CS 173, Spring 19 Examlet 5, white

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) How many different 12-letter strings can be made be rearranging the letters in the word ``apalachicola''? Show your work.

   Solution: There are 12 letters total to rearrange, with 4 copies of a, 2 copies of l, and 2 copies of c. So the total number of possibilities is

$$\frac{12!}{4!2!2!}$$

2. (10 points) Check the (single) box that best characterizes each item.

If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = 2x$ then the set of all even integers is the _____ of f.

domain ☐ co-domain ☐ image ☑ none of these ☐

$f : \mathbb{N}^2 \to \mathbb{N}$
$f(p, q) = pq$

onto ☑ not onto ☐ not a function ☐

$g : (\mathbb{Z}^+)^2 \to \mathbb{Z}^+$
$g(x, y) = \gcd(x, y)$

one-to-one ☐ not one-to-one ☑ not a function ☐

We painted 12 mailboxes. There were 5 colors to choose from and each mailbox is painted with a single color. By the pigeonhole principle, there is a color that appears on exactly two mailboxes.

true ☐ false ☑

$\exists y \in \mathbb{Z},\ \forall x \in \mathbb{Z},\ y \leq x$

true ☐ false ☑

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1. (5 points) Hermione Grainger has 7000 socks in her magically expanding drawer. The socks are colored purple, magenta, and shocking pink. How many socks must she pull out of the drawer before she is guaranteed to have two socks of the same color. Briefly justify your answer.

   Solution: She needs to pull out four socks. By the pigeonhole principle, four socks and only three colors means that two must have the same color.

2. (10 points) Check the (single) box that best characterizes each item.

A function is onto if and only if its image is the same as its co-domain.	true	$\boxed{\surd}$	false	$\square$		
$g : \mathbb{R} \to [-1, 1]$ $g(x) = \sin(x)$	onto	$\boxed{\surd}$	not onto	$\square$	not a function	$\square$
$g : \mathbb{R}^2 \to \mathbb{R}^2$ $g(x, y) = (y, 3x)$	one-to-one	$\boxed{\surd}$	not one-to-one	$\square$	not a function	$\square$
Each elf has exactly one gift: charm, strength, or stamina. If there are 10 elves, the pigeonhole principle says that at least three elves have charm.	true	$\square$	false	$\boxed{\surd}$		
$\exists t \in \mathbb{N},\ \forall p \in \mathbb{Z}^+,\ \gcd(p, t) = p$	true	$\boxed{\surd}$	false	$\square$		

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1. (5 points) 8 presidential candidates (including Bernie and Hilary) need to line up for a photo. The new editor would like Bernie and Hilary to stand next to each other. How many different ways can we arrange the eight people?

   Solution: Bernie and Hilary can be moved as a unit. So we have 7 objects to permute, so 7! permutations. However, Bernie might stand either to the left or to the right of Hilary. So the total number of possibilities is $2 \cdot 7!$.

2. (10 points) Check the (single) box that best characterizes each item.

   Suppose $f : A \to B$. For all $x, y \in A$, if $x = y$, then $f(x) = f(y)$. onto ☐ one-to-one ☐ neither ☑ $\surd$

   $g : \mathbb{R} \to [0, 1]$
   $g(x) = \sin(x)$ onto ☐ not onto ☐ not a function ☑ $\surd$

   $f : \mathbb{R} \to \mathbb{Z}$
   $f(x) = x$ one-to-one ☐ not one-to-one ☐ not a function ☑ $\surd$

   Each dorm room is given an integer access code between 1 and 10 (inclusive). According to the pigeonhole principle, if there are 21 dorm rooms, then every access code must be shared by at least two rooms. true ☐ false ☑ $\surd$

   $\exists y \in \mathbb{Z},\ \forall x \in \mathbb{Z},\ x - y < 100$ true ☐ false ☑ $\surd$

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1. (5 points) To make exam grading anonymous and therefore hopefully more fair, each of the 200 students in CS 241 has been assigned a unique 3-character exam code. The character set is $\{\alpha, \beta, \gamma, \delta\}$. Use the Pigeonhole Principle to explain what's wrong with this plan.

   Solution: Since there are four distinct characters, there are $4^3 = 64$ different 3-character codes. Since there are more students than codes, the pigeonhole principle implies that there is at least one pair of students with the same code.

2. (10 points) Check the (single) box that best characterizes each item.

   If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = 2x$ then the real numbers is the _____ of f.
   domain ☐ co-domain [√] image ☐ none of these ☐

   $f : \mathbb{Z} \to \mathbb{Z}$
   $f(x) = x + 3$ (x even),
   $f(x) = x - 22$ (x odd)
   onto ☐ not onto [√] not a function ☐

   $g : \mathbb{N}^2 \to \mathbb{N}$
   $g(x, y) = \gcd(x, y)$
   one-to-one ☐ not one-to-one ☐ not a function [√]

   Each elf has exactly one gift: charm, strength, or stamina. If there are 10 elves, there must be at least three elves with the same gift.
   true [√] false ☐

   $\exists y \in \mathbb{R}^+,\ \forall x \in \mathbb{R}^+,\ xy = 1$
   ($\mathbb{R}^+$ is the positive real numbers.)
   true ☐ false [√]

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Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Suppose that $|A| = p$ and $|B| = q$. How many different functions are there from A to B?

   Solution: q^p

2. (10 points) Check the (single) box that best characterizes each item.

   A function is one-to-one if and only if each value in the co-domain has at most one pre-image. true ☑ false ☐

   $g : \mathbb{R}^2 \to \mathbb{R}$
   $g(x, y) = \lfloor x \rfloor + y$ onto ☑ not onto ☐ not a function ☐

   $g : \mathbb{Z}^2 \to \mathbb{Z}^2$
   $g(x, y) = (y, 3x)$ one-to-one ☑ not one-to-one ☐ not a function ☐

   Each ACM shirt has one of 6 trendy slogans. I bought 13 ACM shirts. At least three of these shirts must have the same slogan. true ☑ false ☐

   $\forall x \in \mathbb{Q},\ \exists m, n \in \mathbb{Z},\ x = \frac{m}{n}$ true ☑ false ☐

Name:____

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Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) 15 men and 15 women showed up to this week's meeting of the UIUC Swing Dance Society. How many different ways can we form all of them into pairs, each pair containing one man and one woman?

   Solution: We're constructing a bijection from the women to the men (or vice versa). Since there are 15 people in each set, there are 15! bijections.

2. (10 points) Check the (single) box that best characterizes each item.

Suppose $f : A \to B$. For all $x \in A$, there is a $y \in B$, $f(x) = y$.	onto ☐	one-to-one ☐	neither ☑
$g : (\mathbb{Z}^+)^2 \to \mathbb{Z}^+$ $g(x, y) = \gcd(x, y)$	onto ☑	not onto ☐	not a function ☐
$f : \mathbb{N} \to \mathbb{R}$ $f(x) = x^2 + 2$	one-to-one ☑	not one-to-one ☐	not a function ☐
Each ACM shirt has one of 6 trendy slogans. I bought 13 ACM shirts. There is a slogan that appears on at least two shirts.	true ☑	false ☐	
$\forall x \in \mathbb{Z},\ \exists y \in \mathbb{N},\ x^2 = y$	true ☑	false ☐	

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1. (5 points) Suppose that $|A| = 2$ and $|B| = 3$. How many onto functions are there from A to B? Briefly justify or show work.

   Solution: There are no onto functions from A to B, because $|A|$ is smaller than $|B|$.

2. (10 points) Check the (single) box that best characterizes each item.

   If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = 2x$ then the integers is the _____ of f.
   domain ☑ co-domain ☐
   image ☐ none of these ☐

   $f : \mathbb{N}^2 \to \mathbb{Z}$
   $f(p, q) = 2^p 3^q$
   onto ☐ not onto ☑ not a function ☐

   $g : \mathbb{Z} \to \mathbb{Z}$
   $g(x) = x|x|$
   one-to-one ☑ not one-to-one ☐ not a function ☐

   Each elf has exactly one gift: charm, strength, or stamina. If there are 10 elves, the pigeonhole principle says that at least one elf has stamina.
   true ☐ false ☑

   $\forall x \in \mathbb{R}^+,\ \exists y \in \mathbb{R}^+,\ xy = 1$
   ($\mathbb{R}^+$ is the positive real numbers.)
   true ☑ false ☐

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1. (5 points) Prof. Snape is teaching potions to 52 girls and 73 boys. Quiz 1 has integer scores between zero and 100 (inclusive). Assuming no one missed the quiz, what is the probability that two students got the same score? Briefly justify your answer.

   Solution: There are 101 distinct quiz scores and 125 students. By the pigeonhole principle, two students got the same score. So probability 1.0 (or 100% if you prefer that notation).

2. (10 points) Check the (single) box that best characterizes each item.

If $f : A \to B$ is one-to-one, then	$\|A\| \geq \|B\|$ ☐	$\|A\| \leq \|B\|$ ☑	$\|A\| = \|B\|$ ☐
$g : \mathbb{N}^2 \to \mathbb{N}$ $g(x, y) = \gcd(x, y)$	onto ☐	not onto ☐	not a function ☑
$g : \mathbb{R} \to \mathbb{R}^2$ $g(x) = (x, 3x^2 + 2)$	one-to-one ☑	not one-to-one ☐	not a function ☐
We painted 12 mailboxes. There were 5 colors to choose from and each mailbox is painted with a single color. By the pigeonhole principle, there is a color that appears on at least two mailboxes.	true ☑	false ☐	
$\exists m, n \in \mathbb{Z},\ \forall x \in \mathbb{Q},\ x = \frac{m}{n}$	true ☐	false ☑	