Worksheet on Logic

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Definitions from the Lecture

- A proposition is a statement that can either be true (denoted T) or false (denoted F).
- Propositions can be combined using logical operators *not* ($\neg$), *or* ($\vee$), *and* ($\wedge$), *implies* ($\rightarrow$), and *if and only if* ($\leftrightarrow$), to create new propositions. Truth tables for these operations is shown in Figures 1 to 5.
- Two formulas/propositions P and Q are *logically equivalent* (denoted $P \equiv Q$) if they have the same meaning. That is, P and Q evaluate to the same value in all rows of a truth table, or the formula $P \leftrightarrow Q$ evaluated to T in all rows of a truth table.
- The *contrapositive* of an implication $P \rightarrow Q$ is the formula $(\neg Q) \rightarrow (\neg P)$. The contrapositive $(\neg Q) \rightarrow (\neg P)$ is logically equivalent to $P \rightarrow Q$. The *converse* of an implication $P \rightarrow Q$ is the formula $Q \rightarrow P$.
- Predicates can either be *universally* quantified ($\forall x\ P(x)$) or *existentially* quantified ($\exists x\ P(x)$) to create propositions from predicates. Formulas can have multiple quantifiers and the order in which they appear can influence their meaning.
- A *domain of discourse* identifies the set over which predicate variables take values and the meaning of predicates. It plays an important role in determining the truth of propositions.

P	$\neg P$
F	T
T	F

Figure 1: Truth table for $\neg P$

Problem 1. In which time zones are other students in your breakout room?

Problem 2. Your class has a textbook and a final exam. Let P, Q, and

Figure 2: Truth table for $P \vee Q$

P	Q	$P \vee Q$
F	F	F
F	T	T
T	F	T
T	T	T

Figure 3: Truth table for $P \wedge Q$

P	Q	$P \wedge Q$
F	F	F
F	T	F
T	F	F
T	T	T

Figure 4: Truth table for $P \to Q$

P	Q	$P \to Q$
F	F	T
F	T	T
T	F	F
T	T	T

Figure 5: Truth table for $P \leftrightarrow Q$

P	Q	$P \leftrightarrow Q$
F	F	T
F	T	F
T	F	F
T	T	T

R be the following propositions.

P : You get an A on the final exam.
Q : You do every exercise in the book.
R : You get an A in the class.

Translate the following assertions into propositional formulas using P, Q, R and the logical operators $\wedge$, $\neg$, $\to$.

1. You get an A in the class, but you do not do every exercise in the book.
2. You get an A on the final, you do every exercise in the book, and you get an A in the class.
3. To get an A in the class, it is necessary for you to get an A on the final.
4. You get an A on the final, but you don't do every exercise in the book; nevertheless, you get an A in this class.

Problem 3. Consider the following statement

> If $m + n$ is even then either m and n are both even, or m and n are both odd.

Define the following propositions: P to be "$m + n$ is even"; E_m to be "m is even"; E_n to be "n is even"; O_m to be "m is odd"; and O_n to be "n is odd".

1. Rewrite the statement using logical operators, and the propositions P, E_m, E_n, O_m, O_n.
2. What is the negation of an implication $R \to S$, where R and S are propositions?

3. Negate the formula obtained in part 1, moving all negations (e.g. "not") onto individual propositions. *Hint:* Use the observation in part 2 and then use de Morgan's laws to push the negations inside.

4. Translate the formula obtained in part 3 into English.

5. Construct the contrapositive of the given statement (in english).

6. Construct the converse of the given statement (in english).

Problem 4. For each of the pairs below determine if they are equivalent or not. To prove that a pair of formulas is not equivalent, provide a truth assignment under which the two formulas evaluate to different values. To prove that a pair of formulas is equivalent, construct a truth table showing that the formulas evaluate to the same value in all cases.

1. $P \to (Q \to R)$ and $(P \to Q) \to R$.

2. $P \vee (Q \wedge R)$ and $(P \vee Q) \wedge (P \vee R)$

Problem 5. Consider a new logical operator $\oplus$ whose truth table is given as follows.

P	Q	$P \oplus Q$
F	F	F
F	T	T
T	F	T
T	T	F

Express $P \oplus Q$ in an equivalent form using only $\neg$, $\wedge$, and $\vee$.