

Name: \_\_\_\_\_

NetID: \_\_\_\_\_ Lecture:    A    B

Discussion:    Thursday    Friday    9    10    11    12    1    2    3    4    5    6

1. (8 points) Consider the following grammar  $G$ , with start symbol  $S$  and terminals  $a$  and  $b$ .

$$S \rightarrow a S a \mid b S b \mid a S b \mid b S a \mid a \mid b$$

Amy claims that this generates all non-empty strings containing a's and/or b's. Is this correct? Justify your answer.

2. (4 points) Check the (single) box that best characterizes each item.

Total number of leaves in a full and complete 5-ary tree of height  $h$      $5^h$       $\leq 5^h$       $\geq 5^h$       $5^{h+1} - 1$

The level of a leaf node in a full and complete binary tree of height  $h$ .    0     1      $h - 1$       $\leq h$       $h$

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1. (8 points) Here is a grammar with start symbol  $S$  and terminal symbols  $a$  and  $b$ . Draw three parse trees for the string **abba** that match this grammar.

$$S \rightarrow SS \mid aS \mid Sa \mid b$$

2. (4 points) Check the (single) box that best characterizes each item.

A full  $m$ -ary tree with  $i$  internal nodes has  $mi + 1$  nodes total.

always sometimes never 

A binary tree of height  $h$  has at least  $2^{h+1} - 1$  nodes.

true false

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1. (8 points) Consider the following grammar  $G$

$$S \rightarrow S b S \mid a \mid c d$$

$S$  is the only start symbol. The terminal symbols are  $a$ ,  $b$ ,  $c$ , and  $d$ .

Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar  $G$  whose leaves have this sequence of labels, or else explain briefly why  $G$  cannot generate this sequence of leaf labels.

aaacd

bbbbbb

2. (4 points) Check the (single) box that best characterizes each item.

The mathematical symbol for an empty (zero-length) string  $\emptyset$   e   $\epsilon$   NULL

Number of bit strings of length  $\leq k$ .  $2^k$    $2^k - 1$    $2^{k-1}$    $2^{k+1} - 1$

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1. (8 points) Min's virus detection code needs to generate all strings of the form  $a^n b^n$ . That is, all strings that consist of a sequence of one or more a's followed by the same number of b's. Write a context-free grammar G that will do this.

2. (4 points) Check the (single) box that best characterizes each item.

The number of nodes in a binary tree of height  $h$

|                    |                          |                    |                          |
|--------------------|--------------------------|--------------------|--------------------------|
| $\geq 2^h$         | <input type="checkbox"/> | $2^{h+1} - 1$      | <input type="checkbox"/> |
| $\leq 2^{h+1} - 1$ | <input type="checkbox"/> | $\geq 2^{h+1} - 1$ | <input type="checkbox"/> |

A tree node is a descendent of itself.

|        |                          |           |                          |       |                          |
|--------|--------------------------|-----------|--------------------------|-------|--------------------------|
| always | <input type="checkbox"/> | sometimes | <input type="checkbox"/> | never | <input type="checkbox"/> |
|--------|--------------------------|-----------|--------------------------|-------|--------------------------|

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1. (8 points) Consider the following grammar  $G$

$$S \rightarrow a S b \mid b S b \mid c$$

$S$  is the only start symbol. The terminal symbols are  $a$ ,  $b$ , and  $c$ .

Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar  $G$  whose leaves have this sequence of labels, or else explain briefly why  $G$  cannot generate this sequence of leaf labels.

ababb

babcbbb

2. (4 points) Check the (single) box that best characterizes each item.

The level of the root node in a tree of height  $h$ .    0     1      $h - 1$       $h$       $h + 1$

A tree node is a proper ancestor of itself.    always     sometimes     never

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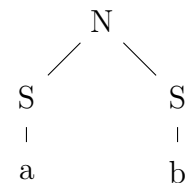
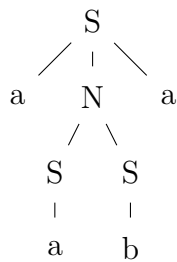
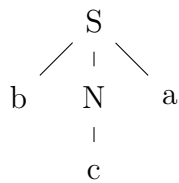
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1. (8 points) Here is a grammar with start symbol  $S$  and terminal symbols  $a$ ,  $b$ , and  $c$ . Circle the trees that match the grammar.

$$S \rightarrow a N a \mid b N b \mid a \mid b$$

$$N \rightarrow S S \mid c$$



2. (4 points) Check the (single) box that best characterizes each item.

A binary tree of height  $h$  has at least  $2^h - 1$  nodes.

true

false

A full  $m$ -ary tree with  $i$  internal nodes has \_\_\_\_\_ nodes total.

$mi - 1$

$mi$

$mi + 1$

$\leq mi + 1$

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- (8 points) Consider the following grammar  $G$

$$S \rightarrow a S b \mid b S b \mid a \mid b$$

$S$  is the only start symbol. The terminal symbols are  $a$  and  $b$ .

Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar  $G$  whose leaves have this sequence of labels, or else explain briefly why  $G$  cannot generate this sequence of leaf labels.

bababbb

aaaab

- (4 points) Check the (single) box that best characterizes each item.

The number of leaves in a binary tree of height  $h$   $2^h$    $2^{h+1} - 1$    $\geq 2^h$    $\leq 2^h$

The number of paths between two distinct nodes in an  $n$ -node tree. Paths in opposite directions count as the same.  $n$    $2n$    $\frac{n(n-1)}{2}$    
 $n(n-1)$    $n^2$    $\frac{n(n+1)}{2}$

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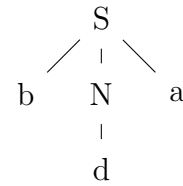
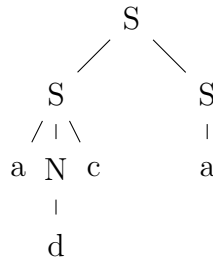
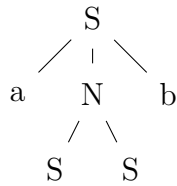
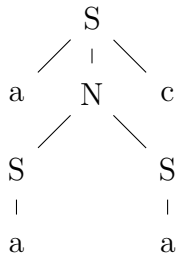
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1. (8 points) Here is a grammar with start symbol  $S$  and terminal symbols  $a, b, c,$  and  $d$ . Circle the trees that match the grammar.

$$S \rightarrow a N b \mid a N c \mid a$$

$$N \rightarrow S S \mid d$$



2. (4 points) Check the (single) box that best characterizes each item.

The diameter of a tree of height  $h$ .     $\leq h$       $h$       $h + 1$

$2h$       $\leq 2h$

The number of nodes in a full complete binary tree of height  $h$      $\geq 2^h$       $2^{h+1} - 1$

$\leq 2^{h+1} - 1$       $\geq 2^{h+1} - 1$