CS 173, Spring 19 Examlet 12, white

Name: _____

NetID: _____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

Let $f : \mathbb{Z}_{12} \to \mathbb{P}(\mathbb{Z}_{12})$ be defined by $f(x) = \{y \in \mathbb{Z}_{12} \mid y^2 = x\}$. Let $S = \{f(x) \mid x \in \mathbb{Z}_{12}\}$.

(3 points) $S =$

(Write elements of $\mathbb{Z}_{12}$ as plain integers, without brackets.)

(3 points) Is S a partition of $\mathbb{Z}_{12}$? Check the partition properties that are satisfied.

No Empty set ☐ No Partial Overlap ☐ Covers base set ☐

(7 points) Suppose that A_1, A_2, ..., A_n are non-empty subsets of A, and let $P = \{A_1, A_2, \ldots, A_n\}$. Also suppose that $A_1 \cap A_2 \cap \ldots \cap A_n = \emptyset$ and $A_1 \cup A_2 \cup \ldots \cup A_n = A$. Is P a partition of A? Explain why or why not.

(2 points) Check the (single) box that best characterizes each item.

If $f : \mathbb{R} \to \mathbb{P}(\mathbb{Z})$ then $f(17)$ is

an integer ☐ a set of integers ☐ undefined ☐

a power set ☐ one or more integers ☐

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(7 points) Suppose that A is a set and P is a collection of subsets of A. Using precise language and/or notation, state the conditions P must satisfy to be a partition of A.

(2 points) $\{\{p, q\} \ : \ p \in \mathbb{Z}^+, q \in \mathbb{Z}^+, \text{ and } pq = 6\} \ =$

(6 points) Check the (single) box that best characterizes each item.

$\{\{a, b\}, c\} = \{a, b, c\}$ true ☐ false ☐

If $f : \mathbb{N} \to \mathbb{P}(\mathbb{Q})$ then $f(3)$ is a rational ☐ a set of rationals ☐ undefined ☐ a power set ☐ one or more rationals ☐

$\binom{k}{k-1}$ 1 ☐ 2 ☐ k-1 ☐ k ☐ undefined ☐

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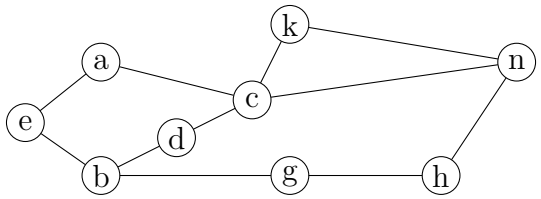
Graph G is at right.
V is the set of nodes. E is the set of edges.



Let $M : (V, \mathbb{N}) \to \mathbb{P}(V)$ such that $M(x, n) = \{y \in V \mid$ there is a path of length n from x to $y\}$.
Let $P(x) = \{M(x, n) \mid n \in \mathbb{N}\}$.

(3 points) $M(c, 2) =$

(3 points) Is $P(c)$ a partition of V? Check the partition properties that are satisfied.

No Empty set ☐ No Partial Overlap ☐ Covers base set ☐

(7 points) Let $f : X \to Y$ be any function, and let A and B be subsets of X. For any subset S of X define its image $f(S)$ by $f(S) = \{f(s) \in Y \mid s \in S\}$. Is it the case that $f(A) \cap f(B) = f(A \cap B)$? Informally explain why this is true or give a concrete counter-example showing why it is not.

(2 points) Check the (single) box that best characterizes each item.

$\{4, 5, 6\} \cap \{6, 7\}$ 6 ☐ $\{6\}$ ☐ $\{\{6\}\}$ ☐

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(7 points) Suppose that $f : A \to B$ is a function. Let's define $T : B \to \mathbb{P}(A)$ by $T(m) = \{x \in A \mid f(x) = m\}$. Then let $P = \{T(m) \mid m \in B\}$. Under what conditions is P a partition of A? Briefly justify your answer.

(2 points) $\{p + q^2 \mid p \in \mathbb{Z},\ q \in \mathbb{Z},\ 1 \leq p \leq 2 \text{ and } 1 \leq q \leq 3\} =$

(6 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(A) \cap \mathbb{P}(B) = \emptyset$

always ☐ sometimes ☐ never ☐

Set B is a partition of a finite set A. Then

$|B| \leq 2^{|A|}$ ☐ $|B| \leq |A|$ ☐

$|B| = 2^{|A|}$ ☐ $|B| \leq |A + 1|$ ☐

Pascal's identity states that $\binom{n}{k}$ is equal to

$\binom{n-1}{k} + \binom{n-1}{k-1}$ ☐ $\binom{n-1}{k} + \binom{n-1}{k+1}$ ☐ $\binom{n-1}{k} + \binom{n-2}{k}$ ☐

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Let $f : \mathbb{R} \to \mathbb{P}(\mathbb{R})$ such that $f(x) = \{p \in \mathbb{R} \mid \lfloor x \rfloor = \lfloor p \rfloor\}$. Let $T = \{f(x) \mid x \in \mathbb{R}\}$.

(3 points) Describe (at a high level) the elements of $f(7)$:

(3 points) Is T a partition of $\mathbb{R}$? Check the partition properties that are satisfied.

No Empty set ☐ No Partial Overlap ☐ Covers base set ☐

(7 points) Define $f : \mathbb{Z} \times \mathbb{Z}^+ \to \mathbb{P}(\mathbb{Z})$ by $f(x, k) = \{y \in \mathbb{Z} : x = y + kn \text{ for some } n \in \mathbb{Z}\}$. Suppose that $k|p$. Compare $f(r, k)$ and $f(r, p)$. Justify your answer.

(2 points) Check the (single) box that best characterizes each item.

	$\emptyset$	$\{\emptyset\}$	$\{\{\emptyset\}\}$	$\{\emptyset, \{\emptyset\}\}$
$\mathbb{P}(\emptyset)$	☐	☐	☐	☐

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(7 points) Let $f : \mathbb{Z}^+ \to \mathbb{P}(\mathbb{Z}^+)$ be defined by $f(n) = \{p \in \mathbb{Z}^+ : n|p\}$. Suppose that $f(a) = f(b) \cap f(c)$. Express a in terms of b and c. Briefly justify your answer.

(2 points) $\{\{p\} \mid p \in \{2, 3, 4\}\} =$

(6 points) Check the (single) box that best characterizes each item.

$|\{A \subseteq \mathbb{Z}_4 : |A| \text{ is even}\}|$ 1 ☐ 6 ☐ 7 ☐ 8 ☐ infinite ☐

There is a set A such that $|\mathbb{P}(A)| \leq 2$. true ☐ false ☐

$\binom{n}{1}$ -1 ☐ 0 ☐ 1 ☐ 2 ☐ n ☐ undefined ☐

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Suppose that $A = \{2, 3, 5, 13, 17\}$. Define a function $F : A \to \mathbb{P}(A)$ and a set S by $F(x) = \{y \in A \mid y \text{ is a factor of } x\}$ $S = \{F(x) \mid x \in A\}$

(3 points) $S =$

(3 points) Is S a partition of A? Check the partition properties that are satisfied.

No Empty set ☐ No Partial Overlap ☐ Covers base set ☐

(7 points) Let $f : X \to Y$ be any function, and let A and B be subsets of X. For any subset S of X define its image $f(S)$ by $f(S) = \{f(s) \in Y \mid s \in S\}$. Is it the case that $f(A) \cup f(B) = f(A \cup B)$? Informally explain why this is true or give a concrete counter-example showing why it is not.

(2 points) Check the (single) box that best characterizes each item.

A partition of a set A contains $\emptyset$ always ☐ sometimes ☐ never ☐

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(7 points) Give an example of a partition P of $\mathbb{N}$ where the set P is infinite. Be specific.

(2 points) $\{pq \mid p \in \mathbb{N},\ q \in \mathbb{N},\ p+q=6\} =$

(6 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(A) \cup \mathbb{P}(B) = \mathbb{P}(A \cup B)$ always ☐ sometimes ☐ never ☐

$|\{\emptyset\}|$ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ undefined ☐

$\{4,5\} \cap \{6,7\}$ $\emptyset$ ☐ $\{\emptyset\}$ ☐ nothing ☐ undefined ☐