Homework on Graphs and Trees

Benjamin Cosman, Patrick Lin and Mahesh Viswanathan

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Problem 1. Among connected simple graphs whose sum of vertex degrees is 20:

a) What is the smallest possible number of vertices?

b) What is the largest possible number of vertices?[1]

Briefly justify each answer.

Problem 2. Prove that in a simple graph, if there are any closed walks with odd length, any shortest of them is a cycle.[2]

Problem 3. Suppose every vertex in a graph has degree at least k. Explain why the graph has a path of length k.

Problem 4. Prove that every n-vertex graph other than K_n has chromatic number less than n. [3]

Problem 5. Solve Problem 12.24 in the textbook. (You do not need to turn in the drawing of the graph, but you should still draw it.)

[1] Hint: it may help to think in terms of "connected components" - the biggest possible subsets of the graph that are still connected. For example, a connected graph has 1 connected component (the whole graph), and a graph with n vertices and no edges has n connected components (each vertex is its own).

[2] Hint: Prove by contradiction. A closed walk which is not a cycle can be written as $f\widehat{bgb}h$ for some walks f, g, h and repeated vertex b.

[3] Hint: find an $(n-1)$-coloring.