Name:______________________________________

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is even.

   $T(0) = 5$ $\quad\quad T(n) = 3T(n-2) + n^2$

   (a) The height:

   (b) The number of leaves (please simplify):

   (c) Value in each node at level k:

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   n $\quad n\log(17n)$ $\quad \sqrt{n}+18$ $\quad 8n^2$ $\quad 2^n+n!$ $\quad 2^{\log_4 n}+5^n$ $\quad 0.001n^3+3^n$

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (7 points) Recall that f is $O(g)$ if and only if there are positive reals c and k such that $0 \leq f(x) \leq cg(x)$ for every $x \geq k$. Prof. Snape claims that there is a function f (from the reals to the reals) that can never be involved in a big-O relationship. Is he correct?

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = c$ $T(n) = 2T(n/2) + n$	$\Theta(\log n)$ ☐ $\Theta(n^2)$ ☐	$\Theta(\sqrt{n})$ ☐ $\Theta(n^3)$ ☐	$\Theta(n)$ ☐ $\Theta(2^n)$ ☐	$\Theta(n \log n)$ ☐ $\Theta(3^n)$ ☐
$T(1) = d$ $T(n) = T(n/2) + c$	$\Theta(\log n)$ ☐ $\Theta(n^2)$ ☐	$\Theta(\sqrt{n})$ ☐ $\Theta(n^3)$ ☐	$\Theta(n)$ ☐ $\Theta(2^n)$ ☐	$\Theta(n \log n)$ ☐ $\Theta(3^n)$ ☐
$n^{1.5}$ is	$\Theta(n^{1.614})$ ☐	$O(n^{1.614})$ ☐	neither of these ☐	
$n^{log_3 5}$ grows	faster than n^2 ☐ at the same rate as n^2 ☐	slower than n^2 ☐		

CS 173, Spring 19 Examlet 10, white

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is even.

   $T(8) = 5$ $\quad\quad$ $T(n) = 3T(n-2) + c$

   (a) The height:

   (b) The number of nodes at level k:

   (c) Value in each node at level k:

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $3n^2$ $\quad$ $\frac{n \log n}{7}$ $\quad$ $(10^{10^{10}})n$ $\quad$ $0.001n^3$ $\quad$ $30\log(n^{17})$ $\quad$ $8n! + 18$ $\quad$ $3^n + 11^n$

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (7 points) Suppose that f and g are functions from the reals to the reals. Define precisely what it means for g to be $\Theta(f)$. Your definition can be in terms of other primitives such as $\ll$ and big-O.

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = d$
$T(n) = T(n-1) + c$

$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐

$T(1) = d$
$T(n) = 2T(n/2) + c$

$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐

Suppose f and g produce only positive outputs and $f(n) \ll g(n)$. Will $g(n)$ be $O(f(n))$?

no ☐ sometimes ☐ yes ☐

$n^{log_2 4}$ grows

faster than n^2 ☐ slower than n^2 ☐
at the same rate as n^2 ☐

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a multiple of 3.

   $T(3) = 7$ $\quad\quad T(n) = 2T(n-3) + c$

   (a) The height:

   (b) The number of leaves (please simplify):

   (c) Total work (sum of the nodes) at level k (please simplify):

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   n $\quad n\log(17n)$ $\quad \sqrt{n}+2^n+18$ $\quad 8n^2$ $\quad 2^n+n!$ $\quad 2^{\log_4 n}$ $\quad 0.001n^3+3^n$

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (7 points) Suppose that f, g, and h are functions from the reals to the reals, such that $f(x)$ is $O(h(x))$ and $g(x)$ is $O(h(x))$. Must $f(x)g(x)$ be $O(h(x))$?

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = c$
$T(n) = 3T(n/3) + n$

$\Theta(\log n)$	☐	$\Theta(\sqrt{n})$	☐	$\Theta(n)$	☐	$\Theta(n \log n)$	☐
$\Theta(n^2)$	☐	$\Theta(n^3)$	☐	$\Theta(2^n)$	☐	$\Theta(3^n)$	☐

$T(1) = c$
$T(n) = 2T(n/2) + n^2$

$\Theta(\log n)$	☐	$\Theta(\sqrt{n})$	☐	$\Theta(n)$	☐	$\Theta(n \log n)$	☐
$\Theta(n^2)$	☐	$\Theta(n^3)$	☐	$\Theta(2^n)$	☐	$\Theta(3^n)$	☐

Dividing a problem of size n into k subproblems, each of size n/m, has the best big-Θ running time when

$k < m$	☐	$k = m$	☐
$k > m$	☐	$km = 1$	☐

$n^{log_2 5}$ grows

faster than n^2	☐	slower than n^2	☐
at the same rate as n^2	☐		

Name:____

NetID:____________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 2.

   $T(8) = 7$ $\quad\quad$ $T(n) = 4T\left(\frac{n}{2}\right) + n$

   (a) The height:

   (b) Total work (sum of the nodes) at level k (please simplify):

   (c) The number of leaves (please simplify):

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   3^n $\quad$ $4^{\log_2 n}$ $\quad$ 2^{3n} $\quad$ $3^{\log_2 4}$ $\quad$ $0.1n$ $\quad$ $(5n)!$ $\quad$ $\sqrt{n}$

CS 173, Spring 19 Examlet 10, white

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (7 points) Suppose that f and g are functions from the reals to the reals. Define precisely when $f \ll g$.

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = d$ $T(n) = 3T(n-1) + c$	$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
	$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐
$T(1) = d$ $T(n) = T(n/2) + n$	$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
	$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐
3^n is	$\Theta(5^n)$ ☐	$O(5^n)$ ☐	neither of these ☐	
3^n is	$\Theta(2^n)$ ☐	$O(2^n)$ ☐	neither of these ☐	