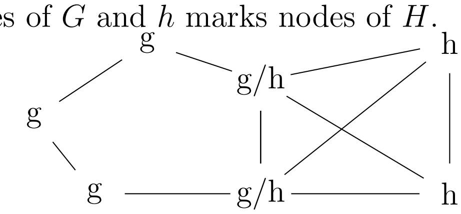
Name:____

NetID:____________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (11 points) If G is a graph, recall that $\chi(G)$ is its chromatic number. Suppose that G is a graph with at least one edge and H is another graph with at least one edge, not connected to G. Now, pick a specific edge e from G and an edge f from H and merge the two edges, creating a combined graph T. For example, suppose that G is C_5 and H is K_4. Then T might look as follows, where g marks nodes of G and h marks nodes of H.



Describe how $\chi(T)$ is related to $\chi(G)$ and $\chi(H)$, justifying your answer. Your answer should handle any choice for G and H.

Solution: $\chi(T) = \max(\chi(G), \chi(H))$

Lower bound: G is a subgraph of T, so $\chi(T) \geq \chi(G)$. Similarly, $\chi(T) \geq \chi(H)$. So $\chi(T) \geq \max(\chi(G), \chi(H))$.

Upper bound: Suppose that $k = \max(\chi(G), \chi(H))$. We can color G with k colors, because $k \geq \chi(G)$. The merged edge in H is already colored. Because $k \geq \chi(H)$, we can extend this to a coloring of H with k colors. So $\chi(T) \leq k = \max(\chi(G), \chi(H))$.

2. (4 points) Check the (single) box that best characterizes each item.

$\sum_{i=1}^{p-1} i$ $\quad \frac{p(p-1)}{2}$ ☑ $\quad \frac{(p-1)^2}{2}$ ☐ $\quad \frac{p(p+1)}{2}$ ☐ $\quad \frac{(p-1)(p+1)}{2}$ ☐

Leal team's bridge held 100 pounds without collapsing. 100 pounds is _____ on how much the bridge can hold.

an upper bound on ☐ exactly ☐
a lower bound on ☑ not a bound on ☐

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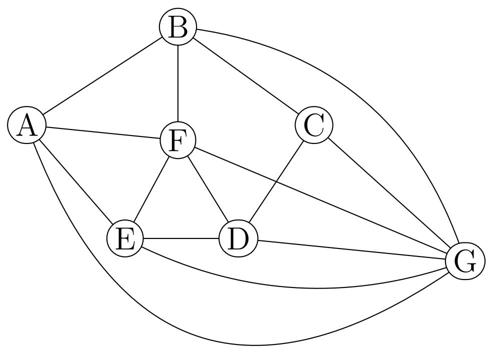
Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (9 points) What is the chromatic number of the graph below? Justify your answer.



Solution: The chromatic number is 4. We can color the graph with four colors as follows: A and D first color, B and E second color, F and C third color, G fourth color. The graph can't be colored with only 3 colors, because The graph contains an odd wheel (rim A,B, C, D, E and hub G).

2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of a graph containing a C_7.	≥ 2 ☐	≥ 3 ☑ √	≤ 3 ☐	can't tell ☐
$\sum_{i=1}^{p-1} i$	$\frac{(p-1)^2}{2}$ ☐	$\frac{(p-1)(p+1)}{2}$ ☐	$\frac{p(p+1)}{2}$ ☐	$\frac{p(p-1)}{2}$ ☑ √

$\tau \leq 1.3$	an upper bound on τ ☑ √	exactly τ ☐
	a lower bound on τ ☐	not a bound on τ ☐

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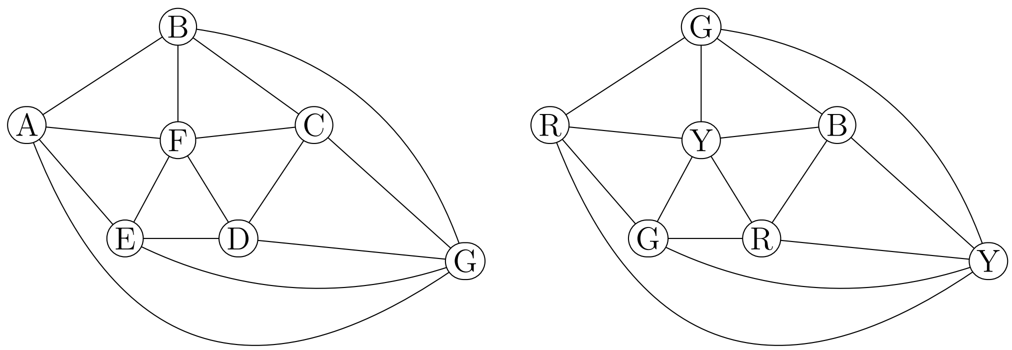
Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (9 points) What is the chromatic number of the graph below? Justify your answer.



Solution: The chromatic number is four. The picture above shows how to color it with four colors (upper bound).

For the lower bound, the graph contains a W_5 whose hub is F and whose rim contains nodes A, B, C, D, E. Coloring a W_5 requires four colors.

2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of a graph containing a W_n.	≤ 3 ☐	≥ 3 ☑ $\surd$	$\geq n$ ☐	can't tell ☐
$\sum_{i=1}^{p-1} \frac{i}{p}$	$\frac{p(p-1)}{2}$ ☐	$\frac{p(p+1)}{2}$ ☐	$\frac{(p+1)}{2}$ ☐	$\frac{(p-1)}{2}$ ☑ $\surd$

Putting 10 people in the canoe caused it to sink. 10 is _____ how many people the canoe can carry.	an upper bound on ☑ $\surd$	exactly ☐
	a lower bound on ☐	not a bound on ☐

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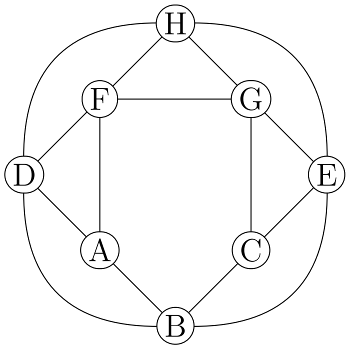
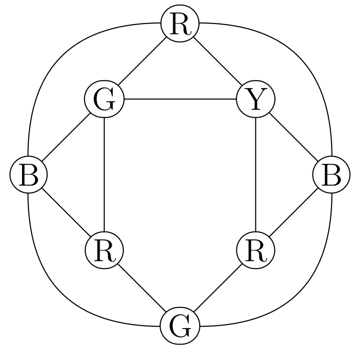
Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (9 points) What is the chromatic number of the graph below? Justify your answer.





Solution: This graph has chromatic number four. The picture above shows that four colors are enough. If you delete node H (and all its edges), you get the special graph presented towards the end of section 10.3 in the textbook, which we know to require four colors.

Alternatively, you can directly argue the lower bound as follows. Suppose we try to color this with three colors. Color the triangle A, B, D with R, G, B respectively. Then F must have color G. Nodes E and C must have colors R and B in some order. But then node G has neighbors with all three colors, so we're stuck. Therefore three colors is not enough.

2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of an connected acyclic graph with 5 nodes. ≤ 2 ☐ $= 2$ [√] ≤ 5 ☐ can't tell ☐

$$\sum_{i=0}^{k-1}(k \cdot i + 2)$$

$\frac{k^2(k+1)}{2} + 2k$ ☐ $\frac{k(k+1)}{2} + 2(k-1)$ ☐

$\frac{k^2(k-1)}{2} + 2k$ [√] $\frac{k(k-1)}{2} + 2(k-1)$ ☐

$\pi \geq 1.3$

an upper bound on π ☐ exactly π ☐

a lower bound on π [√] not a bound on π ☐

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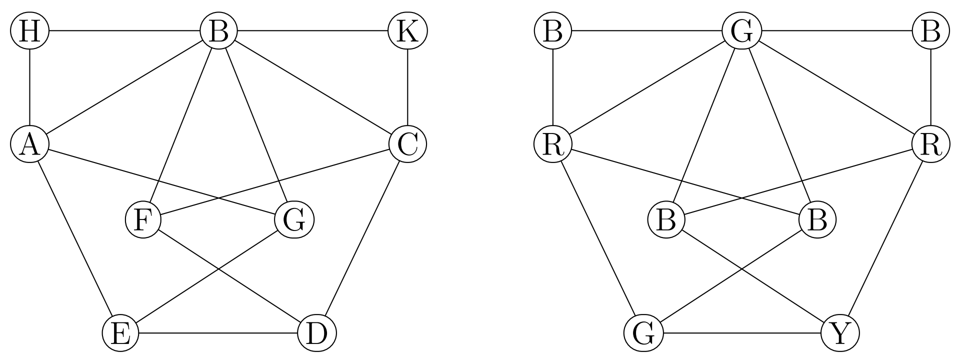
Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (9 points) What is the chromatic number of the graph below? Justify your answer.



Solution: The chromatic number of this graph is four.

The above picture shows that it can be colored with four colors (upper bound).

To show the lower bound, we could notice that if we delete the nodes H and K, we get a graph that we've shown to have chromatic number 4 (see section 10.3 of the textbook). If you want to argue this directly, color the triangle ABG with three colors as shown above. Then E must have the same color (G) as B. F and C must have the other two colors (in either order). This means that D has neighbors of all three colors. So three colors is not enough and therefore four is a lower bound.

2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of a connected graph with 10 nodes. ≤ 2 ☐ $= 2$ ☐ ≥ 2 ☑ can't tell ☐

$\sum_{k=-2}^{n} k^2$ $\sum_{p=0}^{n+2} (p+2)^2$ ☐ $\sum_{p=0}^{n-2} (p-2)^2$ ☐ $\sum_{p=0}^{n+2} (p-2)^2$ ☑ $\sum_{p=0}^{n+2} p^2$ ☐

We have 30 tablespoons of filling. Each bun requires exactly one tablespoon of filling. 30 is _____ on how many buns we can make.

an upper bound on ☑ exactly ☐
a lower bound on ☐ not a bound on ☐

Name:____

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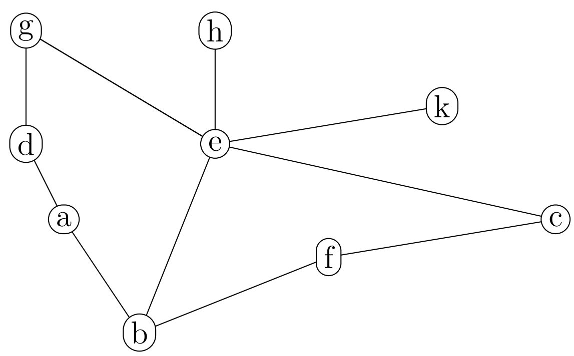
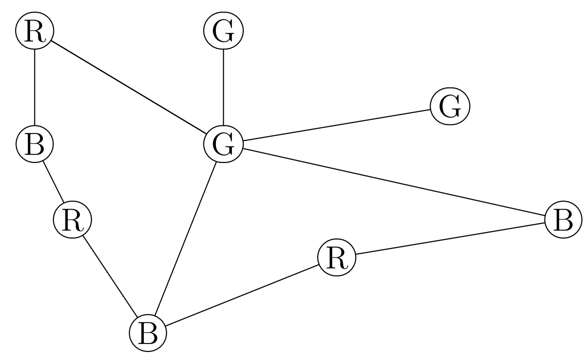
Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (9 points) What is the chromatic number of the graph below? Justify your answer.





Solution: The chromatic number is three. The picture above shows how to color it with three colors (upper bound). For the lower bound, the graph contains a C_5 made up of nodes a, b, e, g, and d.

2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of a graph containing a W_7.

≥ 3 ☐ ≥ 4 ☑ $\sqrt{}$ ≥ 7 ☐ can't tell ☐

$\displaystyle\sum_{k=1}^{n} k!$

$\displaystyle\sum_{p=0}^{n+1}(p+1)!$ ☐ $\displaystyle\sum_{k=0}^{n+1}(k-1)!$ ☐ $\displaystyle\sum_{k=0}^{n-1}(k+1)!$ ☑ $\sqrt{}$ $\displaystyle\sum_{p=0}^{n+1} k!$ ☐

10 people rowed across Lake Tahoe in my canoe. 10 is _____ how many people the canoe can carry.

an upper bound on ☐ exactly ☐

a lower bound on ☑ $\sqrt{}$ not a bound on ☐

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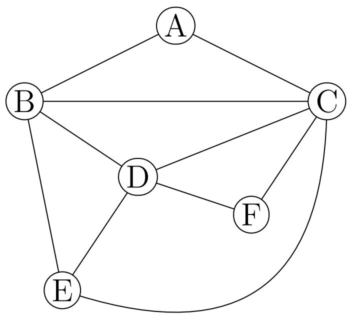
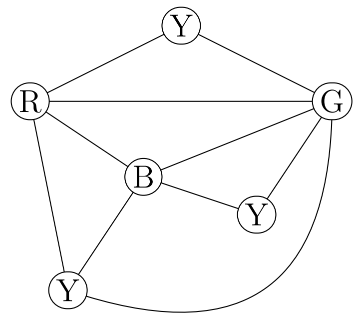
Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (9 points) What is the chromatic number of the graph below? Justify your answer.





Solution: The chromatic number is four. The picture above shows how to color it with four colors (upper bound). For the lower bound, the graph contains a K_4 made up of nodes B, C, D, E.

2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of a graph containing a K_n. $\leq n$ ☐ $= n$ ☐ $\geq n$ ☐ √ can't tell ☐

$\sum_{k=0}^{n} k!$ $\sum_{p=1}^{n+1} (p+1)!$ ☐ $\sum_{k=1}^{n+1} (k-1)!$ ☐ √ $\sum_{k=1}^{n-1} (k+1)!$ ☐ $\sum_{p=1}^{n+1} k!$ ☐

I heated 2 liters of milk in my big pot. 2 liters is _____ how much the pot holds. an upper bound on ☐ exactly ☐ a lower bound on ☐ √ not a bound on ☐

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Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (11 points) Let's define two sets as follows:

$$A = \{(x, y) \in \mathbb{R}^2 : y = x^2 - 4x + 3\}$$
$$B = \{(t + 2, t^2 - 1) : t \in \mathbb{R}\}$$

Prove that $A = B$ by proving two subset inclusions.

Solution:

$\mathbf{B} \subseteq \mathbf{A}$: Let $(x, y) \in B$. Then $(x, y) = (t + 2, t^2 - 1)$ for some real number t. So $x = t + 2$ and $y = t^2 - 1$.

Then $x^2 - 4x + 3 = (t+2)^2 - 4(t+2) + 3 = (t^2 + 4t + 4) - (4t + 8) + 3 = t^2 - 1 = y$. So $y = x^2 - 4x + 3$ and therefore $(x, y) \in A$.

$\mathbf{A} \subseteq \mathbf{B}$: Let $(x, y) \in A$. Then $y = x^2 - 4x + 3$.

Let $t = x - 2$. Then $x = t + 2$. Also $t^2 - 1 = (x - 2)^2 - 1 = x^2 - 4x + 4 - 1 = x^2 - 4x + 3 = y$. So $(x, y) = (t + 2, t^2 - 1)$ for this choice of t. Therefore $(x, y) \in B$.

Since $A \subseteq B$ and $B \subseteq A$, $A = B$, which is what we needed to show.

2. (4 points) Check the (single) box that best characterizes each item.

Chromatic number of $K_{m,n}$. (Assume $m \geq 1$, $n \geq 1$.)	2 ☑ √	3 ☐	4 ☐	can't tell ☐

$\pi \leq 10$	an upper bound on π ☑ √	exactly π ☐
	a lower bound on π ☐	not a bound on π ☐