Worksheet on Counting

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Fall 2020

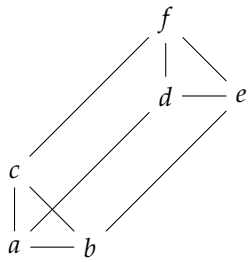
Takeaways from Lecture

- Sum rule: If $A_1, \ldots, A_n$ are disjoint, $\left|\bigcup_{i=1}^{n} A_i\right| = \sum_{i=1}^{n} |A_i|$.
- Product rule: $\left|\bigtimes_{i=1}^{n} A_i\right| = \prod_{i=1}^{n} |A_i|$.
- Bijection rule: If $f : A \to B$ is a bijection, then $|A| = |B|$.
- Generalized product rule: The number of length k sequences where the i-th entry has n_i possibilities is $\prod_{i=1}^{k} n_i$.
- Permutation rule: The number of permutations of distinct objects is $n! = n \cdot (n-1) \cdots 2 \cdot 1$. The number of ways to order k objects out of n is $P(n,k) = \frac{n!}{(n-k)!}$.
- Subset/combination rule: The number of ways to choose a k-element subset from an n-element set is $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

Problem 1. Two standard six-sided dice, one orange and one blue, are rolled. How many ways can the sum of the two rolls be odd?

Problem 2. Consider an integer grid, where we want to walk from $(0,0)$ to $(a+m, b+n)$, by only walking rightwards or upwards, but we have to pass through (a,b) on the way. How many possible such paths are there?

Problem 3. How many self-isomorphisms are there of the following graph?



Problem 4. The roman alphabet has 21 consonants and 5 vowels (for simplicity, the letter `y` is a consonant for this problem).

a) Consider the set of strings of length six where the second and fifth letters are vowels, and the rest are consonants. For example, the string `tiktok` satisfies this property. How many such strings are there? (You do not need to simplify your answer.)

b) Now consider the set of strings of length six formed from two vowels and four consonants. For example, the string `tiktok` satisfies this property, but so does the string `counts`. How many such strings are there?

c) More generally, how many strings can be formed from r consonants and s vowels?