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(7 points) A triomino is a triangular tile with a number on each edge, visible on both front and back. In our set of triominos, the numbers range from 0 to 5. So possible tiles include 5-3-4, 0-4-4, and 3-3-3. A tile is the same if you turn it over or rotate it. So 5-3-4 is the same tile as 3-4-5, 4-5-3, and also 5-4-3. How many distinct tiles are in our set? Briefly justify your answer and/or show work.

(6 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every dinosaur k, if k is blue, then k is not vegetarian or k is friendly.

(2 points) Check the (single) box that best characterizes each item.

The number of bit strings of length 20 with exactly 8 1's.

$\binom{26}{7}$ ☐	$\binom{27}{7}$ ☐	$\binom{20}{8}$ ☐
$\binom{20}{13}$ ☐	$\binom{20}{14}$ ☐	2^{20} ☐

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(7 points) This evening, Ollie the Owl wants to say `hoot` 8 times, `krick` 7 times, and `yeet` 3 times. How many distinct sequences of the 18 noises could he produce? Briefly justify your answer and/or show work.

(8 points) Use proof by contradiction to show that, in a party of n people ($n \geq 2$), there are (at least) two people who danced with the same number of different partners. Assume people always dance in pairs. Don't assume everyone danced.

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(7 points) Each time Nancy presses her 2FA login token, it generate a sequence of 6 decimal digits. What is the chance that this sequence will contain at least one duplicate digit? Give an exact formula; don't try to figure out the decimal equivalent. Briefly justify your answer and/or show work.

(6 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every relish r, if r is orange and r is not spicy, then r is pungent.

(2 points) Check the (single) box that best characterizes each item.

If you want to take 4 classes next semester, out of 35 classes being offered, how many different choices do you have?

$\frac{38!}{35!3!}$ ☐	$\frac{35!}{31!4!}$ ☐	35^4 ☐
4^{35} ☐	$\frac{35!}{31!}$ ☐	4^4 ☐

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(7 points) Suppose a car dealer is planning to buy a collection of Civics, Accords, and Fits (three kinds of cars). The dealer will buy ten cars in total and can buy any number of each type. How many different choices does he have? The collection is unordered, so three Civics and seven Fits is the same as seven Fits and three Civics. Briefly justify your answer and/or show work.

(8 points) Use proof by contradiction to show that if x and y are positive integers, $x^2 - y^2 \neq 6$.

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(7 points) Pearl's start-up sells sets of circular bands for labelling wine glasses. Each band has four beads, each of which can be orange, blue, or silver. Two bands are the same if they can be made to look the same by rotation and/or turning over. E.g. all bands with three blue beads and one silver bead are the same. How many distinct bands can she put in each set? Briefly justify your answer and/or show work.

(6 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every mountain m, if m is tall or m is not in the north, then m has a snow cap.

(2 points) Check the (single) box that best characterizes each item.

Suppose you want to take 5 classes next semester, out of 25 classes being offered. You must take STATS 100. How many different choices do you have?	5^{24} ☐	$\frac{(24+3)!}{24!3!}$ ☐	$\frac{24!}{20!}$ ☐
	24^5 ☐	$\frac{24!}{20!4!}$ ☐	$\frac{25!}{20!5!}$ ☐

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(7 points) Suppose that A is a set containing p elements and B is a set containing n elements. How many functions are there from A to $\mathbb{P}(B)$? How many of these functions are one-to-one? Briefly justify your answer and/or show work.

(8 points) Researchers recorded phone calls between pairs of two (different) people, never repeating the same pair of people. The experiment used n people ($n \geq 2$), but it's possible some of these n people were not in any conversation. Use proof by contradiction to show that two people were in the same number of conversations.

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(7 points) CMU's new robotic hummingbird Merrill travels in 3D. Each command makes it move one foot in a specified cardinal direction e.g. up/down, north/south, or east/west, but not diagonally. How many different sequences of 30 commands will get Merrill from position $(1, 10, 3)$ to position $(10, 6, 20)$? Briefly justify your answer and/or show work.

(6 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

There is a mushroom f such that f is not poisonous or f is blue.

(2 points) Check the (single) box that best characterizes each item.

E is the edge set of a tree with n nodes. $\lvert\mathbb{P}(E)\rvert =$	2^{n-1} ☐	2^n ☐	can't tell ☐
	2^{n+1} ☐	n ☐	$n-1$ ☐

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(7 points) Prof. Howard has 17 tuba players, ordered by ability, who must be distributed among three bands. Stellar will get the best players, Dismal the worst ones, and Normal the group in the middle of the range. Each band must be given at least one tuba player. How many options does he have? Briefly justify your answer and/or show work.

(8 points) Suppose we know that $\sqrt{6}$ is irrational. Use proof by contradiction to show that $\sqrt{2}+\sqrt{3}$ is irrational. (You must use the definition of "rational." You may not use facts about adding/subtracting rational numbers.)