Problem 1 *Verify DFT equations* 15 points
Verify that the *inverse*

$$f[n] := \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} F[k] e^{in\omega_k} \quad \text{for} \quad n = 0, 1, 2, \ldots, N-1 \tag{1}$$

and *forward* Discrete Fourier Transform equations

$$F[k] := \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} f[n] e^{-in\omega_k} \quad \text{where} \quad \omega_k := \frac{2\pi k}{N} \quad \text{and} \quad k = 0, 1, \ldots, N-1 \tag{2}$$

are indeed valid.
Hint: Plug (2) into (1) and try to get the identity or vice versa. See hints in the lecture notes.

Problem 2 *Fourier Series by hand I* 25 points
Consider the following periodic function:

$$f(t) = \begin{cases} 2t, & 0 \le t < T/2 \\ -2(t - T/2), & T/2 \le t < T \end{cases}$$

Suppose $T = 1$.

(a) Plot the periodic waveform in MATLAB for this value of T.
(b) Compute analytically the Fourier Series for it. You are free to use any flavor of Fourier Analysis (complex, trigonometric, etc.) you like.
(c) Reconstruct the waveform (see an example) using the coefficients you computed in Part (b). Truncate the expansion at 60 nonzero terms. In other words, reconstruct the signal using 60 nonzero Fourier coefficients (either 60 complex c_k in total **or** 60 a_k and 60 b_k *each*). Compare with the plot from Part (a).

Problem 3 *Exponential form by hand* 10 points
Consider the periodically falling exponential waveform defined over its period T as:

$$f(t) = e^{-2t}, \qquad 0 < t \le T$$

Letting $T = 2$, compute the Fourier series for this periodic wave. In this case it will be probably easier to use the exponential form, a similar example was discussed in CSSB (Example 3.4).

Problem 4 *Fourier Series by hand II* 20 points
Consider the plot in Figure 1.

(a) Taking advantage of symmetry considerations, compute the Fourier coefficients.
(b) Verify your solution by reconstructing the plot in MATLAB.

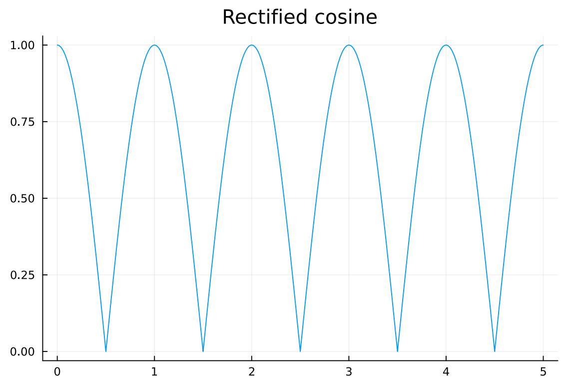

Figure 1: A fully rectified cosine wave

Problem 5 *A Fourier Transform* 20 points

Consider the plot below of an *aperiodic* waveform.

(a) Write down an equation for this waveform in the *cases* format, e.g. like $f(t)$ in Problem 2.

(b) Find its continuous Fourier transform using symmetry considerations. It might be easier to use the trigonometric forms from the textbook here (pp. 135, Eq. 3.26).

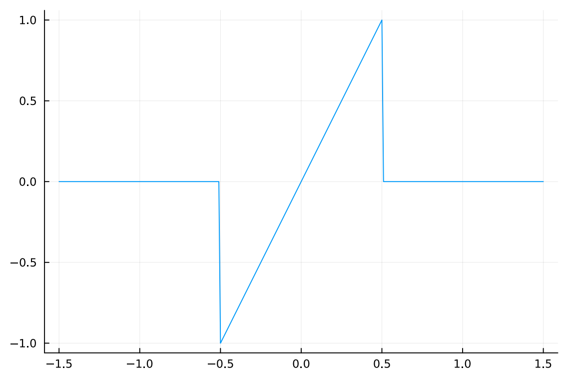

Figure 2: An aperiodic signal

Problem 6 *Even and odd functions* 10 points

Consider the function:

$$f(x) = 10x^3 - 4x^2 + 3x - 8 \tag{3}$$

(a) Is $f(x)$ and odd function or an even function?

(b) Express $f(x)$ as $f(x) = g(x) + h(x)$ where $g(x)$ is even and $h(x)$ is an odd function.