CS 173, Spring 19 Examlet 1, colored

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) State the contrapositive of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every dragon d, if d is green, then d is not large or d is fat.

Solution: For all dragons d, if d is large and d is not fat, then d is not green.

2. (5 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For any student s, if s rides a bicycle, then s wears a helmet or s has no fear of death.

Solution: There is a student s who rides a bicycle but doesn't wear a helmet and fears death.

3. (5 points) Solve $\frac{3}{x} + m = \frac{3}{p}$ for x, expressing your answer as a single fraction. Simplify your answer and show your work.

Solution: Multiplying by xp gives you $3p + mxp = 3x$.

So $3x - mxp = 3p$.

So $x(3 - mp) = 3p$.

So $x = \frac{3p}{3-mp}$.

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1. (5 points) Give a truth table for the following expression and (using your truth table or other means) find a simpler expression equivalent to it.

Solution:

$(p \wedge q) \vee q \equiv q$

p	q	$p \wedge q$	$(p \wedge q) \vee q$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	F

2. (5 points) State the contrapositive of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every cat c, if c is not fierce or c wears a collar, then c is a pet.

Solution: For every cat c, if c is not a pet, then c is fierce and c does not wear a collar.

3. (5 points) Solve $16p^2 - 81 = 0$ for p. Simplify your answer and show your work.

Solution: $16p^2 - 81 = (4p - 9)(4p + 9)$

$(4p - 9)(4p + 9) = 0$ when either $4p = 9$ or $4p = -9$. That is $p = \pm\frac{9}{4}$

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1. (5 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every dragon d, if d is not large, then d is green or d not hungry.

Solution: There is a dragon d such that d is not large but d is not green and d is hungry.

2. (5 points) State the contrapositive of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every tree t, if t is in Illinois and t is not hardy, then t is indoors.

Solution: For every tree t, if t is not indoors, then t is not in Illinois or t is hardy.

3. (5 points) Suppose that k is a positive integer, x is a positive real number, and $\frac{1}{k} = x + \frac{1}{6}$. What are the possible values for k? (Hint: k is an INTEGER.) Briefly explain or show work.

Solution: Observe that we can rearrange the equation as follows:

Since x is positive, $\frac{1}{k} = x + \frac{1}{6}$ implies that $\frac{1}{k} > \frac{1}{6}$. So k must be smaller than 6. But we were told that k was a positive integer. The only positive integers smaller than 6 are 1, 2, 3, 4, and 5.

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1. (5 points) Show that the following two expressions are not logically equivalent, by giving specific values of p, q for which they produce different values.

$p \rightarrow (q \rightarrow p)$

$(p \rightarrow q) \rightarrow p$

Solution: Set p and q to be false.

Then $p \rightarrow (q \rightarrow p)$ is true because its hypothesis is false.

$p \rightarrow q$ is also true. So $(p \rightarrow q) \rightarrow p$ is false because its hypothesis is true but its conclusion (p) is false.

A similar argument works if you set p to be false and q to be true.

2. (5 points) State the contrapositive of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every car c, if c is a Tesla, then c is new or c is not fast.

Solution: For every car c, if c is not new and c is fast, then c is not a Tesla.

3. (5 points) Suppose that k is a positive integer, x is a positive real number, and $\frac{1}{k} + x = \frac{1}{6}$. What are the possible values for k? (Hint: k is an INTEGER.) Briefly explain or show work.

Solution: Observe that we can rearrange the equation as follows:

Since x is positive, $\frac{1}{k} + x = \frac{1}{6}$ implies that $\frac{1}{k} < \frac{1}{6}$. So k must be an integer greater than 6.

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1. (5 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every tree t, if t grows in Canada, then t is not tall or t is a conifer.

Solution: There is a tree t, such that g is tall and t is not a conifer, but t grows in Canada.

2. (5 points) State the contrapositive of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every garbage can c, if c was supplied by the city, then c is small or c has wheels.

Solution: For every garbage can c, if c is large and c does not have wheels, then c was not supplied by the city.

3. (5 points) Solve $3x + 2m = \frac{w}{y}$ for x, expressing your answer as a single fraction. Simplify your answer and show your work.

Solution:

$$\begin{aligned}
3x + 2m &= \frac{w}{y} \\
3x &= \frac{w}{y} - 2m \\
3x &= \frac{w - 2ym}{y} \\
x &= \frac{w - 2ym}{3y}
\end{aligned}$$

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1. (5 points) Give a truth table for the following expression and (using your truth table or other means) find a simpler expression equivalent to it.

Solution:

$(p \to q) \wedge (p \to \neg q) \equiv \neg p$

p	q	$p \to q$	$p \to \neg q$	$(p \to q) \wedge (p \to \neg q)$
T	T	T	F	F
T	F	F	T	F
F	T	T	T	T
F	F	T	T	T

2. (5 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every book b, if b is blue or b is not heavy, then b is not a math book.

Solution: There is a book b, such that b is blue or b is not heavy, but b is a math book.

3. (5 points) Solve $\frac{2m^2 - m - 6}{m-2} = 9$ for m. (Assume $m \neq 2$.)

Solution: Notice that $2m^2 - m - 6 = (m-2)(2m+3)$. So $\frac{2m^2 - m - 6}{m-2} = 2m + 3$.

So our problem reduces to solving $2m + 3 = 9$. That is, $2m = 6$. So $m = 3$.

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1. (5 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

There is a soup s such that s is tasty and s does not contain meat.

Solution: For every soup s, s is not tasty or s contains meat.

2. (5 points) State the contrapositive of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For any bear b, if b is blue and b talks, then b is fuzzy.

Solution: For any bear b, if b is not fuzzy, then b is not blue or b doesn't talk.

3. (5 points) Suppose that G and H are functions whose inputs and outputs are real numbers, defined by $G(x) = x - 2$ and $H(x) = \sqrt{2x+1}$, where the square root function returns only the positive root. Compute the value of $H(G(G(8)))$, showing your work.

Solution: $G(8) = 6$

So $G(G(8)) = 4$

So $H(G(G(8))) = \sqrt{2 \cdot 4 + 1} = \sqrt{9} = 3$

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1. (5 points) Show that the following two expressions are not logically equivalent, by giving specific values of p, q, and r for which they produce different values.

$(p \rightarrow q) \wedge r$

$p \rightarrow (q \wedge r)$

Solution: Set p and r to be false and q to be true. Then $(p \rightarrow q)$ is true (because its hypothesis is false) and $(p \rightarrow q) \wedge r$. But $p \rightarrow (q \wedge r)$ is true because its hypothesis is false.

2. (5 points) State the contrapositive of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every jedi j, if j has a light saber and j is not sick, then j can defeat the Dark Side.

Solution: For every jedi j, if j cannot defeat the Dark Side. then j does not have a light saber or j is sick.

3. (5 points) Suppose that x is an integer and $x^2 + 3x - 18 < 0$. What are the possible values of x? Show your work.

Solution: $x^2 + 3x - 18 = (x + 6)(x - 3)$. So we have $(x + 6)(x - 3) < 0$. So one of $(x + 6)$ and $(x - 3)$ is negative and the other positive. Because $(x + 6)$ is larger, $(x + 6)$ must be the positive one.

So we have $x + 6 > 0$ and $x - 3 < 0$. So $x > -6$ and $x < 3$. Since x is an integer, it must be one of the following values:

$$-5, -4, -3, -2, -1, 0, 1, 2$$