Name:____

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Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(20 points) Suppose that $g : \mathbb{N} \to \mathbb{R}$ is defined by

$g(0) = 0 \qquad g(1) = \frac{4}{3}$

$g(n) = \frac{4}{3}g(n-1) - \frac{1}{3}g(n-2), \quad \text{for } n \geq 2$

Use (strong) induction to prove that $g(n) = 2 - \dfrac{2}{3^n}$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) Let function $f : \mathbb{N} \to \mathbb{Z}$ be defined by

$f(0) = 2$

$f(1) = 7$

$f(n) = f(n-1) + 2f(n-2)$, for $n \geq 2$

Use (strong) induction to prove that $f(n) = 3 \cdot 2^n + (-1)^{n+1}$ for any natural number n.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) Use (strong) induction to prove that the following claim holds:

Claim : For any integer $n \geq 2$, if $p_1, \ldots, p_n$ is a sequence of integers and $p_1 < p_n$, then there is an index j $(1 \leq j < n)$ such that $p_j < p_{j+1}$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) Suppose that $f : \mathbb{Z}^+ \to \mathbb{Z}$ is defined by is defined by

$f(1) = 5$ $\qquad$ $f(2) = -5$

$f(n) = 4f(n-2) - 3f(n-1)$, for all $n \geq 3$

Use (strong) induction to prove that $f(n) = 2 \cdot (-4)^{n-1} + 3$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) Suppose that θ is a constant (but unknown) real number. For any real number p, the angle addition formulas imply the following two equations (which you can assume without proof):

$$\cos(\theta)\cos(p\theta) = \cos((p+1)\theta) + \sin(\theta)\sin(p\theta) \quad (1)$$
$$\cos(\theta)\cos(p\theta) = \cos((p-1)\theta) - \sin(\theta)\sin(p\theta) \quad (2)$$

Suppose that $f : \mathbb{Z}^+ \to \mathbb{Z}$ is defined by

$f(0) = 1$ $\qquad$ $f(1) = \cos(\theta)$

$f(n+1) = 2\cos(\theta)f(n) - f(n-1)$, for all $n \geq 2$.

Use (strong) induction to prove that $f(n) = \cos(n\theta)$ for any natural number n.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) A Zellig graph consists of $2n$ ($n \geq 1$) nodes connected so as to form a circle. Half of the nodes have label 1 and the other half have label -1. As you move clockwise around the circle, you keep a running total of node labels. E.g. if you start at a 1 node and then pass through two -1 nodes, your running total is -1. Use (strong) induction to prove that there is a choice of starting node for which the running total stays ≥ 0.

Hint: remove an adjacent pair of nodes.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

Name:____

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(20 points) (20 points) Suppose that $f : \mathbb{N} \to \mathbb{Z}$ is defined by

$f(0) = 2 \qquad f(1) = 5 \qquad f(2) = 15$

$f(n) = 6f(n-1) - 11f(n-2) + 6f(n-3)$, for all $n \geq 3$

Use (strong) induction to prove that $f(n) = 1 - 2^n + 2 \cdot 3^n$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) Use (strong) induction to prove that, for any integer $n \geq 8$, there are non-negative integers p and q such that $n = 3p + 5q$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step: