Homework on Logic

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Fall 2020

Problem 1. Based on your understanding of logical operators, real numbers, ordering on real numbers, and basic arithmetic operations, determine for each of the statements below whether it is necessarily true or not.

1. Either $1 \leq 2$ or $2 \leq 3$.
2. Either $1 \leq 2$ or $2 > 3$.
3. Both $1 \leq 2$ and $2 \leq 3$.
4. Both $1 \leq 2$ and $2 > 3$.
5. If $2 \neq 1$ then $2 \geq \frac{3}{2}$.
6. If $2 < \frac{3}{2}$ then $2 = 1$.
7. $2 < \frac{3}{2}$ if and only if $2 = 1$.
8. For every non-zero real number x, there is a real number y such that $xy = 1$.
9. There is a non-zero real number x such that for every real number y, $xy = 1$.

Problem 2. What is the converse and contrapositive of the statement below?

> If a user accesses the database then they must have been authenticated.

Problem 3. Construct a truth table for the proposition $(P \wedge Q) \rightarrow (P \vee Q)$.

Problem 4. Use the logical equivalences stated in the lecture notes to conclude that, for any propositions P and Q, $(P \wedge Q) \rightarrow (P \vee Q)$ is logically equivalent to T. Note, do not use truth tables to prove this.

Problem 5. Let $P(x)$ be a predicate over real numbers. Translate the following english sentences into logical formulas using quantifiers.

1. There are at least two real numbers for which predicate P is true.
2. There are at most two real numbers for which predicate P is true.

Problem 6. Find a domain for the variables x, y, z such that the statement

$$\forall x \forall y ((x \neq y) \rightarrow \forall z ((z = x) \vee (z = y)))$$

is true, and another domain such that it is false.

Problem 7. Construct the negation of the following formula, pushing all negations inside (i.e. in your final answer, no negated proposition should contain a quantifier).

$$\exists x \forall y (x + y = 0)$$