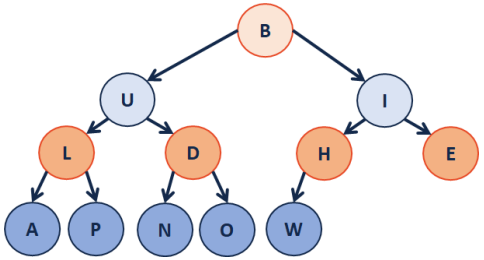
CS 225

#30: Heap Analysis and Disjoint Sets

November 5, 2018 · *Wade Fagen-Ulmschneider*

Q: An optimal buildHeap operation:



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Heap.hpp (partial)

```
1  template <class T>
2  void Heap<T>::buildHeap() {
3    for (unsigned i = parent(size); i > 0; i--) {
4      heapifyDown(i);
5    }
6  }
```

Theorem: The running time of buildHeap on array of size n is:

_____.

Strategy:

Define S(h):

S(h) :=

S(0) =

S(1) =

S(h) =

Proof of S(h) by Induction:

Disjoint Sets

Let **R** be an equivalence relation. We represent R as several disjoint sets. Two key ideas from Monday:

- Each element exists in exactly one set.
- Every set is an equitant representation.
  - Mathematically: $4 \in [0]_R \rightarrow 8 \in [0]_R$
  - Programmatically: find(4) == find(8)

Building Disjoint Sets:

- Maintain a collection $S = \{s_0, s_1, \ldots s_k\}$
- Each set has a representative member.
- ADT:

```
void makeSet(const T & t);
void union(const T & k1, const T & k2);
T & find(const T & k);
```

Implementation #1: Representative Member Array

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]

Operation: find(k)
...running time?

Operation: union(k1, k2)
...running time?

Implementation #2: UpTrees

- Continue to use an array where the index is the key
- The value of the array is:
  - **-1**, if we have found the representative element
  - **The index of the parent**, if we haven't found the rep. element

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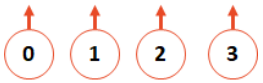
Step-by-step construction of UpTrees:



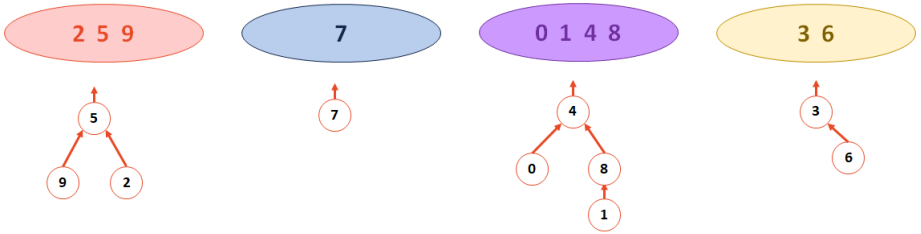
[0]	[1]	[2]	[3]

[0]	[1]	[2]	[3]

[0]	[1]	[2]	[3]

[0]	[1]	[2]	[3]

Example:



4	**8**	**5**	**6**	**-1**	**-1**	**-1**	**-1**	**4**	**5**
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]

...where is the error(s) in this table?

Implementation – DisjointSets::find

DisjointSets.cpp (partial)

```
1  int DisjointSets::find(int i) {
2    if ( s[i] < 0 ) { return i; }
3    else { return _find( s[i] ); }
4  }
```

What is the running time of `find`?

What is the ideal UpTree?

Implementation – DisjointSets::union

DisjointSets.cpp (partial)

```
1  void DisjointSets::union(int r1, int r2) {
2
3
4  }
```

How do we want to union the two UpTrees?

CS 225 – Things To Be Doing:

1. Theory Exam 3 starts Thursday; **Practice Exam Available!**
2. MP5 due tonight at 11:59pm
3. Lab Section: lab_puzzles coming up this week in lab!
4. Daily POTDs are ongoing!