CS 173, Spring 19 Examlet 10, colored

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Use (strong) induction to prove the following claim:

Claim: $\displaystyle\sum_{p=1}^{n} \frac{p}{p+1} \leq \frac{n^2}{n+1}$ for all positive integers n.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Use (strong) induction to prove the following claim:

Claim: $(2n)!^2 < (4n)!$ for all positive integers.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) The operator $\prod$ is like $\sum$ except that it multiplies its terms rather than adding them. So e.g. $\prod_{p=3}^{5}(p+1) = 4 \cdot 5 \cdot 6$. Use (strong) induction to prove the following claim:

Claim: For any positive integer n and any reals $a_1, \ldots, a_n$ between 0 and 1 (inclusive)

$$\prod_{p=1}^{n}(1 - a_p) \geq 1 - \sum_{p=1}^{n} a_p$$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

CS 173, Spring 19 Examlet 10, colored

Name:____

NetID:_____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Use (strong) induction to prove the following claim.

Claim: For any positive integer n, $\sum_{p=1}^{n} \frac{(-1)^{p-1}}{p} > 0$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step: Hint: divide into cases based on whether k is even or odd. Consider removing more than term from the summation in one case.

CS 173, Spring 19 Examlet 10, colored

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Use (strong) induction to prove the following claim:

Claim: $\frac{(2n)!}{n!n!} > 2^n$, for all integers $n \geq 2$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

CS 173, Spring 19 Examlet 10, colored

Name:____

NetID:________________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(15 points) Use (strong) induction to prove the following claim:

Claim: For any natural number n and any real number x, where $0 < x < 1$, $(1-x)^n \geq 1-nx$.

Let x be a real number, where $0 < x < 1$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

CS 173, Spring 19 Examlet 10, colored

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Use (strong) induction to prove the following claim:

Claim: $\sum_{p=1}^{n} \frac{1}{p} \leq \frac{n}{2} + 1$, for any positive integer n.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step: Hint: recall that if $x \leq y$, then $\frac{1}{y} \leq \frac{1}{x}$

Name:____

NetID:________________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(15 points) The operator $\prod$ is like $\sum$ except that it multiplies its terms rather than adding them. So e.g. $\prod_{p=3}^{5}(p+1) = 4 \cdot 5 \cdot 6$. Use (strong) induction to prove the following claim:

Claim: For any positive integer n and any positive reals $a_1, \ldots, a_n$,

$$\prod_{p=1}^{n}(1+a_p) \geq 1 + \sum_{p=1}^{n} a_p$$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step: