

Name: _____

NetID: _____ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

Let's define a relation T between natural numbers follows:

aTb if and only if $a = b + 2k$, where k is a natural number

Working directly from this definition, prove that T is antisymmetric.

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A closed interval of the real line can be represented as a pair (c, r) , where c is the center of the interval and r is its radius. Let $X = \{(c, r) \mid c, r \in \mathbb{R}, r \geq 0\}$ be the set of closed intervals represented this way.

Now, let's define the interval containment \preceq on X as follows

$$(c, r) \preceq (d, q) \text{ if and only if } r \leq q \text{ and } |c - d| + r \leq q.$$

Prove that \preceq is transitive.

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Suppose that T is a relation on the integers which is transitive. Let's define a relation R on the integers as follows:

xRy if and only if there is an integer k such that xTk and kTy .

Prove that R is transitive.

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Let T be the relation defined on \mathbb{Z}^2 by

$$(x, y)T(p, q) \text{ if and only if } x < p \text{ or } (x = p \text{ and } y \leq q)$$

Prove that T is antisymmetric.

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Define the relation \sim on \mathbb{Z} by

$$x \sim y \text{ if and only if } 5 \mid (3x + 7y)$$

Working directly from the definition of divides, prove that \sim is transitive.

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Let $A = \{(x, y, z) \in \mathbb{Z}^3 : x \leq y \leq z\}$. Let's define a relation R on A as follows:

$(a, b, c)R(x, y, z)$ if and only if $a \leq x$ and $z \leq b$.

Working directly from this definition, prove that R is antisymmetric.

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Let $A = \mathbb{Z}^+ \times \mathbb{Z}^+$, i.e. pairs of positive integers.

Define a relation \gg on A as follows:

$(x, y) \gg (p, q)$ if and only if there exists an integer $n \geq 1$ such that $(x, y) = (np, nq)$.

Prove that \gg is antisymmetric.

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Let $A = \{(x, y, z) \in \mathbb{Z}^3 : x \leq y \leq z\}$. Let's define a relation R on A as follows:

$(a, b, c)R(x, y, z)$ if and only if $a \leq x$ and $z \leq b$.

Working directly from this definition, prove that R is transitive.