Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(10 points) Suppose we have a function f defined (for n a power of 4) by

$$\begin{aligned} f(1) &= 0 \\ f(n) &= 2f(n/4) + n \text{ for } n \geq 4 \end{aligned}$$

Your partner has already figured out that

$$f(n) = 2^k f(n/4^k) + n \sum_{p=0}^{k-1} 1/2^p$$

Finish finding the closed form for $f(n)$ assuming that n is a power of 4. Show your work and simplify your answer. Recall that $\log_b n = (\log_a n)(\log_b a)$.

Solution: To find the value of k at the base case, set $n/4^k = 1$. Then $n = 4^k$, so $k = \log_4 n$. Notice also that $2^{\log_4 n} = 2^{\log_2 n \log_4 2} = n^{1/2} = \sqrt{n}$

Substituting this into the above, we get

$$\begin{aligned} f(n) &= 2^{\log_4 n} f(1) + n \sum_{p=0}^{\log_4 n - 1} 1/2^p \\ &= 0 + n(2 - \frac{1}{2^{\log_4 n - 1}}) \\ &= n(2 - \frac{2}{2^{\log_4 n}}) = n(2 - \frac{2}{\sqrt{n}}) = 2(n - \sqrt{n}) \end{aligned}$$

Name:____

NetID:______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(10 points) Suppose we have a function F defined (for n a power of 2) by

$$\begin{aligned} F(2) &= c \\ F(n) &= F(n/2) + n \text{ for } n \geq 4 \end{aligned}$$

Your partner has already figured out that

$$F(n) = F(n/2^k) + \sum_{i=0}^{k-1} n \frac{1}{2^i}$$

Finish finding the closed form for F. Show your work and simplify your answer.

Solution:

To find the value of k at the base case, we need to set $n/2^k = 2$. This means that $n = 2 \cdot 2^k$. So $n = 2^{k+1}$. So $k + 1 = \log n$. So $k = \log n - 1$. Substituting this value into the above equation, we get

$$\begin{aligned} F(n) &= T(n/2^k) + \sum_{i=0}^{k-1} n \frac{1}{2^i} = F(2) + \sum_{i=0}^{\log n - 2} n \frac{1}{2^i} \\ &= c + n \sum_{i=0}^{\log n - 2} \frac{1}{2^i} = c + n(2 - \frac{1}{2^{\log n - 2}}) \\ &= c + n(2 - \frac{1}{2^{\log n} \cdot 2^{-2}}) = c + n(2 - \frac{4}{2^{\log n}}) \\ &= c + n(2 - \frac{4}{n}) = c + 2n - 4 \end{aligned}$$

CS 173, Spring 19 Examlet 8, white

Name:____

NetID:________________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (8 points) Suppose we have a function f defined by

$$\begin{aligned} f(0) &= f(1) = 3 \\ f(n) &= 5f(n-2) + d, \text{ for } n \geq 2 \end{aligned}$$

where d is a constant. Express $f(n)$ in terms of $f(n-6)$ (where $n \geq 6$). Show your work and simplify your answer. You do **not** need to find a closed form for $f(n)$.

Solution:

$$\begin{aligned} f(n) &= 5f(n-2) + d \\ &= 5(5(f(n-4) + d) + d) \\ &= 5(5(5(f(n-6) + d) + d) + d) \\ &= 5^3 f(n-6) + (25 + 5 + 1)d \\ &= 5^3 f(n-6) + 31d \end{aligned}$$

2. (2 points) Check the (single) box that best characterizes each item.

The chromatic number of the 4-dimensional hypercube Q_4	2 ☑	3 ☐	4 ☐	5 ☐

CS 173, Spring 19 Examlet 8, white

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(10 points) Suppose we have a function f defined (for n a power of 4) by

$$\begin{aligned} f(1) &= 0 \\ f(n) &= 2f(n/4) + n \text{ for } n \geq 4 \end{aligned}$$

Express $f(n)$ in terms of $f(n/4^{13})$ (assuming n is large enough that this input hasn't reached the base case). Express your answer using a summation and show your work. Do **not** finish the process of finding the closed form for $f(n)$.

Solution:

$$\begin{aligned} f(n) &= 2f(n/4) + n \\ &= 2(2f(n/4^2) + n/4) + n \\ &= 2(2(2f(n/4^3) + n/4^2) + n/4) + n \\ &= 2^3 f(n/4^3) + n/2^2) + n/2) + n \\ &= 2^k f(n/4^k) + n \sum_{p=0}^{k-1} 1/2^p \\ &= 2^{13} f(n/4^{13}) + n \sum_{p=0}^{12} 1/2^p \end{aligned}$$

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (8 points) Suppose we have a function g defined (for n a power of 3) by

$$\begin{aligned} g(1) &= c \\ g(n) &= 3g(n/3) + n \text{ for } n \geq 3 \end{aligned}$$

Express $g(n)$ in terms of $g(n/3^3)$ (where $n \geq 27$). Show your work and simplify your answer. You do **not** need to find a closed form for $g(n)$.

Solution:

$$\begin{aligned} g(n) &= 3g(n/3) + n \\ &= 3(3g(n/9) + n/3) + n \\ &= 3(3(3g(n/27) + n/9) + n/3) + n \\ &= 27g(n/27) + n + n + n \\ &= 27g(n/27) + 3n \end{aligned}$$

2. (2 points) Check the (single) box that best characterizes each item.

The number of nodes in the 4-dimensional hypercube Q_4	4 ☐	16 ☑	32 ☐	64 ☐

Name:____

NetID:______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(10 points) Suppose we have a function g defined (for n a power of 4) by

$$\begin{aligned} g(1) &= c \\ g(n) &= 2g(n/4) + n \text{ for } n \geq 4 \end{aligned}$$

Your partner has already figured out that

$$g(n) = 2^k g(n/4^k) + n \sum_{p=0}^{k-1} \frac{1}{2^p}$$

Finish finding the closed form for $g(n)$ assuming that n is a power of 4. Show your work and simplify your answer. Recall that $\log_b n = (\log_a n)(\log_b a)$.

Solution: To find the value of k at the base case, set $n/4^k = 1$. Then $n = 4^k$, so $k = \log_4 n$. Notice also that $2^{\log_4 n} = 2^{\log_2 n \log_4 2} = n^{1/2} = \sqrt{n}$

Substituting this into the above, we get

$$\begin{aligned} g(n) &= 2^{\log_4 n} \cdot c + n \sum_{p=0}^{\log_4 n - 1} \frac{1}{2^p} \\ &= 2^{\log_4 n} \cdot c + n(2 - \frac{1}{2^{\log_4 n - 1}}) \\ &= c\sqrt{n} + n(2 - \frac{2}{\sqrt{n}}) \\ &= c\sqrt{n} + 2n - 2\sqrt{n} \\ &= 2n + (c-2)\sqrt{n} \end{aligned}$$

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (8 points) Suppose we have a function g defined (for n a power of 2) by

$$\begin{aligned} g(1) &= 1 \\ g(n) &= 4g(n/2) + n^2 \text{ for } n \geq 2 \end{aligned}$$

Your partner has already figured out that

$$g(n) = 4^k g(n/2^k) + kn^2$$

Finish finding the closed form for g. Show your work and simplify your answer.

Solution:

To find the value of k at the base case, set $n/2^k = 1$. Then $n = 2^k$, so $k = \log_2 n$. Substituting this into the above, we get:

$$\begin{aligned} g(n) &= 4^k g(n/2^k) + kn^2 \\ &= 4^{\log_2 n} g(1) + (\log_2 n)n^2 \\ &= 4^{\log_2 n} + n^2 \log_2 n \\ &= 4^{\log_4 n \log_2 4} + n^2 \log_2 n \\ &= (4^{\log_4 n})^{\log_2 4} + n^2 \log_2 n \\ &= n^{\log_2 4} + n^2 \log_2 n \\ &= n^2 + n^2 \log_2 n \end{aligned}$$

2. (2 points) Check the (single) box that best characterizes each item.

The Fibonacci numbers can be defined recursively by $F(0) = 0$, $F(1) = 1$, and $F(n) = F(n-1) + F(n-2)$ for all integers ... $n \geq 0$ ☐ $n \geq 1$ ☐ $n \geq 2$ ☑

Name:______________________________________

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(10 points) Suppose we have a function g defined (for n a power of 2) by

$$\begin{aligned} g(1) &= 3 \\ g(n) &= 4g(n/2) + n \text{ for } n \geq 2 \end{aligned}$$

Your partner has already figured out that

$$g(n) = 4^k g(n/2^k) + \sum_{p=0}^{k-1} n 2^p$$

Finish finding the closed form for $g(n)$ assuming that n is a power of 2. Show your work and simplify your answer. Recall that $\log_b n = (\log_a n)(\log_b a)$.

Solution: To find the value of k at the base case, set $n/2^k = 1$. Then $n = 2^k$, so $k = \log_2 n$. Notice also that $4^{\log_2 n} = 4^{\log_4 n \log_2 4} = n^2$.

Substituting this into the above, we get

$$\begin{aligned} g(n) &= 4^k g(n/2^k) + n \sum_{p=0}^{k-1} 2^p \\ &= 4^{\log_2 n} \cdot 3 + n \sum_{p=0}^{\log_2 n - 1} 2^p \\ &= 3n^2 + n(2^{\log_2 n} - 1) \\ &= 3n^2 + n(n-1) = 4n^2 - n \end{aligned}$$