

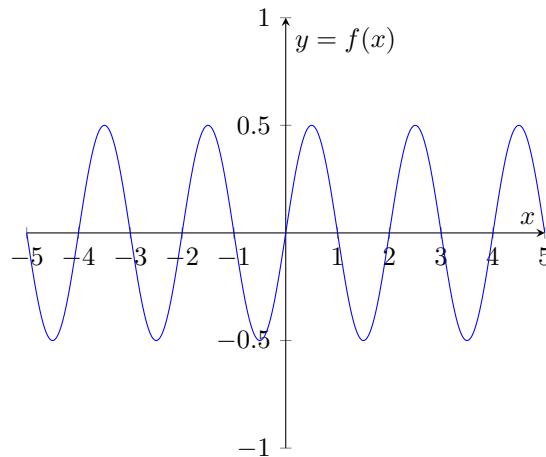
Some constants:

- Elementary charge q : 1.60218×10^{-19} Coulomb
- Boltzmann constant k : 1.38065×10^{-23} Joules / Kelvin

Problem 1 *Elementary operations and sinusoids*

10 points

Consider the following graph of $y = f(x)$. Plot the graphs of:

Figure 1: The graph of $y = f(x)$ for Problem 1

- (a) $z_1(x) = 2f(x/2)$
 (b) $z_2(x) = f(x)/z_1(x)$

Hint: Compare $f(x)$ to the general form of a sinusoid. You don't need to use software for this question.

Problem 2 *Complex numbers*

10 points

- (a) If $(x + iy)^{205} = 1$ then what is $(x + iy)^{204}$? Write the answer in terms of x and y in Cartesian form.
 (b) Let $z, w \in \mathbb{C}$. Recall that the complex conjugate of $z = x + iy$ is $\bar{z} = x - iy$. Using this fact, show that $|zw| = |z||w|$ where $|u|$ denotes the magnitude of the complex number $u \in \mathbb{C}$.

Hint: Consider squaring the equality you want to show and what it means to multiply a complex number with its conjugate.

Problem 3 *Euler Identity*

10 points

- (a) Is i^i a real number or a complex number? Explain your reasoning. If real provide a numerical value; if complex write it in Cartesian form.

- (b) Find the natural logarithm of $z = 1 + \sqrt{3}i$; that is find $\ln(z)$. Is this logarithm unique? If yes explain why, if not give another one.

Problem 4 *Complex numbers redux*

15 points

- (a) Use MATLAB to plot $(1 + i)^n$ on the complex plane for $n = 1, 2, \dots, 6$. Draw the x, y axis and center the plot on the origin.
- (b) Given $z = re^{i\theta}$ in polar form, derive using basic trigonometry the expression for x and y in terms of r and θ in the Cartesian form $z = x + iy$.

Problem 5 *Noise sources*

10 points

A 3 mA current flows through a diode (i.e a semiconductor) and a 20,000 Ω (i.e 20 k Ω) resistor. What is the net current noise, i_n in Amperes? Assume a bandwidth of 1 kHz (i.e. 1×10^3 Hz) and room temperature of 295 K. Which of the two components is responsible for producing the most noise?

Problem 6 *Logistic equation revisited*

15 points

Use MATLAB to evaluate the logistic equation

$$x_{n+1} = rx_n(1 - x_n)$$

for different values of r :

$$r = 1.25, \quad r = 2.25, \quad r = 3.2, \quad \text{and} \quad r = 3.6$$

Evaluate the first 50 generations (use a `for` loop to increment n from 1 to 50) and start with an initial value $x = 0.02$. Plot the population x as a function of generation n . Use `subplot` command to put the four plots together. Label the plots appropriately.

Problem 7 *Logarithms*

15 points

In class we showed that $\log_a(xy) = \log_a x + \log_a y$. Now show that

- (a) $\log_a(x^y) = y \log_a(x)$
- (b) $\log_a(x/y) = \log_a(x) - \log_a(y)$
- (c) For real x, y show that:

$$\int_1^{xy} \frac{1}{t} dt = \int_1^x \frac{1}{t} dt + \int_1^y \frac{1}{t} dt$$

Problem 8 *Plotting sinusoids*

15 points

Construct plots of a 2.5 Hz sine wave and 1.5 Hz cosine wave. Make the peak amplitude of both equal to 20 units. Use a 500-point array. Make the sampling frequency 250 Hz. Plot the two waveforms in different colors superimposed and label both axes. Also plot a zero center line.