spontaneous Parametric Down Conversion (SPDC)



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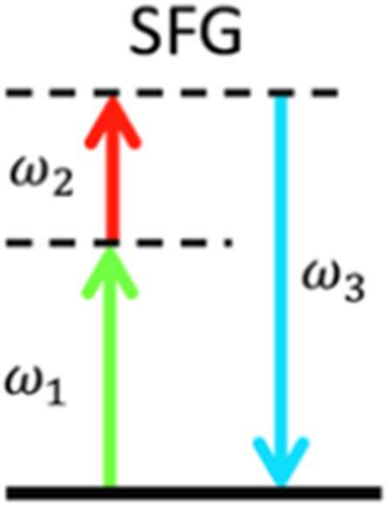
Nonlinear optical process

High-energy photon passing through a nonlinear crystal is split into two lower-energy photons

Process conserves energy and momentum

Efficiency ~ 10^{-6}

Sum Frequency Generation



Mobini, Ehsan & Espinosa, Daniel & Vyas, Kaustubh & Dolgaleva, Ksenia. (2022). AlGaAs Nonlinear Integrated Photonics. Micromachines. 13. 991. 10.3390/mi13070991.

Another nonlinear phenomenon

Two photons interact in a nonlinear medium to produce a single photon

Also conserves energy, which means that the energies of the photons add

Post Selection in Quantum Mechanics

Blasiak, Pawel, et al. "On safe post-selection for Bell tests with ideal detectors: Causal diagram approach." *Quantum* 5 (2021): 575.

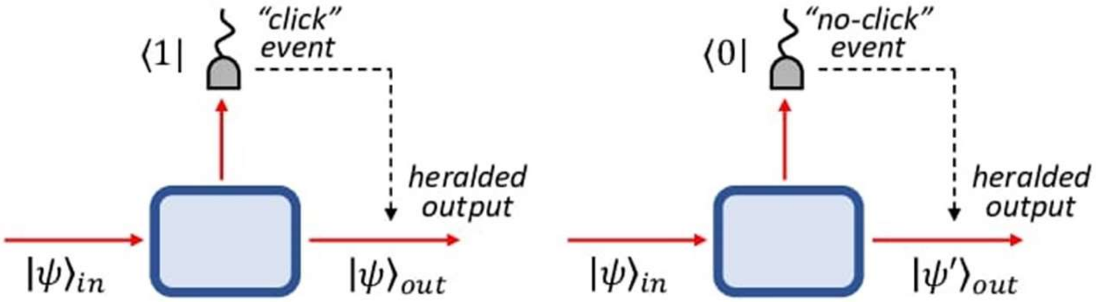
Specific subset of outcomes from an experiment are kept

May be used to detect quantum entanglement

Entanglement is inferred by discarding all events except those that meet specific criteria

Very inefficient

Event Ready Entangled Quantum States



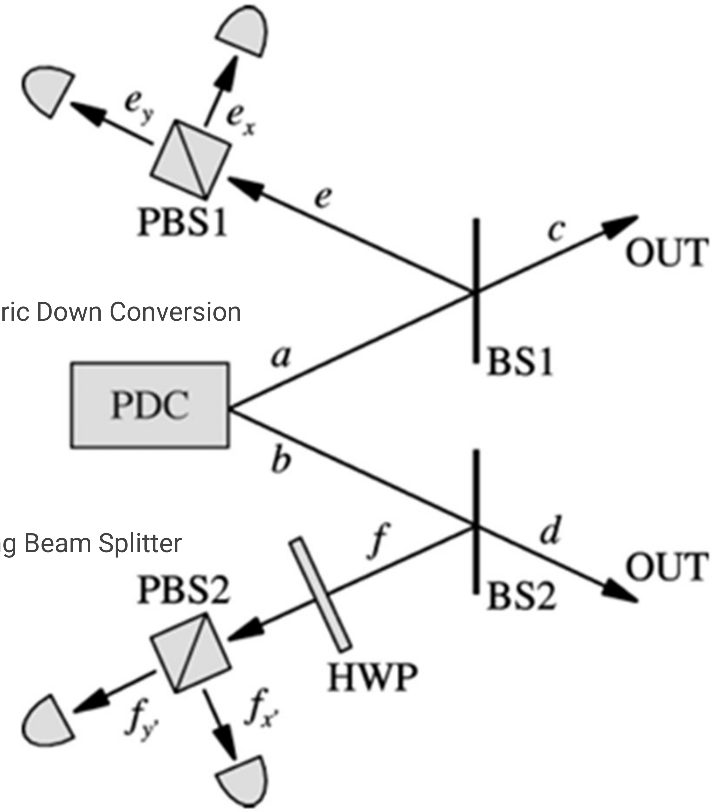
https://www.researchgate.net/figure/Comparison-of-quantum-state-engineering-via-a-conventional-heralding-on-the-detection_fig1_353117045

Post selection can only measure entangled states after the fact

In event-ready states, the entanglement is confirmed by a heralding event

One mechanism of generating event ready entangled states is through entanglement swapping

Heralded generation of entangled photon pairs



Parametric Down Conversion

Polarizing Beam Splitter

3 pairs of entangled photons

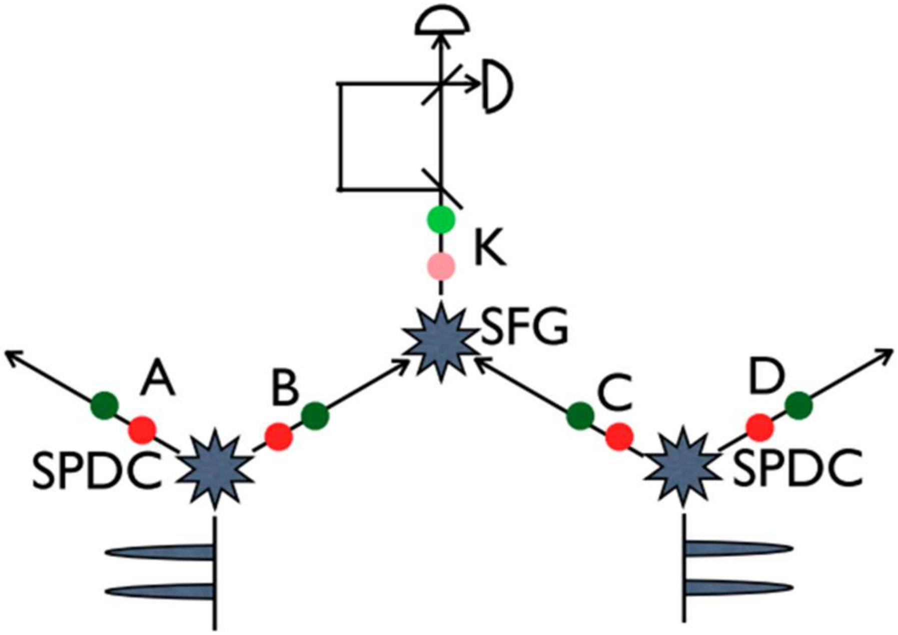
Requires 4 simultaneous detections on Detectors for PBS1 & PBS2

Photons C & D entangled

$$|\Psi_3\rangle = \frac{1}{2}\Big(|HHH\rangle_{\mathrm{a}}|VVV\rangle_{\mathrm{b}} - |HHV\rangle_{\mathrm{a}}|VVH\rangle_{\mathrm{b}} + |VVH\rangle_{\mathrm{a}}|HHV\rangle_{\mathrm{b}} - |VVV\rangle_{\mathrm{a}}|HHH\rangle_{\mathrm{b}}\Big)$$

Śliwa, Cezary, and Konrad Banaszek. "Conditional preparation of maximal polarization entanglement." *Physical Review A* 67.3 (2003): 030101.

Proposed Experimental Setup within Paper



- Two independent SPDC sources generate entangled photon pairs (A-B, C-D).
- Photons B and C are directed into a nonlinear crystal, where SFG occurs, producing a single sum-frequency photon.
- An interferometer is used to erase the which-path information of the photons.

SFG: sum frequency generation
SPDC: spontaneous parametric down conversion

Efficiency of Signal Frequency Generation

$$\eta_{\mathrm{SFG}}^{\mathrm{th}} = \frac{\hat{\eta}\hat{\Delta}v h v L}{\mathrm{tbp}}$$

Experimental Parameters:

$\hat{\Delta}v = 300\mathrm{GHz}\cdot\mathrm{cm}$ → spectral acceptance of crystal

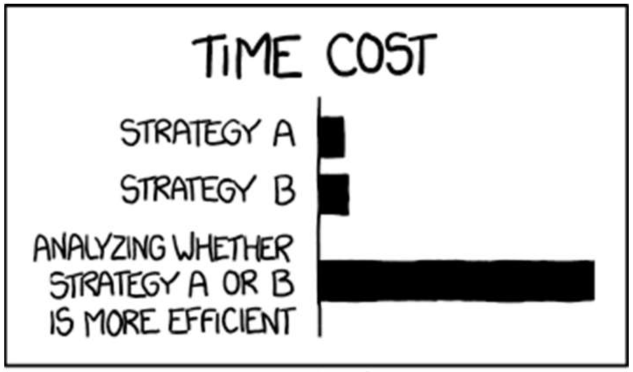
$\hat{\eta} = 15\%/(\mathrm{W}\cdot\mathrm{cm}^2)$ → normalized SFG efficiency

$\mathrm{tbp} = 0.66$ → time bandwidth product

$L = 2.6\mathrm{cm}$ → crystal length

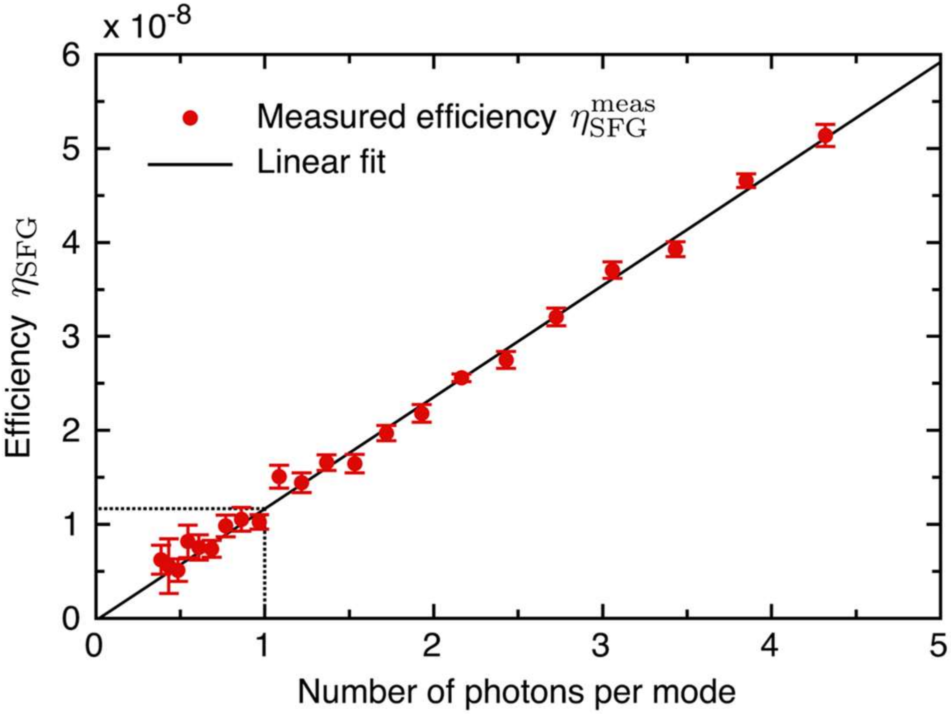
$\lambda_{\mathrm{pump}} = 1557\mathrm{nm}$ → pump wavelength

$\eta_{\mathrm{SFG}}^{\mathrm{th}} \approx 1\times 10^{-8}$ → Expected Efficiency



THE REASON I AM SO INEFFICIENT

Efficiency of SFG



Expected Efficiency : $\eta_{\mathrm{SFG}}^{\mathrm{th}} \approx 1 \times 10^{-8}$

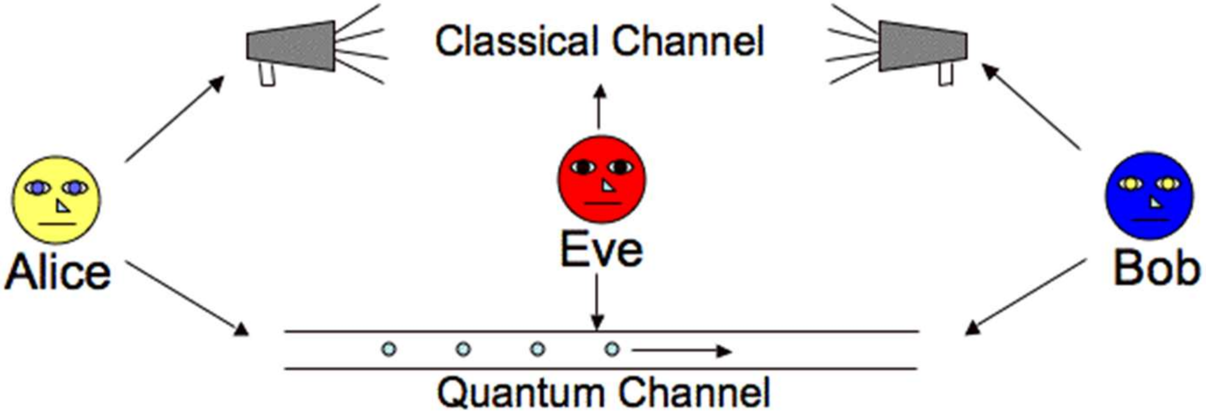
Measured Efficiency : $\eta_{\mathrm{SFG}}^{\mathrm{meas}} = 1.2(0.2) \times 10^{-8}$

With a more appropriate commercial waveguide
$L = 5\mathrm{cm}, \quad \hat{\eta} = 100\%/(\mathrm{W} \cdot \mathrm{cm}^2)$
Efficiency as high as $\eta_{\mathrm{SFG}}^{\mathrm{th}} \approx 1.5 \times 10^{-7}$
could be achieved

Applications: Quantum Key Distribution

Quantum Key Distribution is the method of sending the encrypted key from a sender(Alice) to a receiver(Bob)

Security of the Key relies on the no cloning theorem

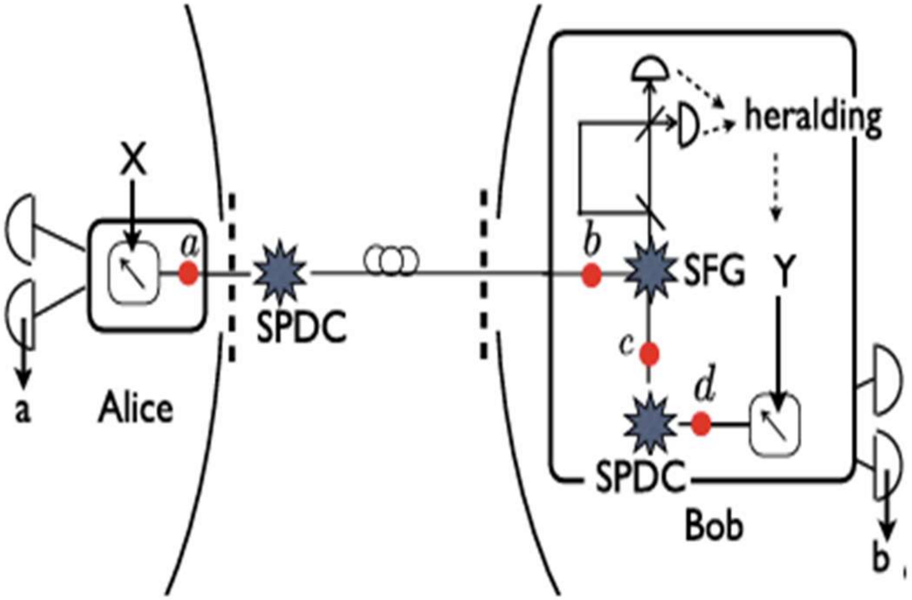


https://www.cse.wustl.edu/~jain/cse571-07/ftp/quantum/

Detection Loophole: The security of the system is compromised when not all photons sent by alice are detected by bob. This occurs when the total loss >20% by the CHSH inequality

Losses occur due to transmission loss and detector loss. Transmission loss limits how far apart Alice and Bob can be

Advantage of SFGs for Quantum Key Distribution



In this setup Bob generates an entangled pair and passes one photon and Alice’s photon to the crystal

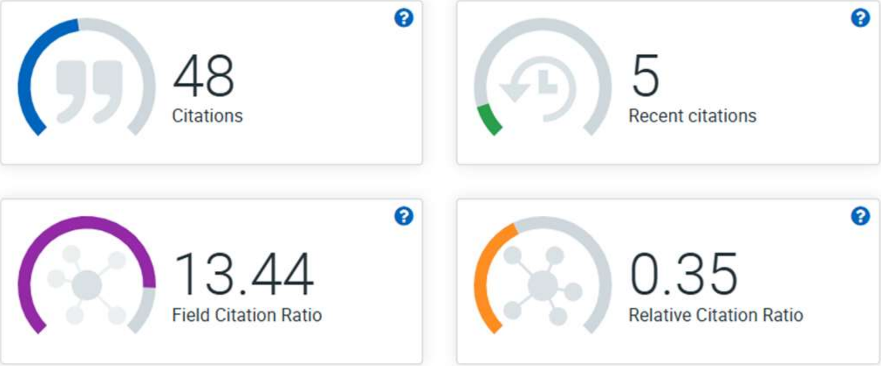
The state of the second photon is only measured when the heralding process occurs.

This helps overcome transmission losses and allows for key distribution over longer distances

Citation Analysis

Cited 48 times

This is considered highly cited in the field



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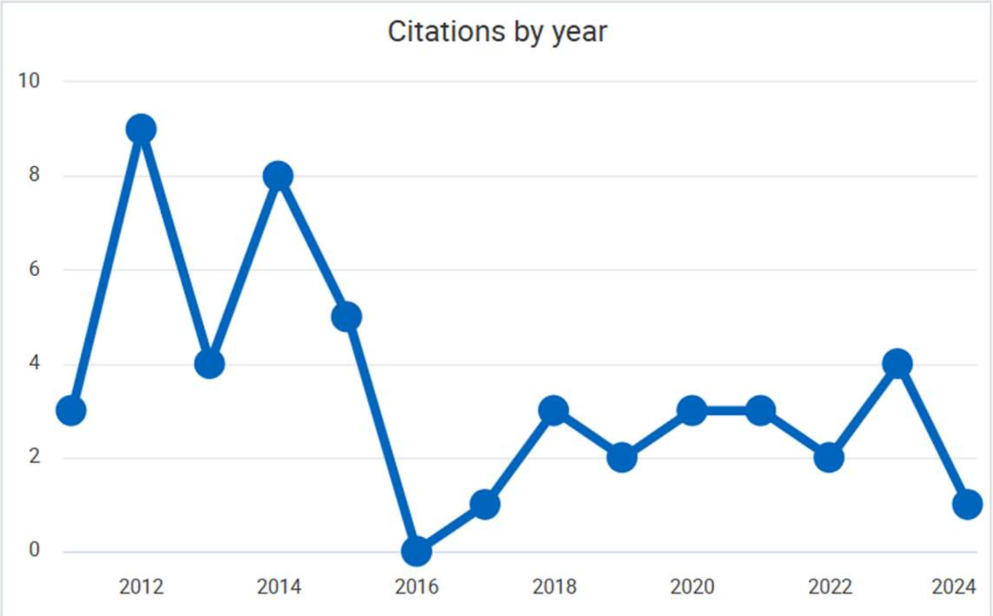
https://badge.dimensions.ai/details/id/pub.1006643302



Critical analysis

Pros

Highlighted how heralding using SFGs is more efficient than 4-four fold coincidence detection using SPDCs only

Discussed applications of entanglement swapping and heralding

Good use of diagrams to highlight a proposed an experimental setup

Cons

Incomplete or missing derivations for some of the theoretical results on fidelity and probabilities of the created entangled states

Supplemental appendices should be added to the paper to allow for better reader understanding

Thank you!

We would like to specifically recognize and appreciate Professor Lance for his feedback and contribution