CS 173, Spring 19 Examlet 7, colored

Name:____

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Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(15 points) Working directly from the definition of divides, use (strong) induction to prove the following claim:

Claim: $n^3 + 5n$ is divisible by 6, for all positive integers n.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(15 points) Use (strong) induction to prove the following claim.

Claim: For any positive integer n, $\sum_{p=1}^{n} \log(p^2) = 2\log(n!)$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(15 points) Let A be a constant integer. Use (strong) induction to prove the following claim. Remember that $0! = 1$.

Claim: For any integer $n \geq A$, $\sum_{p=A}^{n} \frac{p!}{A!(p-A)!} = \frac{(n+1)!}{(A+1)!(n-A)!}$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(15 points) The operator $\prod$ is like $\sum$ except that it multiplies its terms rather than adding them. So e.g. $\prod_{p=3}^{5}(p+1) = 4 \cdot 5 \cdot 6$. Use (strong) induction to prove the following claim:

$\prod_{p=2}^{n}(1 - \frac{1}{p^2}) = \frac{n+1}{2n}$ for any integer $n \geq 2$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(15 points) Use (strong) induction to prove the following claim:

Claim: for all natural numbers n, $\sum_{j=0}^{n} 2(-7)^j = \frac{1-(-7)^{n+1}}{4}$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(15 points) Use (strong) induction to prove the following claim.

Claim: For any positive integer n, $\sum_{p=1}^{n} \frac{1}{\sqrt{p-1}+\sqrt{p}} = \sqrt{n}$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step: (Recall that $\frac{1}{a+b} = \frac{a-b}{(a-b)(a+b)} = \frac{a-b}{a^2-b^2}$.)

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(15 points) Use (strong) induction to prove the following claim:

Claim: $\sum_{j=1}^{n} \frac{1}{j(j+1)} = \frac{n}{n+1}$ for all positive integers n.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(15 points) Use (strong) induction and the fact that $\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$ to prove the following claim:

For all natural numbers n, $\left(\sum_{i=0}^{n} i\right)^2 = \sum_{i=0}^{n} i^3$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step: (Start by removing the top term from the sum on the lefthand side.)