Name:____

NetID:________________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

$A = \{(x, y) \in \mathbb{R}^2 \ : \ y = x^2 - 4\}$

$B = \{(p, q) \in \mathbb{Z}^2 \ : q < 0\}$

$C = \{(a, b) \in \mathbb{R}^2 \ : \ |a| \leq 1\}$

Prove that $A \cap B \subseteq C$.

Solution: Proof: Let $(x, y) \in A \cap B$. Then, $(x, y) \in A$ and $(x, y) \in B$. So, from the definition of A, we know that $y = x^2 - 4$. From the definition of B, we know that $y < 0$ and that x and y are both integers.

$y = x^2 - 4 = (x - 2)(x + 2)$. So since $y < 0$, $-2 < x < 2$. But x is an integer. So the only possible values in this range are -1, 0, and 1. Therefore $|x| \leq 1$. So $(x, y) \in C$, which is what we needed to prove.

Name:____

NetID:________________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

$A = \{(x, y, z) \in \mathbb{R}^3 \mid 0 < x < y - 1\}$

$B = \{(a, b, c) \in \mathbb{R}^3 \mid b^2 + 2 < c^2\}$

$C = \{(p, q, r) \in \mathbb{R}^3 \mid p^2 < r^2\}$

Prove that $A \cap B \subseteq C$.

Solution: Let $(x, y, z) \in A \cap B$. Then $(x, y, z) \in A$, so (x, y, z) is a triple of real numbers with $0 < x < y - 1$. Also $(x, y, z) \in B$, so $y^2 + 2 < z^2$.

We know that $0 < x < y-1$. Since $y-1 > 0$, $y > 0$, so $-2y < 0$. Squaring both sides of $x < y-1$ and using the fact that both sides of the equation are positive, we get $x^2 < y^2-2y+1$. So $x^2 < y^2+1 < y^2+2$. But we know that $y^2 + 2 < z^2$. So we have $x^2 < z^2$, and therefore $(x, y, z) \in C$, which is what we needed to show.

Name:____

NetID:______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

$A = \{\alpha(2,-4) + (1-\alpha)(-3,6)) \mid \alpha \in \mathbb{R}\}$

$B = \{(a,b) \in \mathbb{R}^2 \mid a \geq 1\}$

$C = \{(p,q) \in \mathbb{R}^2 \mid q \leq 0\}$

Prove that $A \cap B \subseteq C$.

Solution: Let (x,y) be a 2D point and suppose that $(x,y) \in A \cap B$. Then $(x,y) \in A$ and $(x,y) \in B$.

Since $(x,y) \in A$, $(x,y) = \alpha(2,-4) + (1-\alpha)(-3,6))$ where α is a real number. So $x = 2\alpha - 3(1-\alpha) = 5\alpha - 3$. And $y = -4\alpha + 6(1-\alpha) = 6 - 10\alpha$.

Since $(x,y) \in B$, we know that $x \geq 1$. So $5\alpha - 3 \geq 1$. Therefore $\alpha \geq \frac{4}{5}$.

Substituting this into the equation for y, we get $y = 6 - 10\alpha \leq 6 - 10\frac{4}{5} = 6 - 8 = -2 \leq 0$. Since $y \leq 0$, $(x,y) \in C$, which is what we needed to show.

CS 173, Spring 19 Examlet 3, colored sheet

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

$A = \{(x, y) \in \mathbb{R}^2 \ : \ y = x^2 - 2x - 1\}$

$B = \{(p, q) \in \mathbb{R}^2 \ : \ |p| \geq 3\}$

$C = \{(m, n) \in \mathbb{R}^2 \ : \ n \geq 0\}$

Prove that $A \cap B \subseteq C$.

Solution: Let $(x, y) \in \mathbb{R}^2$ and suppose that $(x, y) \in A \cap B$. Then $(x, y) \in A$ and $(x, y) \in B$.

Since $(x, y) \in A$, $y = x^2 - 2x - 1$. So $y = x(x - 2) - 1$.

Since $(x, y) \in B$, $|x| \geq 3$. There are two cases:

Case 1: $x \geq 3$. Then $x - 2 \geq 1$. So $y \geq 3 \cdot 1 - 1 = 2$.

Case 2: $x \leq -3$. Then $x - 2 \leq -5$. So $x(x - 2) \geq (-3)(-5) = 15$. Therefore $y = x(x - 2) - 1 \geq 14$.

In both cases, $y \geq 0$. So $(x, y) \in C$, which is what we needed to prove.

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

$A = \{(x, y, z) \in \mathbb{R}^3 \ : y = x^2 - 2x + 11\}$

$B = \{(a, b, c) \in \mathbb{R}^3 \ : \ b \leq c\}$

$C = \{(p, q, r) \in \mathbb{R}^3 \ : \ r \geq 5\}$

Prove that $A \cap B \subseteq C$.

Solution: Let $(x, y, z) \in A \cap B$. Then $(x, y, z) \in A$, so $y = x^2 - 2x + 11$. Also $(x, y, z) \in B$, so $y \leq z$.

We can rewrite the first equation as $y = (x-1)^2 + 10$. $(x-1)^2 \geq 0$ because it's the square of a real number. So $y \geq 10$.

We now have $y \geq 10$ and $y \leq z$. Combining these gives us $z \geq 10$. So $z \geq 5$. Therefore $(x, y, z) \in C$, which is what we needed to show.

Name:____

NetID:________________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

$A = \{(x, y, z) \in \mathbb{R}^3 \ : \ |x + y + z| = 20\}$

$B = \{(a, b, c) \in \mathbb{N}^3 \ : \ a + b < 5\}$

$C = \{(p, q, r) \in \mathbb{R}^3 \ : \ r > 10\}$

Prove that $A \cap B \subseteq C$.

Solution: Let $(x, y, z) \in A \cap B$. Then $(x, y, z) \in A$, so $|x + y + z| = 20$. Also $(x, y, z) \in B$, so $x + y < 5$ and x, y, and z are all natural numbers.

Since x, y, and z are natural numbers, they can't be negative. So $x + y + z$ isn't negative. Therefore $x + y + z = |x + y + z| = 20$. So $z = 20 - (x + y)$.

Since $z = 20 - (x + y)$ and $x + y < 5$, $z > 15$. So $z > 10$, which means that $(x, y, z) \in C$. This is what we needed to show.

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

$A = \{(x, y) \in \mathbb{R}^2 \ : \ xy \leq -7\}$

$B = \{(p^3, p^2) \ : \ p \in \mathbb{R}\}$

$C = \{(a, b) \in \mathbb{R}^2 \ : \ a < 0\}$

Prove that $A \cap B \subseteq C$.

Solution: Proof: Let $(x, y) \in A \cap B$. Then, $(x, y) \in A$ and $(x, y) \in B$. So, from the definition of A, we know that $xy \leq -7$. From the definition of B, we know that $x = p^3$ and $y = p^2$, for some real number p.

Since $xy \leq 7 < 0$, we know x and y have opposite signs and neither is zero. Since $y = p^2$, we know that y is positive. So x must be negative.

Since x is negative, $(x, y) \in C$, which is what we needed to prove.

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

$A = \{a(1,0) + b(3,1) + c(2,4) \; : \; a, b, c \text{ are positive reals and } a + b + c = 1\}$

$B = \{(x, y) \in \mathbb{R}^2 \; : \; x \leq 3 \text{ and } y \geq 0\}$

Prove that $A \subseteq B$.

Solution: Let $(x, y) \in A$. By the definition of A, $(x, y) = a(1,0) + b(3,1) + c(2,4)$, where a, b, and c are positive reals and $a + b + c = 1$.

Then $(x, y) = (a + 3b + 2c, b + 4c)$. So $x = a + 3b + 2c$ and $y = b + 4c$.

We know that a, b, and c are positive, so $b + 4c$ must be positive. So $y \geq 0$.

Since a and c are positive and $a + b + c = 1$, we have

$$x = a + 3b + 2c \leq 3a + 3b + 3c = 3(a + b + c) = 3$$

So $y \geq 0$ and $x \leq 3$. Therefore (x, y) is in B, by the definition of the set B.