CS 173, Spring 19 Examlet 13

Name:____

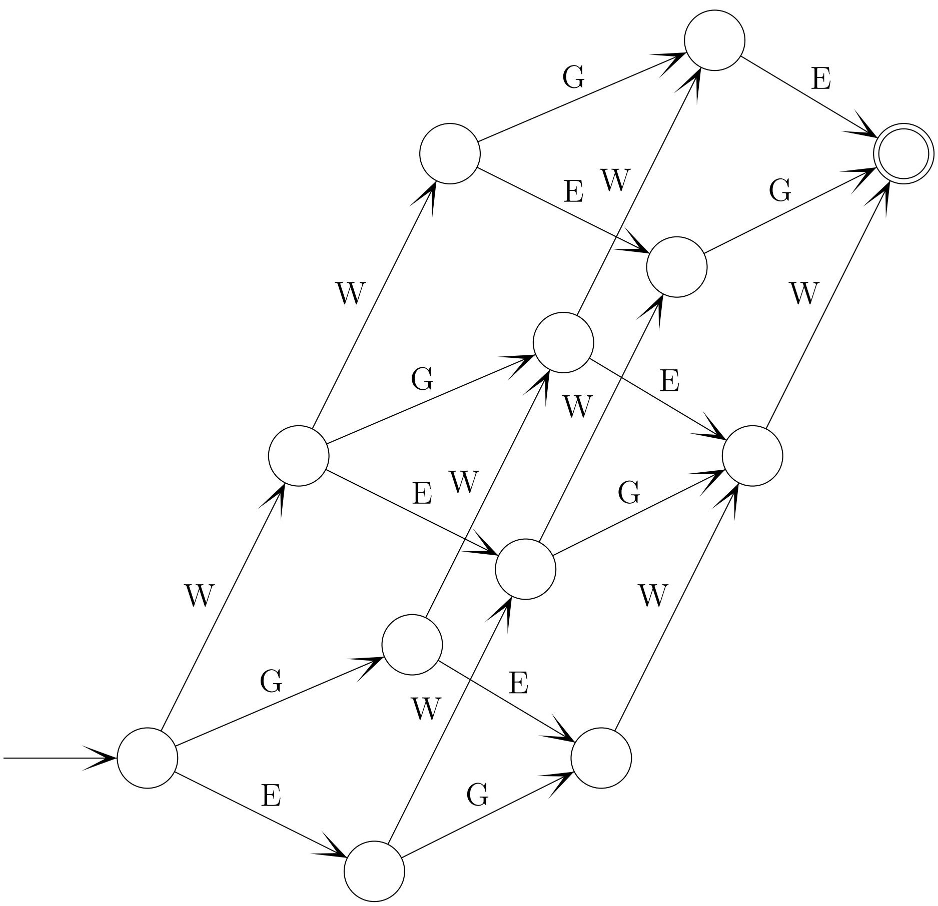
NetID:______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) When senators enter the Magical Senate, the scanner outputs W for a wizard, E for an elf, G for a goblin. The Magical Senate cannot do business unless at least two wizards, one elf, and one goblin are present. Draw a state machine that reads a sequence of characters from the scanner. When it has seen enough participants of each type, it should enter an end state and stay there.

For readability, your drawing should show only transitions that move from one state to another. The house elves will add all required self-loops later, when they implement this in hardware. Your design should use no more than 16 states and, if you can, no more than 13.

Solution:



Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(5 points) Amber Industries makes a line of necklaces featuring a mama brown bear leading an infinite line of baby bears. The baby bears get smaller and smaller, so the whole necklace is only 6 inches long. Each baby bear has either brown or cream fur, with no restrictions on the pattern of baby bear colors in the necklace. Is the set of possible necklaces countable or uncountable? Briefly justify your answer.

Solution: This set is uncountable. An infinite line of baby bears is essentially the same as the natural numbers. At each position, we're picking one of two values. So this is similar to the set of functions from the natural numbers to $\{0, 1\}$, which we know to be uncountable.

(10 points) Check the (single) box that best characterizes each item.

The interval $[2, 3]$ of the real line. finite ☐ countably infinite ☐ uncountable ☑ $\surd$

If $f : A \to B$ is one-to-one $|A| < |B|$ ☐ $|A| \leq |B|$ ☑ $\surd$ $|A| = |B|$ ☐

All infinite sequences of emojis. finite ☐ countably infinite ☐ uncountable ☑ $\surd$

All partitions of the set of natural numbers less than 10,000. finite ☑ $\surd$ countably infinite ☐ uncountable ☐

We can build a program that decides whether an input program halts. true ☐ false ☑ $\surd$ not known ☐

Name:____

NetID:________________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(5 points) Hermione Grainger has 7000 socks in her magically expanding drawer. The socks are colored purple, magenta, shocking pink, and neon green. How many socks must she pull out of the drawer before she is guaranteed to have two socks of the same color. Briefly justify your answer.

Solution: She needs to pull out five socks. By the pigeonhole principle, five socks and only four colors means that two must have the same color.

(10 points) Check the (single) box that best characterizes each item.

If Mickey Mouse is president of the US, then $\pi = 3$.	true ☑	false ☐	undefined ☐
$(5 \times 5) \equiv 1 \pmod 6$	true ☑	false ☐	
W_7 is a subgraph of graph H. 7 is _____ the chromatic number of H.	an upper bound on ☐ a lower bound on ☐	exactly ☐ not a bound on ☑	
$\forall x \in \mathbb{R},\ \exists y \in \mathbb{R},\ xy = 1$	true ☐	false ☑	
A function from $\mathbb{R}$ to $\mathbb{R}$ is strictly increasing if and only if it is one-to-one.	true ☐	false ☑	

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(5 points) Recall that the symmetric difference of two sets A and B written $A \oplus B$ contains all the elements that are in one of the two sets but not the other. That is $A \oplus B = (A - B) \cup (B - A)$. For any set of integers A, let $[A] = \{B \in \mathbb{P}(\mathbb{Z}) \mid A \oplus B \text{ is finite } \}$

Explain clearly what is in $[\{1, 2, 3\}]$. Also, is it true that $[\mathbb{Z}] = [\mathbb{N}]$? Briefly justify your answer.

Solution:

$[\{1, 2, 3\}]$ contains all finite sets of integers.

$\mathbb{N}$ is in $[\mathbb{N}]$. However, $\mathbb{N}$ can't be in $[\mathbb{Z}]$, since the symmetric difference between $\mathbb{Z}$ and $\mathbb{N}$ includes all the negative integers and therefore is infinite. So $[\mathbb{Z}]$ cannot be equal to $[\mathbb{N}]$.

(10 points) Check the (single) box that best characterizes each item.

$n^{log_4 2}$ grows — faster than n^2 ☐ — slower than n^2 ☑ — at the same rate as n^2 ☐

Number of nodes at level k in a full complete binary tree. — 2^k ☑ — $2^k - 1$ ☐ — $2^{k+1} - 1$ ☐ — 2^{k-1} ☐

Shorthand for the n-dimensional hypercube. — C_n ☐ — H_n ☐ — Z_n ☐ — Q_n ☑

$T(1) = d$
$T(n) = 2T(n-1) + c$

$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐	$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐
$\Theta(n^{\log_3 2})$ ☐	$\Theta(n^{\log_2 3})$ ☐	$\Theta(2^n)$ ☑	$\Theta(3^n)$ ☐

$\mathbb{P}(A \cap B) \subseteq \mathbb{P}(A \cup B)$ — always ☑ — sometimes ☐ — never ☐