CS 173, Spring 19 Examlet 10, white

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is even.

   $T(0) = 5 \qquad T(n) = 3T(n-2) + n^2$

   (a) The height: $\frac{n}{2}$

   (b) The number of leaves (please simplify): $3^{\frac{n}{2}} = (\sqrt{3})^n$

   (c) Value in each node at level k: $(n-2k)^2$

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $n \qquad n\log(17n) \qquad \sqrt{n}+18 \qquad 8n^2 \qquad 2^n+n! \qquad 2^{\log_4 n}+5^n \qquad 0.001n^3+3^n$

   Solution:

   $\sqrt{n}+18 \ll n \ll n\log(17n) \ll 8n^2 \ll 0.001n^3+3^n \ll 2^{\log_4 n}+5^n \ll 2^n+n!$

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1. (7 points) Recall that f is $O(g)$ if and only if there are positive reals c and k such that $0 \leq f(x) \leq cg(x)$ for every $x \geq k$. Prof. Snape claims that there is a function f (from the reals to the reals) that can never be involved in a big-O relationship. Is he correct?

   Solution: He is right. Our definition will never be satisfied if one (or both) of the functions produces only negative outputs.

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = c$
$T(n) = 2T(n/2) + n$

$\Theta(\log n)$	☐	$\Theta(\sqrt{n})$	☐	$\Theta(n)$	☐	$\Theta(n \log n)$	☑ √
$\Theta(n^2)$	☐	$\Theta(n^3)$	☐	$\Theta(2^n)$	☐	$\Theta(3^n)$	☐

$T(1) = d$
$T(n) = T(n/2) + c$

$\Theta(\log n)$	☑ √	$\Theta(\sqrt{n})$	☐	$\Theta(n)$	☐	$\Theta(n \log n)$	☐
$\Theta(n^2)$	☐	$\Theta(n^3)$	☐	$\Theta(2^n)$	☐	$\Theta(3^n)$	☐

$n^{1.5}$ is

$\Theta(n^{1.614})$	☐	$O(n^{1.614})$	☑ √	neither of these	☐

$n^{log_3 5}$ grows

faster than n^2	☐	slower than n^2	☑ √
at the same rate as n^2	☐		

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Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is even.

   $T(8) = 5 \qquad T(n) = 3T(n-2) + c$

   (a) The height: $\frac{n}{2} - 4$

   (b) The number of nodes at level k: 3^k

   (c) Value in each node at level k: c

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $3n^2 \qquad \frac{n \log n}{7} \qquad (10^{10^{10}})n \qquad 0.001n^3 \qquad 30\log(n^{17}) \qquad 8n! + 18 \qquad 3^n + 11^n$

   Solution:

   $30\log(n^{17}) \ll (10^{10^{10}})n \ll \frac{n \log n}{7} \ll 3n^2 \ll 0.001n^3 \ll 3^n + 11^n \ll 8n! + 18$

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1. (7 points) Suppose that f and g are functions from the reals to the reals. Define precisely what it means for g to be $\Theta(f)$. Your definition can be in terms of other primitives such as $\ll$ and big-O.

   Solution: g is $\Theta(f)$ if and only if g is $O(f)$ and f is $O(g)$.

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = d$
$T(n) = T(n-1) + c$

$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ √	$\Theta(n \log n)$ ☐
$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐

$T(1) = d$
$T(n) = 2T(n/2) + c$

$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ √	$\Theta(n \log n)$ ☐
$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐

Suppose f and g produce only positive outputs and $f(n) \ll g(n)$. Will $g(n)$ be $O(f(n))$?

no √ sometimes ☐ yes ☐

$n^{log_2 4}$ grows

faster than n^2 ☐ slower than n^2 ☐
at the same rate as n^2 √

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Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a multiple of 3.

   $T(3) = 7 \qquad T(n) = 2T(n-3) + c$

   (a) The height: $\frac{n}{3} - 1$

   (b) The number of leaves (please simplify): $2^{\frac{n}{3}-1}$

   (c) Total work (sum of the nodes) at level k (please simplify): There are 2^k nodes at level k, each containing value c. So the total work is $c2^k$.

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $n \qquad n\log(17n) \qquad \sqrt{n}+2^n+18 \qquad 8n^2 \qquad 2^n+n! \qquad 2^{\log_4 n} \qquad 0.001n^3+3^n$

   Solution:

   $2^{\log_4 n} \ll n \ll n\log(17n) \ll 8n^2 \ll \sqrt{n}+2^n+18 \ll 0.001n^3+3^n \ll 2^n+n!$

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1. (7 points) Suppose that f, g, and h are functions from the reals to the reals, such that $f(x)$ is $O(h(x))$ and $g(x)$ is $O(h(x))$. Must $f(x)g(x)$ be $O(h(x))$?

   Solution: This is false.

   Suppose that $f(x) = g(x) = h(x) = x$. Then $f(x)$ is $O(h(x))$ and $g(x)$ is $O(h(x))$, but $f(x)g(x) = x^2$ is not $O(h(x))$.

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = c$
$T(n) = 3T(n/3) + n$

$\Theta(\log n)$	☐	$\Theta(\sqrt{n})$	☐	$\Theta(n)$	☐	$\Theta(n \log n)$	√
$\Theta(n^2)$	☐	$\Theta(n^3)$	☐	$\Theta(2^n)$	☐	$\Theta(3^n)$	☐

$T(1) = c$
$T(n) = 2T(n/2) + n^2$

$\Theta(\log n)$	☐	$\Theta(\sqrt{n})$	☐	$\Theta(n)$	☐	$\Theta(n \log n)$	☐
$\Theta(n^2)$	√	$\Theta(n^3)$	☐	$\Theta(2^n)$	☐	$\Theta(3^n)$	☐

Dividing a problem of size n into k subproblems, each of size n/m, has the best big-Θ running time when

$k < m$	√	$k = m$	☐
$k > m$	☐	$km = 1$	☐

$n^{log_2 5}$ grows

faster than n^2	√	slower than n^2	☐
at the same rate as n^2	☐		

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Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 2.

   $T(8) = 7$ $\qquad T(n) = 4T\left(\frac{n}{2}\right) + n$

   (a) The height: $\log_2 n - 3$

   (b) Total work (sum of the nodes) at level k (please simplify): There are 4^k nodes at level k. Each one contains the value $\frac{n}{2^k}$. So the total for the level is $2^k n$.

   (c) The number of leaves (please simplify): $4^{\log_2 n - 3} = \frac{1}{4^3} 4^{\log_2 n}$
   $4^{\log_2 n} = 4^{\log_4 n \log_2 4} = (4^{\log_4 n})^{\log_2 4} == n^{\log_2 4} = n^2$
   So the number of leaves is $\frac{1}{4^3} n^2$.

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $3^n \qquad 4^{\log_2 n} \qquad 2^{3n} \qquad 3^{\log_2 4} \qquad 0.1n \qquad (5n)! \qquad \sqrt{n}$

   Solution:

   $3^{\log_2 4} \ll \sqrt{n} \ll 0.1n \ll 4^{\log_2 n} \ll 3^n \ll 2^{3n} \ll (5n)!$

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1. (7 points) Suppose that f and g are functions from the reals to the reals. Define precisely when $f \ll g$.

   Solution: $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = d$	$\Theta(\log n)$	☐	$\Theta(\sqrt{n})$	☐	$\Theta(n)$	☐	$\Theta(n \log n)$	☐
$T(n) = 3T(n-1) + c$	$\Theta(n^2)$	☐	$\Theta(n^3)$	☐	$\Theta(2^n)$	☐	$\Theta(3^n)$	√
$T(1) = d$	$\Theta(\log n)$	☐	$\Theta(\sqrt{n})$	☐	$\Theta(n)$	√	$\Theta(n \log n)$	☐
$T(n) = T(n/2) + n$	$\Theta(n^2)$	☐	$\Theta(n^3)$	☐	$\Theta(2^n)$	☐	$\Theta(3^n)$	☐

3^n is	$\Theta(5^n)$	☐	$O(5^n)$	√	neither of these	☐
3^n is	$\Theta(2^n)$	☐	$O(2^n)$	☐	neither of these	√