Problem 1 *Phase difference of sine waves (See Ex. 2.6 & 2.7 of CSSB)* 15 points

(a) Suppose two 5-Hz sine waves have a phase difference of $\pi/3$ radians. What is the time delay t_d between them?
(b) Suppose we wish to double their frequency but maintain the same t_d as above. What should be the new phase difference to achieve this?
(c) Find the time delay between $x_1(t) = \cos(10t + 30)$ and $x_2(t)\sin(10t - 40)$. Note the phases are in *degrees*.

Problem 2 *RMS value of a sawtooth wave* 10 points

Calculate *analytically* the RMS value of triangle wave with period T, maximum amplitude V and rise time t_0 (see figure below: sawtooth wave is in blue).

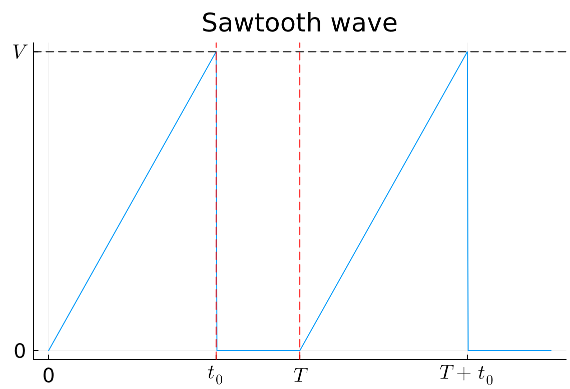

Figure 1: Figure for problem involving RMS of sawtooh wave.

Problem 3 *Plotting a sawtooth wave* 15 points

Using MATLAB (or Python) write function to take as input V, T and $t_0 < T$ and return a sawtooth wave like in Figure 1. Plot a sawtooth wave where:

- V is the value obtained by repeatedly summing the digits of your birthday till it is a single digit. For example, 07/04/1776 gives:

$$0 + 4 + 0 + 7 + 1 + 7 + 7 + 6 = 32 \implies 3 + 2 = 6$$

- T is the value obtained by repeatedly summing the digits of your UIN as above.
- t_0 is set to one-third of T.

Label and scale the axes appropriately to show features of the sawtooth wave (at least 2 cycles).

Problem 4 *Summing sinusoids* 10 points

(a) Convert $x(t) = -5\cos(5t) + 6\sin(5t)$ into a single sinusoid, i.e. $A\sin(5t + \theta)$
(b) Convert $x(t) = 3\sin(2t + \pi/3)$ into sine and cosine components.

Problem 5 *Complex exponentials* 10 points

In Example 2.10 of CSSB we find the following code:

```
1 % Example 2.10 Demonstration of the Euler Identity in MATLAB
2 clear all; close all;
3 fs = 500;                    % Sampling frequncy
4 t = (0:1/fs:1);              % Time vector
5 z = exp(-j*2*pi*t);          % Complex sinusiod
6 plot(t,real(z),'k'); hold on;
7 plot(t,imag(z),':k','LineWidth',2);
8 xlabel('Time (sec)', 'FontSize',14);
9 ylabel('y(t)','FontSize',14);
```

which is used to plot a sine and cosine wave. Appropriately modify lines 5–7 to get a single cosine with phase $\pi/4$ radians and amplitude 5 units. In particular, you still should use a complex exponential in line 5 but can remove irrelevant calls to plot in lines 6–7. Plot this wave. Remember to size and label your axes appropriately.

Problem 6 *Running statistics* 15 points

In class we discussed calculating the standard deviation of a set of samples $X := \{x_1, x_2, \ldots, x_{n-1}\}$ via the variance:

$$\sigma^2 := \frac{1}{n-2}\sum_{k=1}^{n-1}(x_k - \mu)^2, \qquad \mu := \frac{1}{n-1}\sum_{k=1}^{n-1} x_k \tag{1}$$

Suppose we added a new element x_{n+1} to X. To compute the new variance using (1) we would need to: recompute the mean, subtract this mean from every element, compute new squared deviations, sum them, ... This is grossly inefficient. Imagine having tens of millions of samples!

Manipulate the definitions in (1) to provide a new equations for the mean & standard deviation which allows us to keep a *running* mean and estimated standard deviation.

Hint: Assume we have already seen the samples $\{x_1, x_2, \ldots, x_{n-1}\}$ and have already computed μ_{n-1} and σ^2_{n-1} for this set. Consider the arrival of a new sample x_n. Provide *update* equations for: (a) μ_n that depends only the previous μ_{n-1} and the new arrival x_n, (b) σ^2_n that depends only on the previous mean μ_{n-1}, previous variance σ^2_{n-1}, the new arrival x_n and the new μ_n from (a).

Problem 7 *Mean of the set vs. average of the means* 10 points

Suppose we have p sets $X = \{X_1, X_2, \ldots X_p\}$ **each** of size $n_1, n_2, \ldots n_p$. Let μ be a function that takes a collection[1] S and returns the mean of that collection $\mu(S)$, or in function notation, $\mu : S \mapsto \mu(S)$. Does the below equation hold?

$$\mu(X) = \frac{1}{p}\sum_{k=1}^{p}\mu(X_k)$$

If yes prove it, if not provide at least a counter example and give a condition for it to be true.

[1]We use *collection* here because it does not matter if the input to $\mu(\cdot)$ is a *single* set or a *set of sets* ... it just returns the mean value of whatever is given to it.

Problem 8 *Correlations* 15 points

Show that $\sin(2\pi t)$ and $\cos(2\pi t)$ are orthogonal: (a) analytically and (b) in MATLAB.