CS 173, Spring 19 Examlet 8, white

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(10 points) Suppose we have a function f defined (for n a power of 4) by

$$\begin{aligned} f(1) &= 0 \\ f(n) &= 2f(n/4) + n \text{ for } n \geq 4 \end{aligned}$$

Your partner has already figured out that

$$f(n) = 2^k f(n/4^k) + n\sum_{p=0}^{k-1} 1/2^p$$

Finish finding the closed form for $f(n)$ assuming that n is a power of 4. Show your work and simplify your answer. Recall that $\log_b n = (\log_a n)(\log_b a)$.

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(10 points) Suppose we have a function F defined (for n a power of 2) by

$$\begin{aligned} F(2) &= c \\ F(n) &= F(n/2) + n \text{ for } n \geq 4 \end{aligned}$$

Your partner has already figured out that

$$F(n) = F(n/2^k) + \sum_{i=0}^{k-1} n \frac{1}{2^i}$$

Finish finding the closed form for F. Show your work and simplify your answer.

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1. (8 points) Suppose we have a function f defined by

$$\begin{aligned} f(0) &= f(1) = 3 \\ f(n) &= 5f(n-2) + d, \text{ for } n \geq 2 \end{aligned}$$

where d is a constant. Express $f(n)$ in terms of $f(n-6)$ (where $n \geq 6$). Show your work and simplify your answer. You do **not** need to find a closed form for $f(n)$.

2. (2 points) Check the (single) box that best characterizes each item.

The chromatic number of the 4-dimensional hypercube Q_4 2 ☐ 3 ☐ 4 ☐ 5 ☐

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(10 points) Suppose we have a function f defined (for n a power of 4) by

$$\begin{aligned} f(1) &= 0 \\ f(n) &= 2f(n/4) + n \text{ for } n \geq 4 \end{aligned}$$

Express $f(n)$ in terms of $f(n/4^{13})$ (assuming n is large enough that this input hasn't reached the base case). Express your answer using a summation and show your work. Do **not** finish the process of finding the closed form for $f(n)$.

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1. (8 points) Suppose we have a function g defined (for n a power of 3) by

$$\begin{aligned} g(1) &= c \\ g(n) &= 3g(n/3) + n \text{ for } n \geq 3 \end{aligned}$$

Express $g(n)$ in terms of $g(n/3^3)$ (where $n \geq 27$). Show your work and simplify your answer. You do **not** need to find a closed form for $g(n)$.

2. (2 points) Check the (single) box that best characterizes each item.

The number of nodes in the 4-dimensional hypercube Q_4	4 ☐	16 ☐	32 ☐	64 ☐

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(10 points) Suppose we have a function g defined (for n a power of 4) by

$$\begin{aligned} g(1) &= c \\ g(n) &= 2g(n/4) + n \text{ for } n \geq 4 \end{aligned}$$

Your partner has already figured out that

$$g(n) = 2^k g(n/4^k) + n \sum_{p=0}^{k-1} \frac{1}{2^p}$$

Finish finding the closed form for $g(n)$ assuming that n is a power of 4. Show your work and simplify your answer. Recall that $\log_b n = (\log_a n)(\log_b a)$.

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1. (8 points) Suppose we have a function g defined (for n a power of 2) by

$$\begin{aligned} g(1) &= 1 \\ g(n) &= 4g(n/2) + n^2 \text{ for } n \geq 2 \end{aligned}$$

Your partner has already figured out that

$$g(n) = 4^k g(n/2^k) + kn^2$$

Finish finding the closed form for g. Show your work and simplify your answer.

2. (2 points) Check the (single) box that best characterizes each item.

The Fibonacci numbers can be defined recursively by $F(0) = 0$, $F(1) = 1$, and $F(n) = F(n-1) + F(n-2)$ for all integers ... $n \geq 0$ ☐ $n \geq 1$ ☐ $n \geq 2$ ☐

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(10 points) Suppose we have a function g defined (for n a power of 2) by

$$\begin{aligned} g(1) &= 3 \\ g(n) &= 4g(n/2) + n \text{ for } n \geq 2 \end{aligned}$$

Your partner has already figured out that

$$g(n) = 4^k g(n/2^k) + \sum_{p=0}^{k-1} n2^p$$

Finish finding the closed form for $g(n)$ assuming that n is a power of 2. Show your work and simplify your answer. Recall that $\log_b n = (\log_a n)(\log_b a)$.