Name:____

NetID:____________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(18 points) A Mouse tree is a binary tree containing 2D points such that:

- Each leaf node contains $(3,1)$, $(-2,-5)$, or $(2,2)$.
- An internal node with one child labelled (a,b) has label $(a+1,b-1)$.
- An internal node with two childen labelled (x,y) and (a,b) has label $(x+a,y+b)$.

Use (strong) induction to prove that the point in the root node of any Mouse tree is on or below the line $x=y$.

The induction variable is named ______ and it is the ________________ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

Name:____

NetID:________________________ **Lecture: A B**

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(18 points) A palindrome is a string that is the same if you reverse it. For example, `abbabba` and `abaaba` are palindromes. The empty (zero-length) string ϵ counts as a palindrome.

Here is a grammar G, with start symbol S and terminal symbols a and b.

$$S \rightarrow a\,S\,a \mid b\,S\,b \mid a \mid b \mid \epsilon$$

Use (strong) induction to prove that any palindrome made out of characters a and b can be generated by grammar G. That is, show how to build parse trees for these strings. Hint: remove the first and last character from the string.

The induction variable is named ______ and it is the ________________ of/in the string.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

Name:____

NetID:________________________ **Lecture:** **A** **B**

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(18 points) Recall that a node in a full binary tree must have either 0 or 2 children. A Shark tree is a full binary tree in which each node is colored orange or blue, such that:

- If v is a leaf node, then v is colored orange.
- If v has two children of the same color, then v is colored blue.
- If v has two children of different colors, then v is colored orange.

Use (strong) induction to show that the root of a Shark tree is blue if and only if the tree has an even number of leaves. You may assume basic divisibility facts e.g. the sum of two odd numbers is even.

The induction variable is named ______ and it is the ________________ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

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Name:____

NetID:________________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(18 points) A Horse tree is a binary tree whose nodes contain integers such that

- Every leaf node contains 5, 17, or 23.
- A node with one child contains the same number as its child.
- A node with two children contains the value $x(y+1)$, where x and y are the values in its children.

Use strong induction to prove that the value in the root of a Horse tree is always positive.

The induction variable is named ______ and it is the ________________ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

Name:____

NetID:________________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(18 points) Here is a grammar G, with start symbols N and P, and terminal symbols a and b.

$$
\begin{aligned}
N &\rightarrow P\ a \mid b\ b \\
P &\rightarrow P\ N \mid a
\end{aligned}
$$

Use (strong) induction to prove that any tree matching (aka generated by) grammar G has an even number of leaves if and only if its root has label N.

The induction variable is named ______ and it is the ________________ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

CS 173, Spring 19 Examlet 9, colored

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(18 points) Recall that a node in a full binary tree is either a leaf or has exactly two children. A Possum tree is a full binary tree whose leaves are all orange and whose root is blue.

Use (strong) induction to prove that a Possum tree contains a blue node with (at least) one orange child.

The induction variable is named ______ and it is the ________________ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(18 points) Here is a grammar G, with start symbol S and terminal symbols a and p.

$$S \rightarrow S\,S \mid p\,S\,p \mid p\,p \mid a\,a$$

Use (strong) induction to prove that any tree matching (aka generated by) grammar G has an even number of nodes with label p. Use $P(T)$ as shorthand for the number of p's in a tree T.

The induction variable is named ______ and it is the ________________ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

Name:____

NetID:_______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(18 points) Recall that a node in a full binary tree is either a leaf or has exactly two children. A Snake tree is a full binary tree whose leaves are all blue and whose root is orange.

Use (strong) induction to prove that a Snake tree contains an orange node with two blue children.

The induction variable is named ______ and it is the ________________ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step: