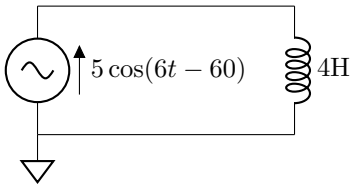
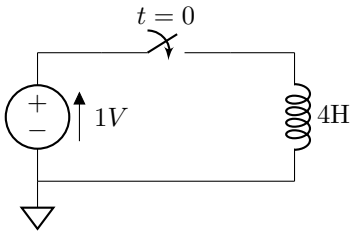
Problem 1 *Ideal circuit* 20 points

Consider the following circuit. Find the current through the inductor if its inductance is 4H.



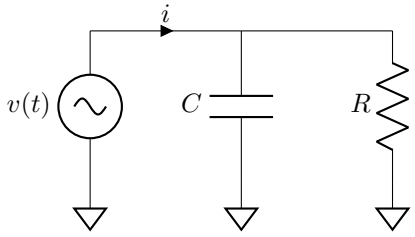
Problem 2 *Real vs. ideal circuits* 20 points

Suppose the sinusoidal voltage above is replaced with a 1V source switched on at $t = 0$. Find the current through the inductor. Is this a sensible answer? If not explain why.



Problem 3 *Windkessel model* 20 points

Assume the Windkessel model shown below applies to a frog.



If the left heart is replaced by a sinusoidal pump with a pressure output $v(t) = \cos(2\pi t)$ mmHg what is the resulting blood flow? If the frequency is changed to 4 Hz what would be the blood flow? Use $R = 1.05$ mmHg·s/ml and $C = 1.1$ ml/mmHg.

Problem 4 *Mechanical system* 20 points

Consider the mass-spring damper system below.

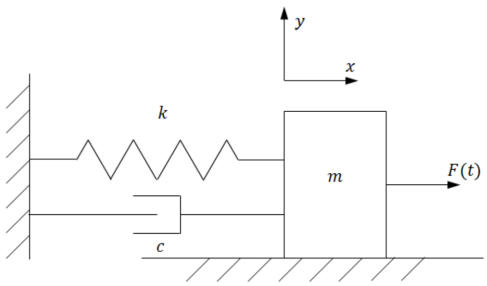


Figure 1: Simple mass-spring damper system

(a) Draw the free body diagram and write down the equation of motion for the displacement x of the mass from its equilibrium ignoring frictional effects[1].
(b) Suppose a force of 2 N is applied from $t = 0$ seconds onwards. Using the Final Valute Theorem, show that the total displacement of the mass under this force in steady state doesn't depend on the damper.

Problem 5 *Beats & resonance* 20 points

Suppose we replace the above configuration with the one below where F is now a function for the *displacement*.

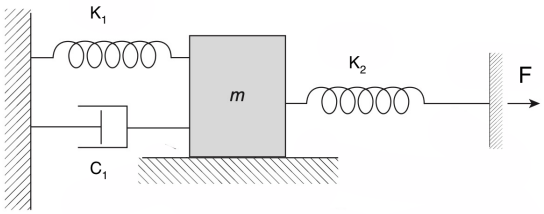


Figure 2: Similar to CSSB Example 13.12

(a) Rederive now the equation for the motion of the mass m once again ignoring frictional effects.
(b) Let $k_1 = k_2 = 5$ N/m and $c_1 = 0$. Find an equation for the time varying velocity of mass m of 1 kg if the motion of the right end is given by $F(t) = 0.1\cos(3t)$ m. Assume zero initial conditions.
(c) Now Let $k_1 = 4$ N/m and $k_2 = 5$ N/m. Redo part (b) for these values.
(d) Plot separately the velocity profile from part (a) and (b) in MATLAB for 100 seconds each. What do you observe?

[1]Recall that the total friction is sum of *static* and *dynamic* friction. The damper takes care of modeling dynamic friction while we ignore static friction.