CS 173, Spring 19 Examlet 2, white

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For all positive integers a, b, and c, if $\gcd(a, bc) = 1$, then $\gcd(a, b) = 1$ and $\gcd(a, c) = 1$.

   Solution: This is true. If $\gcd(a, bc) = 1$, then a doesn't share any prime factors with bc. Since the prime factors of b are a subset of these, they also can't overlap with the prime factors of a. Similarly for c.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(1568, 546)$. Show your work.

   Solution:

   $1568 - 546 \times 2 = 1568 - 1092 = 476$

   $546 - 476 = 70$

   $476 - 70 \times 6 = 476 - 420 = 56$

   $70 - 56 = 14$

   $56 - 14 \times 3 = 0$

   So the GCD is 14.

3. (4 points) Check the (single) box that best characterizes each item.

   For any positive integers p and q, if $\text{lcm}(p, q) = pq$, then p and q are relatively prime. true [√] false [ ]

   Zero is a factor of 7. true [ ] false [√]

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1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For all positive integers a, b, and c, $\gcd(ca, cb) = c \cdot \gcd(a, b)$

   Solution: This is true.

   c divides both ca and cb. So $\gcd(ca, cb)$ must have the form cm, where m is an integer. But then cm is the largest integer that divides both ca and cb if and only if m is the largest integer that divides both a and b.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(2015, 837)$. Show your work.

   Solution:

   $2015 - 837 \times 2 = 2015 - 1674 = 341$

   $837 - 341 \times 2 = 837 - 682 = 155$

   $341 - 155 \times 2 = 341 - 310 = 31$

   $155 - 31 \times 5 = 0$

   So the GCD is 31.

3. (4 points) Check the (single) box that best characterizes each item.

$\gcd(0, 0)$	0	☐	k	☐	undefined	☑
$25 \equiv 4 \pmod 7$	true	☑	false	☐		

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1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   For any positive integer k, $(k-1)^2 \equiv 1 \pmod{k}$.

   Solution: This is true. Notice that $(k-1)-(-1) = k$. So $k-1 \equiv (-1) \pmod{k}$. Therefore $(k-1)^2 \equiv (-1)^2 \equiv 1 \pmod{k}$.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(1183, 351)$. Show your work.

   Solution:

   $1183 - 3 \times 351 = 1183 - 1053 = 130$

   $351 - 2 \times 130 = 351 - 260 = 91$

   $130 - 91 = 39$

   $91 - 3 \times 39 = 91 - 78 = 13$

   $39 - 3 \times 13 = 0$

   So the GCD is 13.

3. (4 points) Check the (single) box that best characterizes each item.

   If a and b are positive and $r = \text{remainder}(a, b)$, then $\gcd(b, r) = \gcd(b, a)$ true [√] false [ ]

   $7 \mid -7$ true [√] false [ ]

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1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For all positive integers a, b, and c, if $\gcd(a, b) = 1$ and $\gcd(b, c) = 1$, then $\gcd(a, c) = 1$.

   Solution: This is false. Consider $a = c = 3$ and $b = 2$. Then a and b have no common factors, i.e. $\gcd(a, b) = 1$. Also b and c have no common factors, i.e. $\gcd(b, c) = 1$. But $\gcd(a, c) = 3$.

2. (6 points) Write pseudocode (iterative or recursive) for a function gcd(a,b) that implements the Euclidean algorithm. Assume both inputs are positive.

   Solution:

```
gcd(a,b)
    x=a
    y=b
    while (b > 0)
        r = remainder(a,b)
        a = b
        b = r
    return a
```

3. (4 points) Check the (single) box that best characterizes each item.

$\gcd(p, q) = \frac{pq}{\text{lcm}(p, q)}$ (p and q positive integers)	always ☑	sometimes ☐	never ☐
$-7 \equiv 13 \pmod 6$	true ☐	false ☑	

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1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   For any positive integers a, b, and c, if $a \mid bc$, then $a \mid b$ or $a \mid c$

   Solution: This is false. Consider $a = 6$, $b = 3$, $c = 2$. Then $a|bc$, but a doesn't divide either b or c.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(1702, 1221)$. Show your work.

   Solution: $1702 - 1221 = 481$

   $1221 - 481 \times 2 = 1221 - 962 = 259$

   $481 - 259 = 222$

   $259 - 222 = 37$

   $222 - 6 \times 37 = 0$

   So the GCD is 37.

3. (4 points) Check the (single) box that best characterizes each item.

   If a and b are positive integers and $r = \text{remainder}(a, b)$, then $\gcd(b, r) = \gcd(r, a)$ — true ☐ false ☑ (√)

   $29 \equiv 2 \pmod 9$ — true ☑ (√) false ☐

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1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For all positive integers a, b, and c, if $\gcd(a,b) = n$ and $\gcd(a,c) = p$, then $\gcd(a,bc) = np$.

   Solution: This is false. Consider $a = b = c = 3$. Then if $\gcd(a,b) = 3$ and $\gcd(a,c) = 3$, but $\gcd(a,bc)$ is 3, not 9.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(2380, 391)$. Show your work.

   Solution:

   $2380 - 391 \times 6 = 2380 - 2346 = 34$

   $391 - 34 \times 11 = 391 - 374 = 17$

   $34 - 17 \times 2 = 0$

   So the GCD is 17.

3. (4 points) Check the (single) box that best characterizes each item.

   For any positive integers p, q, and k, if $p \equiv q \pmod{k}$, then $p^2 \equiv q^2 \pmod{k}$ — true [√] false [ ]

   $2 \mid -4$ — true [√] false [ ]

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Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For any positive integers p and q, $p \equiv q \pmod 1$.

   Solution: This is true. $p \equiv q \pmod 1$ is equivalent to $p - q = n \times 1 = n$ for some integer n. But we can always find an integer that will make this equation balance!

2. (6 points) Use the Euclidean algorithm to compute $\gcd(7917, 357)$. Show your work.

   Solution:

   $7917 - 22 \times 357 = 63$

   $357 - 5 \times 63 = 42$

   $63 - 42 = 21$

   $42 - 2 \times 21 = 0$

   So the GCD is 21.

3. (4 points) Check the (single) box that best characterizes each item.

   If a and b are positive integers and $r = \text{remainder}(a, b)$, then $\gcd(a, b) = \gcd(r, a)$ — true ☐ false ☑ (√)

   $-2 \equiv 2 \pmod 4$ — true ☑ (√) false ☐

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1. (5 points) Is the following claim true? Informally explain why it is false, or give a concrete example showing that it is not.

   There is an integer n such that $n \equiv 5 \pmod 6$ and $n \equiv 6 \pmod 7$?

   Solution: This is true. Consider $n = 41$. $41 \equiv 5 \pmod 6$ and $41 \equiv 6 \pmod 7$.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(1224, 850)$. Show your work.

   Solution:

   $1224 - 850 = 374$

   $850 - 374 \times 2 = 850 - 748 = 102$

   $374 - 102 \times 3 = 374 - 306 = 68$

   $102 - 68 = 34$

   $68 - 34 \times 2 = 0$

   So the GCD is 34.

3. (4 points) Check the (single) box that best characterizes each item.

   Two positive integers p and q are relatively prime if and only if $\gcd(p, q) = 1$. true [√] false [ ]

   $0 \mid 0$ true [√] false [ ]