Do Black Holes have Singularities?

Team 2: Autumn Bauman, Jude Bedessem, Genessa Benton, Nathaniel Bowden, Brook Burbridge

R. P. Kerr. “Do Black Holes have Singularities?” 2023. arXiv: 2312.00841 [gr-gc]. URL: arxiv.org/abs/2312.00841

Do Black Holes have Singularities?

Team 2: Autumn Bauman, Jude Bedessem, Genessa Benton, Nathaniel Bowden, Brook Burbridge

Do Black Holes have Singularities?

R. P. Kerr

University of Canterbury, Christchurch
Lifschitz Prof. ICRANet, Pescara

December 5, 2023

General Relativity (GR) Basics

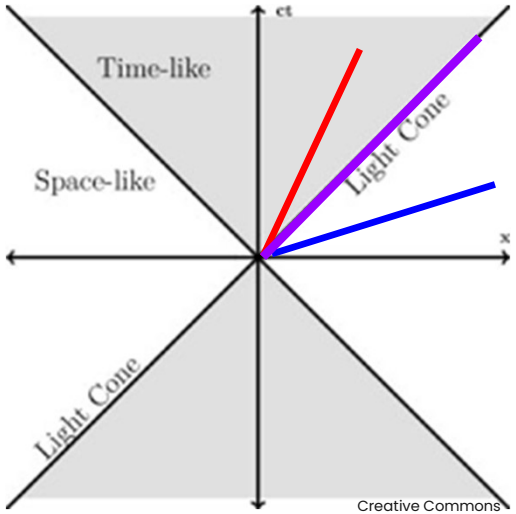
How mass curves spacetime, and how objects move in curved spacetime

$$g_{\mu\nu}^{\text{Minkowski}} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Metric - How spacetime is curved, how to transform from one coordinate system to another

Geodesic - 'Straight lines' through a curved spacetime

- Time-like: A curve where events are causally linked
- Space-like: Spacetime events separated by more than $\Delta x = c\Delta t$
- Null curves: Curves where $x = ct$, light rays

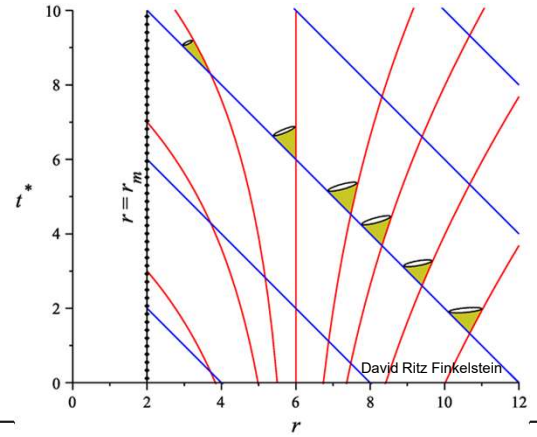


Schwarzschild Metric

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) c^2 dv^2 + 2c\, dv\, dr + r^2 d\Omega^2$$

$$v = t + r + 2GM \ln|r - 2GM|$$



First and most basic solution to Einstein's field equations

Singularity: Where geodesic curves terminate

Two singularities:

- R = 0: "Real" singularity, cannot be coordinate-transformed away, similar to Coulomb potential
- R = 2GM/c^2: Coordinate singularity! Can be removed via a coordinate transformation

Schwarzschild Metric: Most basic solution to Einstein's field equations

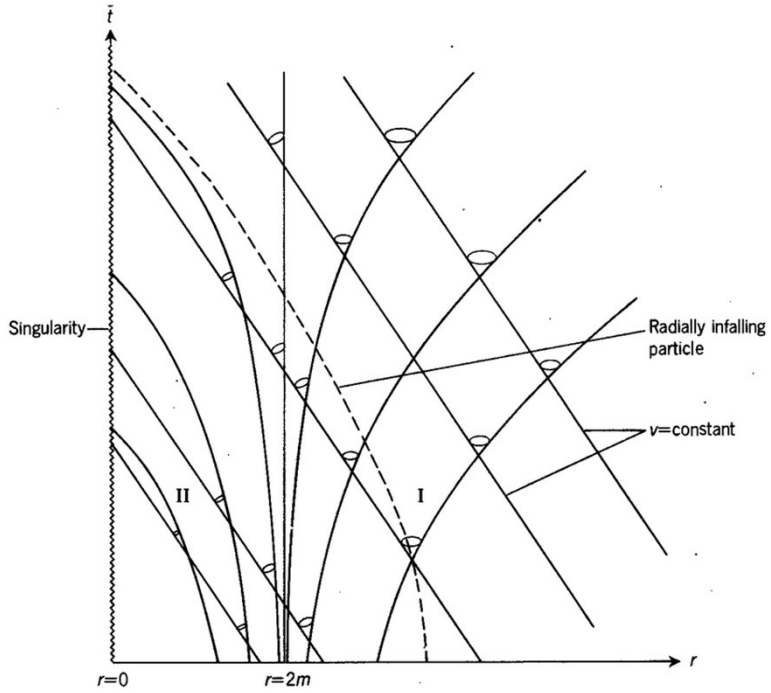
$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) c^2 dv^2 + 2c\, dv\, dr + r^2 d\Omega^2$$

Singularity: Where geodesic curves terminate

Two singularities:

- R = 0: "Real" singularity, cannot be coordinate-transformed away, similar to Coulomb potential
- R = 2GM/c^2: Coordinate singularity! Can be removed via a coordinate transformation



Eugene Kogan

Affine Length - Distance Along a Geodesic

- Time-like: Gives proper time τ, which is the time the object following the geodesic measures
- Space-like: More like an arc-length for how far the curve has progressed
- Null: Characterizes how far along the geodesic the object is

FALL: Finite affine length light (ray)

Null geodesics that terminate!

How could a light ray have a finite affine length?
Penrose + Hawking argue only singularities!

Penrose-Hawking Singularity Theorems

Geodesic incompleteness: a condition where all light and particle-like geodesics cannot be extended beyond a certain proper time or affine length, leading to FALLs.

The Penrose theorem argued that FALLs occur whenever:

- Gravity is attractive, never repulsive
- Mass is non-negative

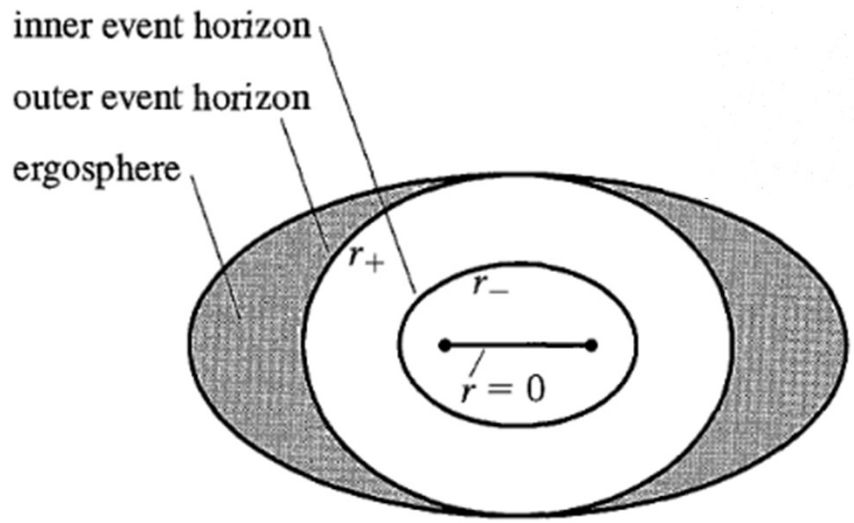
The Hawking theorem argued similarly, but for the whole universe. That the Big Bang had infinity density and was a singularity, with similar (but more stringent) energy conditions.

Roy Kerr finds issue with these proofs, and asks in his 2023 paper with the same name,
Do Black Holes have Singularities?

Kerr Metric

Variation in geometry from Schwarzschild black hole:

- **Ring Singularity**
- **Inner and Outer Horizon**
- **Ergosphere**



S. Carroll, Spacetime and Geometry (2014)

Kerr Metric: describes a black hole w/ angular momentum

$$ds^2 = -\left(1 - \frac{2GMr}{\rho^2}\right)dt^2 - \frac{2GMar \cdot \sin^2\theta}{\rho^2}(dtd\vartheta + d\vartheta dt)$$

$$+ \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \frac{\sin^2\theta}{\rho^2}\left[\left(r^2 + a^2\right)^2 - a^2\Delta\sin^2\theta\right]d\vartheta^2$$

$$\Delta(r) = r^2 - 2GMr + a^2$$

$$\rho^2(r, \theta) = r^2 + a^2\cos^2\theta$$

Kerr Metric: describes a black hole w/ angular momentum

$$ds^2 = -\left(1 - \frac{2GMr}{\rho^2}\right)dt^2 - \frac{2GMar \cdot \sin^2\theta}{\rho^2}(dtd\vartheta + d\vartheta dt)$$

$$+ \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \frac{\sin^2\theta}{\rho^2}\left[\left(r^2 + a^2\right)^2 - a^2\Delta\sin^2\theta\right]d\vartheta^2$$

$$\Delta(r) = 0 \rightarrow r_{+/-} = GM \,{}^{+}\!/_{-}\, \sqrt{G^2M^2a^2}$$

$$\rho^2(r, \theta) = 0 \rightarrow r = 0 \;\; AND \;\; \theta = \frac{\pi}{2}$$

Kerr Metric

Is the outer horizon where all geodesics become null?

Norm of the time-translation killing vector:

$$K^{\mu}K_{\mu} = -\frac{1}{\rho^2}\left(\Delta - a^2 sin^2\theta\right)$$

Plug in solution for outer horizon r_+ :

$$K_{\mu}K^{\mu} = \frac{a^2}{\rho^2}\sin^2\theta \geq 0$$ **(Spacelike)**

Solution to $K^{\mu}K_{\mu} = 0$:

$$r = GM + \sqrt{G^2M^2 - a^2\cos^2\theta}$$ **(Ergosphere)**

Argument for Singularity

Affine Parameter: a path that measures progress along a geodesic, preserving its intrinsic properties

Affine Length: distance along a curve based on an affine parameter

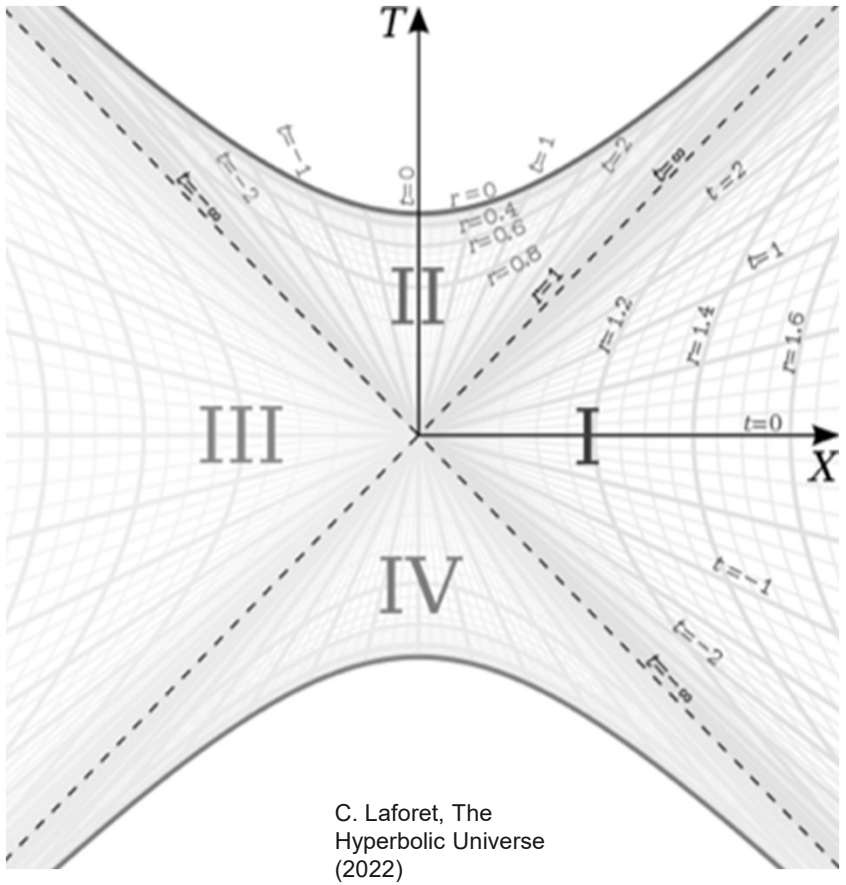
Finite Affine Length (FAL): light rays terminate at a finite value (e.g. singularity)

Trapped Surface: Boundary where nothing physical escapes

Argument for Singularity

According to Penrose-Hawking...

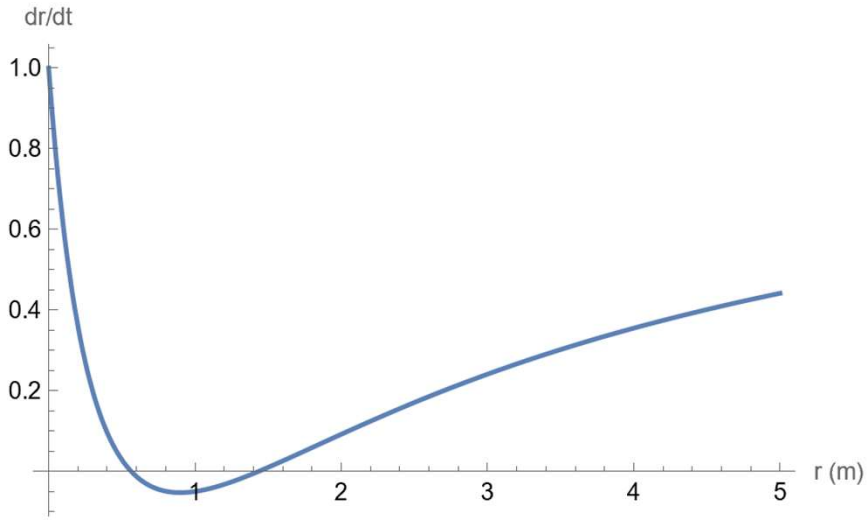
1. Light beyond trapped surface has a **FALL**
2. The **FALL** terminates <u>at</u> the singularity
3. Particles between inner and outer horizons fall to singularity



C. Laforet, The Hyperbolic Universe (2022)

Kerr's Counterexample

- There exists:
  - Families of light rays through every point,
  - Bounded affine parameter or finite affine length,
  - Don't end on the ring singularity,
- Axial ray
  - Ingoing: $\frac{dr}{dt} = -1.$
  - Outgoing: $\frac{dr}{dt} = \frac{r^2 - 2mr + a^2}{r^2 + 2mr + a^2}$



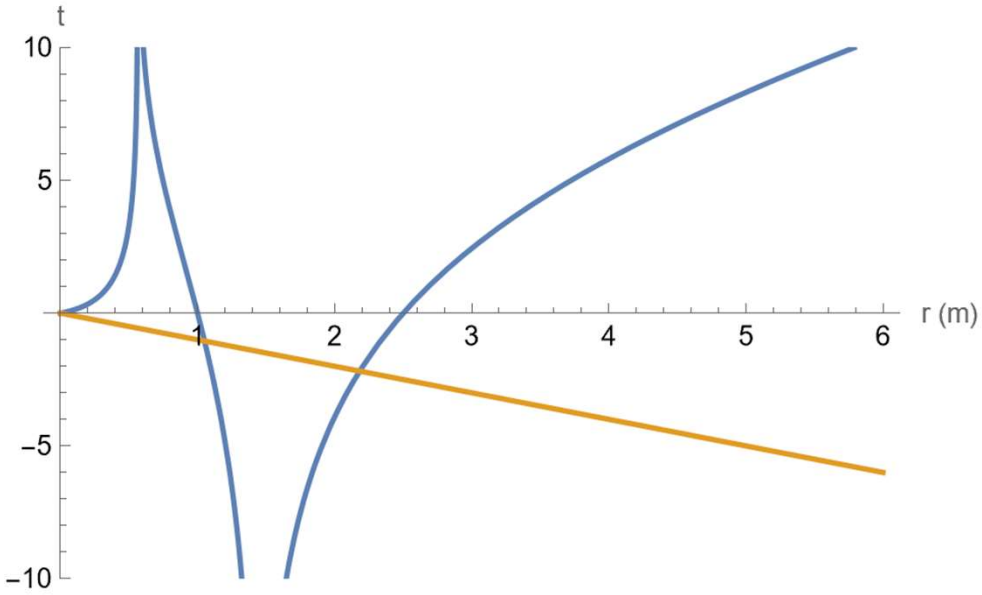
Outgoing Slope for a=0.9. Units of r are m. Generated in Mathematica

Kerr's Counterexample

- Axial ray
  - **Ingoing**: $\frac{dr}{dt} = -1.$
  - ***Outgoing***: $\frac{dr}{dt} = \frac{r^2 - 2mr + a^2}{r^2 + 2mr + a^2}$
- **Outgoing** Light ray between horizons has finite length:

$$2\sqrt{m^2 - a^2}$$

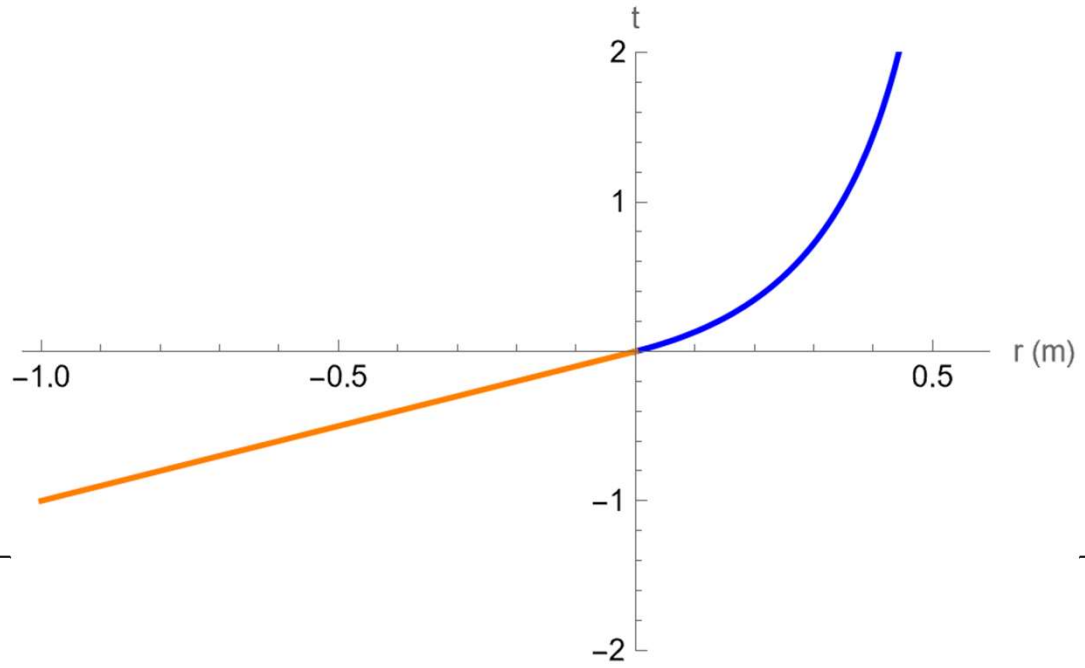
- It does not end at the singularity, contradicting the theorems



Light-lines for a=0.9. **Blue are for outgoing**, **orange for ingoing**. Kerr-Schild coordinates Generated in Mathematica.

What about ingoing light?

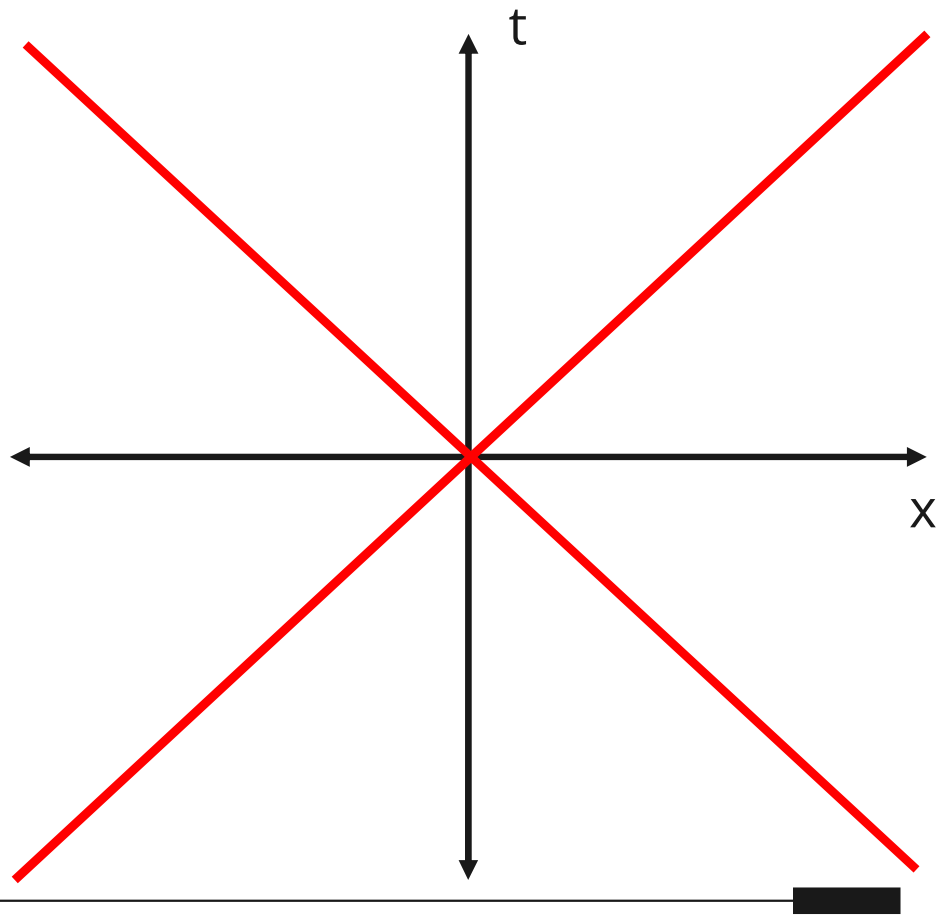
- Kerr metric: ingoing ray goes to a nonphysical branch of spacetime
- Physical solution (a real collapsed star): ingoing and outgoing may connect



Criticisms

Physical Criticism

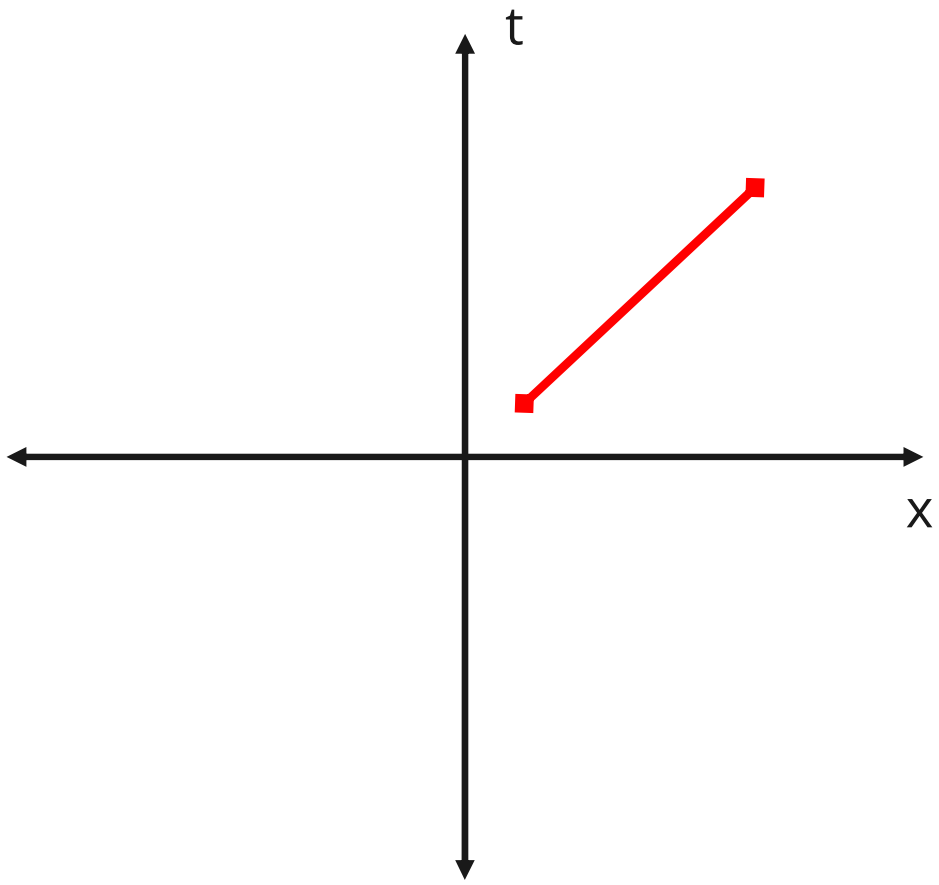
- Inextensible Geodesics example



Criticisms

Physical Criticism

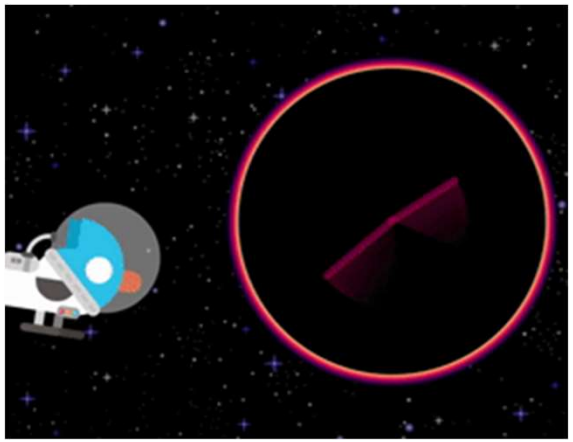
- Inextensible Geodesics example
- Extensible FALL example



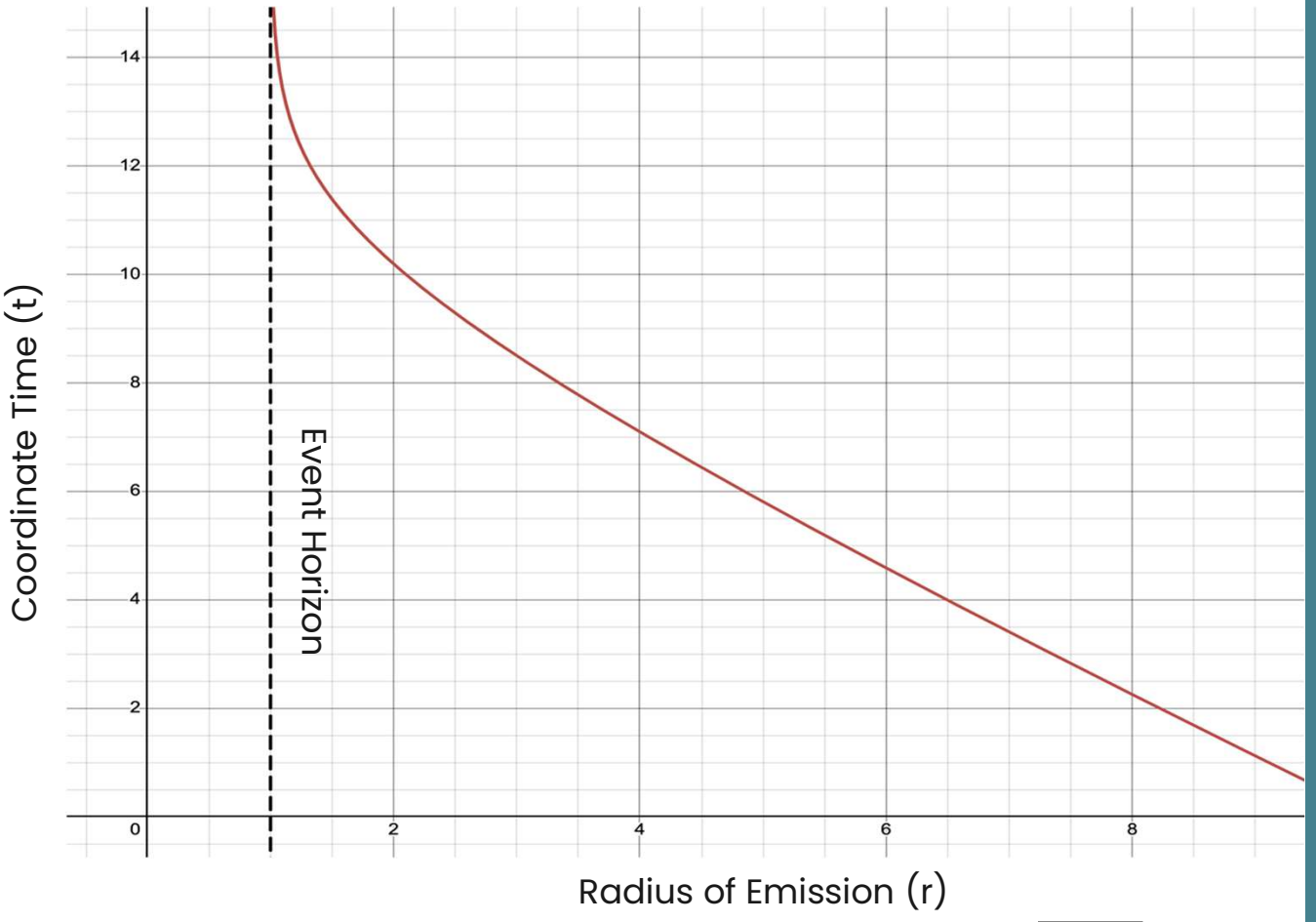
Criticisms

Schwarzschild Analogy

- Stationary observer thinks it takes infinite time for object to fall into a black hole



GIF courtesy Kurzgesagt



Slide 19

1 So, I think the Kerr-Schild coordinates used do not have coordinate singularities and are the analog of the Eddington coordinates for a non-rotating BH (pg. 11h-12a)

He also talks about Boyer Lindquist coordinates (p. 14a) which have coordinate singularities like this Schwarzschild issue, so I think this is not a problem

Nathaniel Bowden, 12/9/2024

1 He says that it reduces to Eddington coordinates as a->0, but it looks like there is a coordinate singularity just from your plots. Either way, I'm not going to claim that there is a coordinate singularity, just that it's not immediately obvious that the geodesic is inextensible.

The Boyer Lindquist extension is related to the grafting of Kerr spacetimes across the singularity. His argument for why these paths need to terminate seems to be a physical one, not a mathematical one? I was planning to say that it's not clear if taking the extended geodesic inside the inner horizon, and measuring its affine length through the negative space past the singularity is meaningful in the context of the singularity theorems.

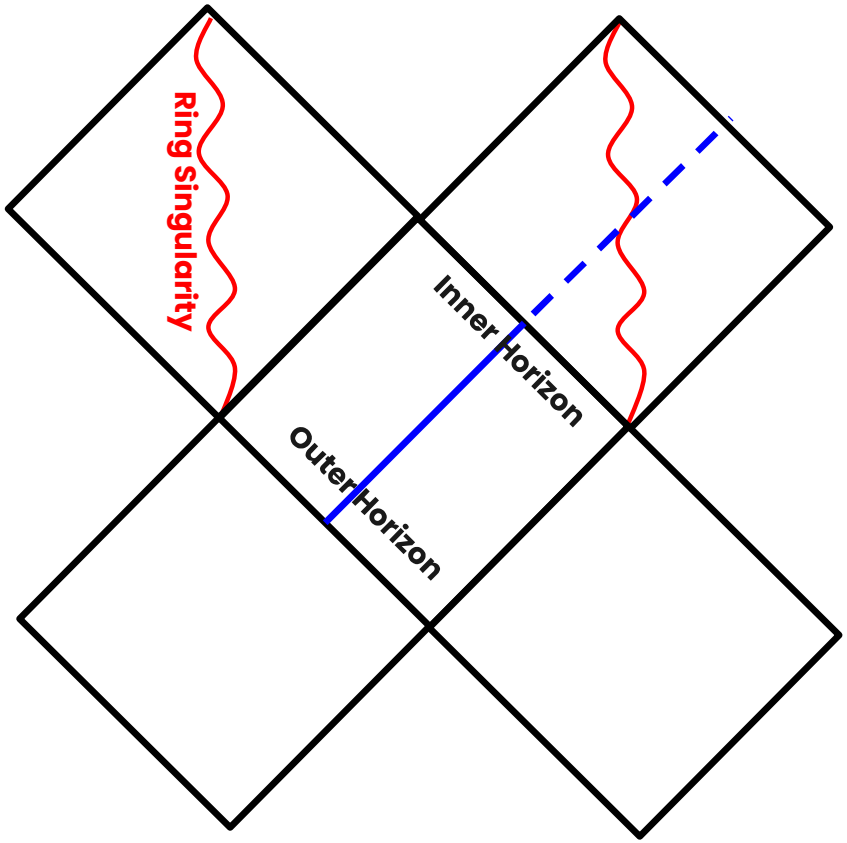
He doesn't really address either of these things.

Jude Bedessem, 12/9/2024

Criticisms

Physical Criticism

- In trapped region FALL is extendible
- Non-physical space beyond singularity
- Unclear which Kerr argues is his contradiction



Criticisms

Penrose and Stephen Hawking[2] then asserted that these must end in actual singularities. When they could not prove this they decreed it to be self evident. It is shown that there are counterexamples through

Nobody has constructed any reason, let alone proof for this. The singularity believers need to show why it is true, not just quote the Penrose assumption.

This has not been done and so all the proofs of the various singularity theorems are incomplete. they always were since nobody could prove that FALL's imply singularities.

Criticisms

STARTS WITH A BANG — DECEMBER 5, 2023

"Singularities don't exist," claims black hole pioneer Roy Kerr

The brilliant mind who discovered the spacetime solution f
physically exist. Is he right?

Physics Forums INSIGHTS BLOG FORUMS

Forums › Physics › Special and General Relativity ›

Kerr disputes singularities in Kerr Black Holes

Dec 5, 2023

e article discusses the challenges to the conventi
re rotating black holes described by the Kerr solut
vancements suggesting that singularities may not
dels that could reshape our understanding of blac

dvisor Homework Helper

:3,522
:1,633

r 1963, Roy Kerr described the geometry of uncharged rotating black holes (ie, Kerr Black
Kerr geometry included (or was presumed to include) a feature called a ring singularity.

parently staring at the blackboard for 60 years, Roy just got up and drew a large and emphatic
question mark over that ring singularity thing.

In a 20 page article published on ResearchGate, Roy states:

The consensus view for sixty years has been that all black holes have singularities. There is no direct proof of this

Black Hole Singularities "Faith, not science!" Prominent Physicist Claims

New Research By Roy Kerr Suggests Black Holes Doesn't Have Singularity

Citation Analysis (Published)

Cited in 20 publications and preprints

Most citing works reference it as casting doubt on singularity existence

“We should note though that certain singularity theorems do not actually guarantee that a singularity will occur”
Some Singular Spacetimes and Their Possible Alternatives DOI:10.3390/particles7040054

“In our specific scenario, there is no presence of a central singularity … It might be argued that the presence of singularity is uncertain, as has been shown very recently by Kerr, where he demonstrates the lack of evidence supporting the existence of singularities within black holes formed by real physical spacetime.”
Geodesic Structure of Generalized Vaidya Spacetime through the K-Essence DOI:10.3390/universe9120510

Citation Analysis (Preprints)

Remove point-mass concept - remove singularities from GR argues that point masses are assumed in GR and by modifying this concept singularities could be eliminated

"If acceptable to Roy, the present paper and those of [31] and [33] could be adequate cases."

Black Holes and Baryon Number Violation: Unveiling the Origins of Early Galaxies and the Low-Mass Gap argue a modification of the Higgs field could help explain baryon number violation

"Much of the scientific community has been hesitant to abandon the traditional concept of BH singularities (…) However, this perspective may be shifting in light of Roy Kerr's recent paper"
arXiv:2407.18165

Others are more skeptical, singularities still occur and challenge Kerr's argument in a paper proposing analog simulation of a singularity in a Bose-Einstein condensate

"We here probe [...] a potential Penrose-type singularity [...] For clarity, we should emphasize here that the Penrose singularity theorem is not a statement about the divergence of curvature scalars"
Probing Penrose-type singularities in sonic black holes arXiv:2407.18165

Conclusion

Singularities were an early fascination in GR, some were removed but others stuck around

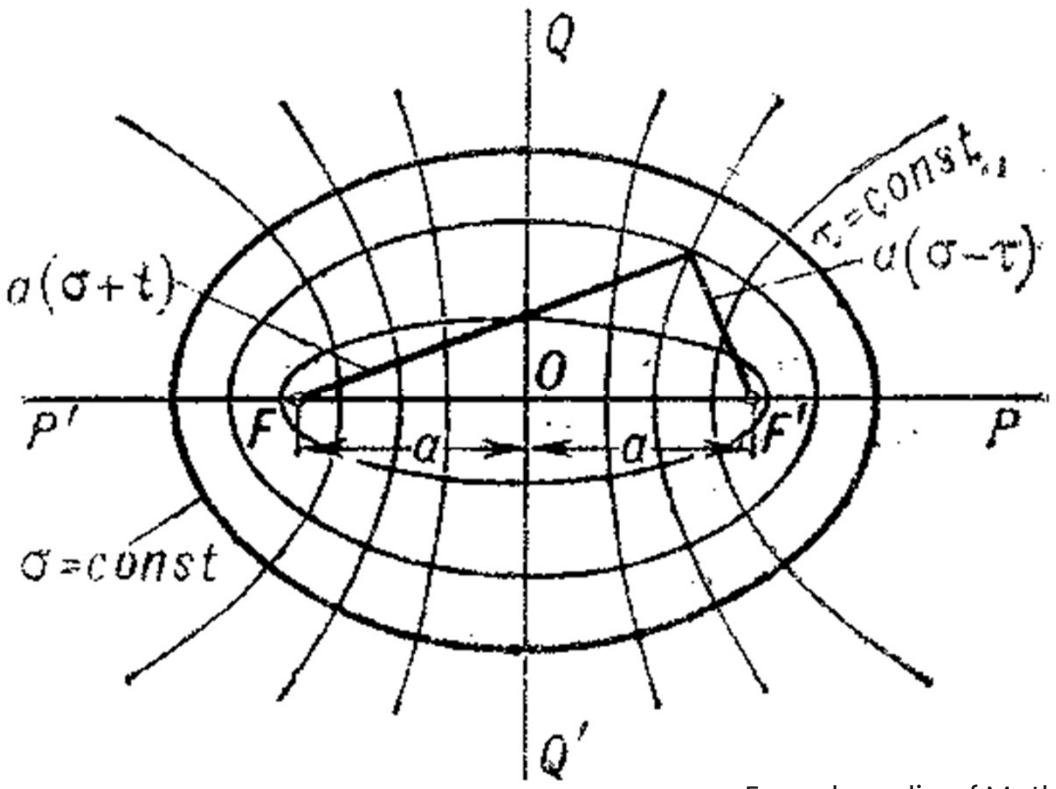
Penrose and Hawking showed that black holes imply the existence of FALLs which they assert must end in a singularity

Kerr provides counterexamples of FALLs that don't terminate in a singularity, however these counterexamples are not flawless

Reception of the paper has been mixed with some arguing that his argument is nonphysical while others use it as supporting evidence for alternative spacetime geometries and quantum gravity theories

Backup Slides

Elliptic Coordinates



Encyclopedia of Mathematics