Name:______________________________________

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (8 points) Consider the following grammar G, with start symbol S and terminals a and b.

$$S \rightarrow a\,S\,a \mid b\,S\,b \mid a\,S\,b \mid b\,S\,a \mid a \mid b$$

Amy claims that this generates all non-empty strings containing a's and/or b's. Is this correct? Justify your answer.

Solution:

Amy is wrong. This grammar only generates strings of odd length.

2. (4 points) Check the (single) box that best characterizes each item.

Total number of leaves in a full and complete 5-ary tree of height h	5^h ☑	$\leq 5^h$ ☐	$\geq 5^h$ ☐	$5^{h+1}-1$ ☐

The level of a leaf node in a full and complete binary tree of height h.	0 ☐	1 ☐	$h-1$ ☐	$\leq h$ ☐	h ☑

Name:____

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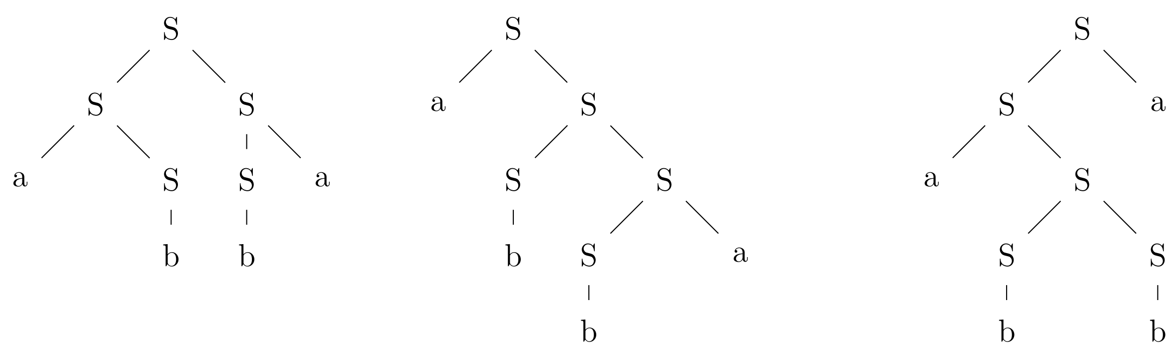
Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (8 points) Here is a grammar with start symbol S and terminal symbols a and b. Draw three parse trees for the string `abba` that match this grammar.

$$S \rightarrow \quad S\,S \mid a\,S \mid S\,a \mid b$$

Solution:



2. (4 points) Check the (single) box that best characterizes each item.

A full m-ary tree with i internal nodes has $mi + 1$ nodes total.	always $\boxed{\surd}$	sometimes ☐	never ☐
A binary tree of height h has at least $2^{h+1} - 1$ nodes.	true ☐	false $\boxed{\surd}$	

Name:____

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Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (8 points) Consider the following grammar G

$$S \rightarrow S\ b\ S \ \mid\ a\ \mid\ c\ d$$

S is the only start symbol. The terminal symbols are a, b, c, and d.

Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why G cannot generate this sequence of leaf labels.

`aaacd`

Solution: In grammar G, making strings with more than two leaves requires using the first rule (SbS) which produces a b. This string can't be generated by G because it is more than two characters long with no b in it.

`bbbbb`

Solution: Impossible. Since the only terminal in the string is b, the only rule we could be using is $S \rightarrow SbS$. But each time we use this rule, the count of S nodes without children increases by one. This is a problem, since S nodes can't be leaves.

2. (4 points) Check the (single) box that best characterizes each item.

The mathematical symbol for an empty (zero-length) string	$\emptyset$ ☐	e ☐	ϵ ☑	NULL ☐
Number of bit strings of length $\leq k$.	2^k ☐	$2^k - 1$ ☐	2^{k-1} ☐	$2^{k+1} - 1$ ☑

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1. (8 points) Min's virus detection code needs to generate all strings of the form $a^n b^n$. That is, all strings that consist of a sequence of one or more a's followed by the same number of b's. Write a context-free grammar G that will do this.

   Solution:

   G has start symbol S, terminals a and b, and the following rules:

   $S \rightarrow \; a\ S\ b \;|\; a\ b$

2. (4 points) Check the (single) box that best characterizes each item.

The number of nodes in a binary tree of height h	$\geq 2^h$	☐	$2^{h+1} - 1$	☐
	$\leq 2^{h+1} - 1$	☑ $\surd$	$\geq 2^{h+1} - 1$	☐

A tree node is a descendent of itself.	always	☑ $\surd$	sometimes	☐	never	☐

Name:____

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Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (8 points) Consider the following grammar G

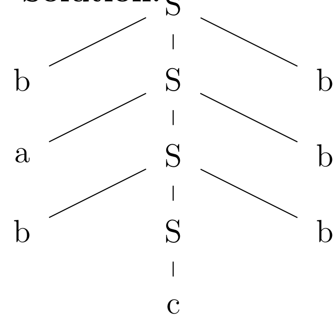
$$S \rightarrow a\ S\ b \ |\ b\ S\ b \ |\ c$$

S is the only start symbol. The terminal symbols are a, b, and c.

Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why G cannot generate this sequence of leaf labels.

`ababb`
Solution:
This is impossible. In strings produced by G, the middle character must be a c.

`babcbbb`
Solution:



2. (4 points) Check the (single) box that best characterizes each item.

The level of the root node in a tree of height h. 0 [√] 1 [ ] $h-1$ [ ] h [ ] $h+1$ [ ]

A tree node is a proper ancestor of itself. always [ ] sometimes [ ] never [√]

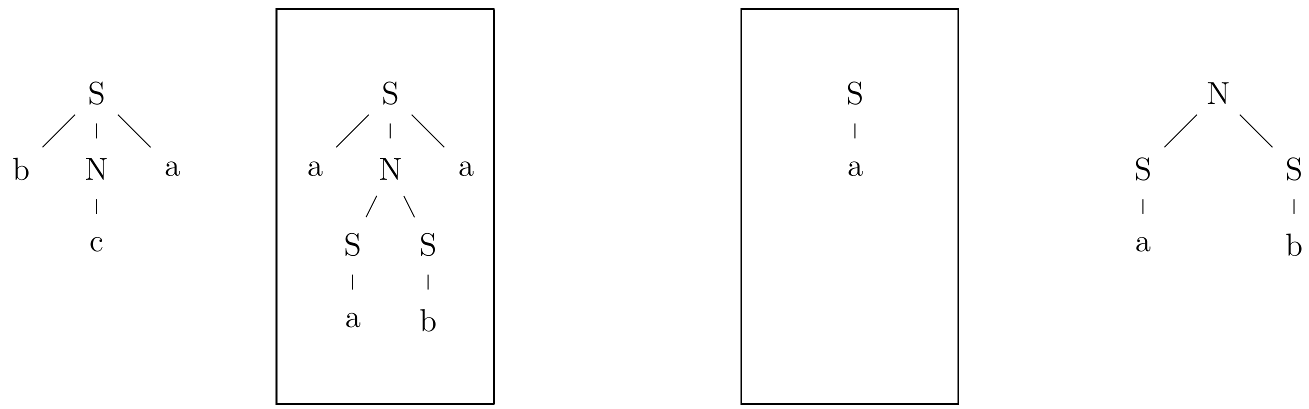
Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (8 points) Here is a grammar with start symbol S and terminal symbols a, b, and c. Circle the trees that match the grammar.

$$
\begin{aligned}
S &\rightarrow a\ N\ a \ \mid\ b\ N\ b \ \mid\ a \mid\ b \\
N &\rightarrow S\ S \ \mid\ c
\end{aligned}
$$



2. (4 points) Check the (single) box that best characterizes each item.

A binary tree of height h has at least $2^h - 1$ nodes. true ☐ false ☑

A full m-ary tree with i internal nodes has ____ nodes total.

$mi - 1$ ☐ mi ☐

$mi + 1$ ☑ $\leq mi + 1$ ☐

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Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (8 points) Consider the following grammar G

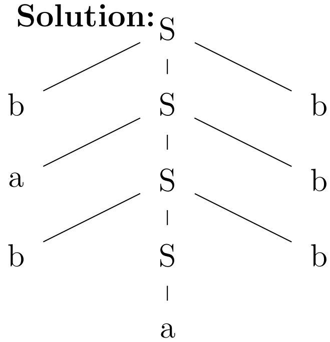
$$S \rightarrow a\ S\ b \ |\ b\ S\ b \ |\ a \ |\ b$$

S is the only start symbol. The terminal symbols are a and b.

Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why G cannot generate this sequence of leaf labels.

`bababbb`

Solution:



`aaaab`

Solution:

This is impossible. In a string produced by grammar G, all characters after the middle of the string must be b's.

2. (4 points) Check the (single) box that best characterizes each item.

The number of leaves in a binary tree of height h

2^h	$2^{h+1}-1$	$\geq 2^h$	$\leq 2^h$
☐	☐	☐	☑

The number of paths between two distinct nodes in an n-node tree. Paths in opposite directions count as the same.

n ☐	$2n$ ☐	$\frac{n(n-1)}{2}$ ☑
$n(n-1)$ ☐	n^2 ☐	$\frac{n(n+1)}{2}$ ☐

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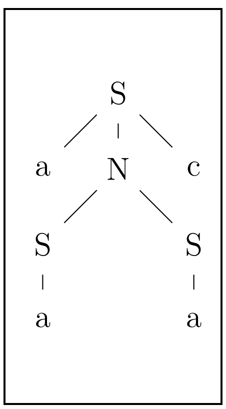
Name: ______________________________

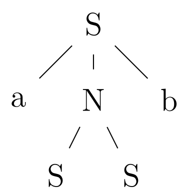
NetID: ____________________ **Lecture:** A B

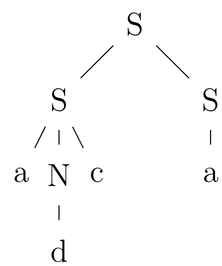
Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

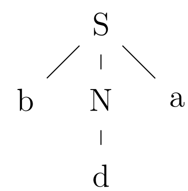
1. (8 points) Here is a grammar with start symbol S and terminal symbols a, b, c, and d. Circle the trees that match the grammar.

$$
\begin{aligned}
S &\rightarrow a\,N\,b \mid a\,N\,c \mid a \\
N &\rightarrow S\,S \mid d
\end{aligned}
$$









2. (4 points) Check the (single) box that best characterizes each item.

The diameter of a tree of height h.

- $\leq h$ ☐
- h ☐
- $h+1$ ☐
- $2h$ ☐
- $\leq 2h$ ☑ √

The number of nodes in a full complete binary tree of height h

- $\geq 2^h$ ☐
- $2^{h+1}-1$ ☑ √
- $\leq 2^{h+1}-1$ ☐
- $\geq 2^{h+1}-1$ ☐