CS 173, Spring 19 Examlet 5, colored

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (10 points) Suppose that $f : \mathbb{Z} \to \mathbb{Z}$ is onto. Let's define $g : \mathbb{Z}^2 \to \mathbb{Z}$ by $g(x, y) = f(x - 7)f(y)$. Prove that g is onto.

2. (5 points) Using precise mathematical words and notation, define what it means for a function $g : C \to M$ to be "onto." You must use explicit quantifiers. Do not assume the reader knows what the image of the function is.

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1. (10 points) Suppose that $f : \mathbb{Z} \to \mathbb{Z}$ is one-to-one. Let's define $g : \mathbb{Z}^2 \to \mathbb{Z}^2$ by $g(x, y) = (2f(x) + f(y), f(x) - f(y))$. Prove that g is one-to-one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) derivatives or the behavior of increasing functions.

2. (5 points) Using precise mathematical words and notation, define what it means for a function $g : C \to M$ to be "one-to-one." You must use explicit quantifiers; do not use words like "unique".

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1. (10 points) Suppose that A and B are sets. Suppose that $f : B \to A$ and $g : A \to B$ are functions such that $f(g(x)) = x$ for every $x \in A$. Prove that f is onto.

2. (5 points) Give an example of a function $f : \mathbb{N} \to \mathbb{N}$ which is one-to-one but not onto. Your answer must include a specific formula.

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1. (10 points) Suppose that $f : \mathbb{Z} \to \mathbb{Z}$ is onto. Let's define $g : \mathbb{Z}^2 \to \mathbb{Z}$ by $g(x, y) = f(x) + 2f(y) - 6$. Prove that g is onto.

2. (5 points) $A = \{0, 2, 4, 6, 8, 10, 12, \ldots\}$, i.e. the even integers starting with 0.

   $B = \{1, 4, 9, 16, 25, 36, 49, \ldots\}$, i.e. perfect squares starting with 1.

   Give a specific formula for a bijection $f : A \to B$. (You do not need to prove that it is a bijection.)

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1. (10 points) Suppose that $f : A \to B$ and $g : B \to C$ are one-to-one. Prove that $g \circ f$ is one-to-one.

2. (5 points) Using precise mathematical words and notation, define what it means for a function $g : \mathbb{R} \to \mathbb{R}$ to be "increasing." You must use explicit quantifiers.

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1. (10 points) Let P be the set of pairs of positive integers. Suppose that $f : P \to \mathbb{R}^2$ is defined by $f(x, y) = (\frac{x}{y}, x + y)$. Prove that f is one-to-one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) derivatives or the behavior of increasing functions.

2. (5 points) Give an example of a function $f : \mathbb{N} \to \mathbb{N}$ which is onto but not one-to-one. Your answer must include a specific formula.

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1. (10 points) Suppose that $f : \mathbb{Z}^2 \to \mathbb{Z}$ is defined by $f(x, y) = 3x + 5y$. Prove that f is onto.

2. (5 points) $A = \{0, 1, 4, 9, 16, 25, 36, \ldots\}$, i.e. perfect squares starting with 0.
   $B = \{2, 4, 6, 8, 10, 12, 14, \ldots\}$, i.e. the even integers starting with 2.
   Give a specific formula for a bijection $f : A \to B$. (You do not need to prove that it is a bijection.)

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1. (10 points) Let's define the function $f : \mathbb{R}^+ \to \mathbb{R}$ by $f(x) = \frac{4x-1}{2x+5}$. ($\mathbb{R}^+$ is the positive reals.) Prove that f is one to one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) derivatives or the behavior of increasing functions.

2. (5 points) Using precise mathematical words and notation, define what it means for a function $g : \mathbb{R} \to \mathbb{R}$ to be "strictly increasing." You must use explicit quantifiers.