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(7 points) A triomino is a triangular tile with a number on each edge, visible on both front and back. In our set of triominos, the numbers range from 0 to 5. So possible tiles include 5-3-4, 0-4-4, and 3-3-3. A tile is the same if you turn it over or rotate it. So 5-3-4 is the same tile as 3-4-5, 4-5-3, and also 5-4-3. How many distinct tiles are in our set? Briefly justify your answer and/or show work.

Solution: All three sides have the same number: 6 tiles.

Two sides have the same number: 6 choices for the duplicated number, five choices for the single number. So 30 different tiles.

All three sides have different numbers: There are $\binom{6}{3} = 20$ ways to pick the three numbers. The order does not matter because we can get from any order to any other order by rotating and/or turning the tile over.

Total number of tiles is 56.

(6 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every dinosaur k, if k is blue, then k is not vegetarian or k is friendly.

Solution: There is a dinosaur k such that k is blue but k is vegetarian and k is not friendly.

(2 points) Check the (single) box that best characterizes each item.

The number of bit strings of length 20 with exactly 8 1's.

$\binom{26}{7}$ ☐	$\binom{27}{7}$ ☐	$\binom{20}{8}$ ☑ ($\surd$)
$\binom{20}{13}$ ☐	$\binom{20}{14}$ ☐	2^{20} ☐

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(7 points) This evening, Ollie the Owl wants to say `hoot` 8 times, `krick` 7 times, and `yeet` 3 times. How many distinct sequences of the 18 noises could he produce? Briefly justify your answer and/or show work.

Solution: In the sequence, he has $\binom{18}{8}$ choices for when to say `hoot`. After he has made that decision, he has $\binom{10}{7}$ options for when to say `krick`. And then `yeet` goes in the other three positions. So the total number of options is $\binom{18}{8}\binom{10}{7}$

(8 points) Use proof by contradiction to show that, in a party of n people ($n \geq 2$), there are (at least) two people who danced with the same number of different partners. Assume people always dance in pairs. Don't assume everyone danced.

Solution: Suppose not. That is, suppose that each of the n people danced with a different number of partners. Notice that the minimum number of partners is 0 and the maximum number is $n-1$. Since there are exactly n numbers between 0 and $n-1$, there's some person P who danced 0 people and another person Q who danced with $n-1$ people. But this is a contradiction. If Q danced with $n-1$ people, then Q must have danced with P, contradicting the fact that P danced with no one.

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(7 points) Each time Nancy presses her 2FA login token, it generate a sequence of 6 decimal digits. What is the chance that this sequence will contain at least one duplicate digit? Give an exact formula; don't try to figure out the decimal equivalent. Briefly justify your answer and/or show work.

Solution: There are 10^6 sequences of 6 decimal digits. There are $\frac{10!}{4!}$ sequences of 6 distinct decimal digits. So the chance of getting at least one duplicate digit is

$$1 - \frac{10!}{4!10^6}$$

.

(6 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every relish r, if r is orange and r is not spicy, then r is pungent.

Solution: There is a relish r, such that r is orange and r is not spicy but r is not pungent.

(2 points) Check the (single) box that best characterizes each item.

If you want to take 4 classes next semester, out of 35 classes being offered, how many different choices do you have?

$\frac{38!}{35!3!}$ ☐	$\frac{35!}{31!4!}$ ☑ $\surd$	35^4 ☐
4^{35} ☐	$\frac{35!}{31!}$ ☐	4^4 ☐

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(7 points) Suppose a car dealer is planning to buy a collection of Civics, Accords, and Fits (three kinds of cars). The dealer will buy ten cars in total and can buy any number of each type. How many different choices does he have? The collection is unordered, so three Civics and seven Fits is the same as seven Fits and three Civics. Briefly justify your answer and/or show work.

Solution: Using the formula for combinations with repetition, there are

$$\binom{10+2}{2}$$

choices.

(8 points) Use proof by contradiction to show that if x and y are positive integers, $x^2 - y^2 \neq 6$.

Solution: Suppose not. That is, suppose that there are positive integers x and y such that $x^2 - y^2 = 6$. Factoring the lefthand side, we get $(x-y)(x+y) = 10$. $(x-y)$ and $(x+y)$ must be integers since x and y are integers.

Because x and y are positive, $x+y$ is positive Since $(x-y)(x+y)$ is positive, $x-y$ must also be positive.

There are only two ways to factor 10 into two positive numbers: $2 \cdot 3$ or $1 \cdot 6$. In both cases, exactly one of the factors is odd, so the sum of the two factors is odd. But the sum of $(x-y)$ and $(x+y)$ is $2x$, which is even.

We have found a contradiction, so the original claim must have been correct.

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(7 points) Pearl's start-up sells sets of circular bands for labelling wine glasses. Each band has four beads, each of which can be orange, blue, or silver. Two bands are the same if they can be made to look the same by rotation and/or turning over. E.g. all bands with three blue beads and one silver bead are the same. How many distinct bands can she put in each set? Briefly justify your answer and/or show work.

Solution: All beads are the same color: 3 choices.

Three beads are the same color, one color is different: three choices for the first color, two for the second, so 6 choices total.

Two beads are the same color, two beads are a second color: 3 choices for the color that's left out, two choices for the pattern (are same-colored beads adjacent?), so 6 choices total.

Beads of all three colors: 3 choices for which color is represented twice, two choices for the pattern, so 6 choices total.

So there are $6 + 6 + 6 + 3 = 21$ distinct bands.

(6 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every mountain m, if m is tall or m is not in the north, then m has a snow cap.

Solution: There is a mountain m, such that m is tall or m is not in the north, but m does not have a snow cap.

(2 points) Check the (single) box that best characterizes each item.

Suppose you want to take 5 classes next semester, out of 25 classes being offered. You must take STATS 100. How many different choices do you have?	5^{24} ☐	$\frac{(24+3)!}{24!3!}$ ☐	$\frac{24!}{20!}$ ☐
	24^5 ☐	$\frac{24!}{20!4!}$ ☑	$\frac{25!}{20!5!}$ ☐

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(7 points) Suppose that A is a set containing p elements and B is a set containing n elements. How many functions are there from A to $\mathbb{P}(B)$? How many of these functions are one-to-one? Briefly justify your answer and/or show work.

Solution: $\mathbb{P}(B)$ contains 2^n elements. So the total number of functions from A to $\mathbb{P}(B)$ is $(2^n)^p$. The number of one-to-one functions is $\frac{2^n!}{(2^n-p)!}$.

(8 points) Researchers recorded phone calls between pairs of two (different) people, never repeating the same pair of people. The experiment used n people ($n \geq 2$), but it's possible some of these n people were not in any conversation. Use proof by contradiction to show that two people were in the same number of conversations.

Solution: Suppose not. That is, suppose that each of the n people was in a different number of conversations. For each person, the minimum number of conversations is zero and the maximum is $n-1$. Since there are exactly n numbers between 0 and $n-1$, there's some person P who wasn't in any conversation and another person Q who was in $n-1$ conversations. But this is a contradiction. If Q talked to $n-1$ people, then Q must have talked to P, contradicting the fact that P didn't talk to anyone.

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(7 points) CMU's new robotic hummingbird Merrill travels in 3D. Each command makes it move one foot in a specified cardinal direction e.g. up/down, north/south, or east/west, but not diagonally. How many different sequences of 30 commands will get Merrill from position $(1, 10, 3)$ to position $(10, 6, 20)$? Briefly justify your answer and/or show work.

Solution: We'll need 9 commands that increase the first coordinate, 4 that decrease the second coordinate, and 17 that increase the third coordinate. There are $\binom{30}{9}$ ways to pick which of the 30 commands changes the first coordinate, and then $\binom{21}{4}$ choices for when to change the second coordinate. So our total choices are

$$\binom{30}{9}\binom{21}{4}$$

.

(6 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

There is a mushroom f such that f is not poisonous or f is blue.

Solution: For every mushroom f, f is poisonous and f is not blue.

(2 points) Check the (single) box that best characterizes each item.

E is the edge set of a tree with n nodes. $\|\mathbb{P}(E)\| =$	2^{n-1} ☑	2^n ☐	can't tell ☐
	2^{n+1} ☐	n ☐	$n-1$ ☐

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(7 points) Prof. Howard has 17 tuba players, ordered by ability, who must be distributed among three bands. Stellar will get the best players, Dismal the worst ones, and Normal the group in the middle of the range. Each band must be given at least one tuba player. How many options does he have? Briefly justify your answer and/or show work.

Solution: There are 16 positions between adjacent tuba players in the ability ordering. We need to pick two of these to be the division points. So there are $\binom{16}{2}$ possibilities

(8 points) Suppose we know that $\sqrt{6}$ is irrational. Use proof by contradiction to show that $\sqrt{2}+\sqrt{3}$ is irrational. (You must use the definition of "rational." You may not use facts about adding/subtracting rational numbers.)

Solution: Suppose not. That is, suppose that $\sqrt{2}+\sqrt{3}$ is rational. Then there are integers p and q (q non-zero) such that $\sqrt{2}+\sqrt{3}=\frac{p}{q}$.

Squaring both sides of this equation gives $2+2\sqrt{6}+3=\frac{p^2}{q^2}$. So $2\sqrt{6}=\frac{p^2}{q^2}-5=\frac{p^2-5q^2}{q^2}$. So $\sqrt{6}=\frac{p^2}{q^2}-5=\frac{p^2-5q^2}{2q^2}$. But notice that p^2-5q^2 and $2q^2$ are both integers since p and q are integers. So this means that $\sqrt{6}$ is the ratio of two integers and therefore rational. But we know that $\sqrt{6}$ is not rational.

Since our original assumption led to a contradiction, $\sqrt{2}+\sqrt{3}$ must be irrational.