Homework on Induction

Benjamin Cosman, Patrick Lin and Mahesh Viswanathan

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Problem 1. Solve Problem 5.16 parts (g)-(l) from our MCS textbook.

Problem 2. In each of the following problems from our MCS textbook, they present a bogus proof and you have to find the flaw. Note that what they call "strong induction" is just what we've been calling "induction". Also, they use a slightly different template where instead of assuming a property holds from 0 through $k-1$ and then proving it for k, they assume it holds for 0 through n and then prove it holds for $n+1$. This is an entirely equivalent way of formulating an inductive proof (just consider our $k-1$ to be their n); the flaw is elsewhere.

a) Solve Problem 5.22

b) Solve Problem 5.26

Problem 3. $n!$, read as "n factorial", is defined on positive integers as $1! = 1$, and for any larger n, $n! = n \cdot (n-1)!$. So for example, $3! = 3 \cdot 2! = 3 \cdot 2 \cdot 1! = 3 \cdot 2 \cdot 1 = 6$. Pick an appropriate z (your choice), and then prove (by induction) that for every $n \geq z$, $2^n < n!$.[1]

[1] Hint: first figure out an appropriate z by playing around with several small values for n

Problem 4. A summation adds up the given function for each value in the range shown on the large sigma - for example,

$$\sum_{i=1}^{n} i^2$$

just means

$$1^2 + 2^2 + ... + n^2$$

Prove by induction that for integer $n \geq 1$,

$$\sum_{i=1}^{n} \frac{1}{(2i+1)(2i-1)} = \frac{n}{2n+1}$$

Problem 5. A rectangular chocolate bar is divided into x rows of y squares each. You can break a bar between any two rows or two columns to get two smaller rectangular bars. For example, a 3 by 6 bar could be broken to get a 1 by 6 bar and a 2 by 6 bar. How many breaks total does it take to break the initial bar all the way down into (xy) 1 by 1 bars?[2] (For example, an initial 2 by 2 bar takes 3 breaks.) Prove your answer by induction.[3]

[2] Hint: Even though you will *prove* your answer using induction, it is probably easiest to *discover* the answer by just playing around with small examples.

[3] Hint: Proceed by induction on the total number of squares in the bar.