

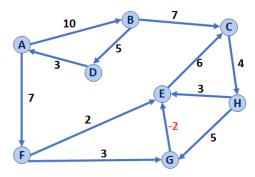
## #42: Floyd-Warshall's Algorithm

December 10, 2018 · Wade Fagen-Ulmschneider

# From Friday:

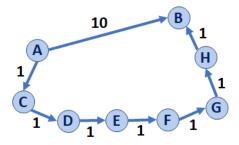
- Graphs with a negative-weight **cycle** have no finite shortest path. (We can always take the cycle one more time to get an even shorter path!)
- Graphs with a negative-weight **edge without a negative-weight cycle** DO have a finite shortest path!
- Does Dijkstra's algorithm find it?

**Dijkstra:** What if we have a minimum-weight edge, without having a negative-weight cycle?



...what assumption does Dijkstra's algorithm make?

**Q:** Can we transform the graph by adding  $+\mathbf{k}$  to every edge?



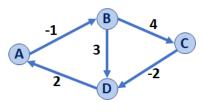
**Dijkstra:** What is the running time?

## Floyd-Warshall Algorithm

Floyd-Warshall's Algorithm is an alternative to Dijkstra in the presence of negative-weight edges (but <u>not</u> negative weight cycles).

## **Algorithm Design:**

- **Goal:** Find the shortest path from vertex **u** to **v**.
- **Setup:** Create an n×n matrix that maintains the best known path between every pair of vertices:



- o Initialize (u, u) to o.
- Initialize all edges present on the graph to their edge weight.
- Initialize all other edges to +infinity.

	A	В	C	D
A				
В				
C				
D				_

- For every vertex  $\mathbf{k}$ , consider which of the following are shorter:
  - o path(u, v) or -
  - o  $path(u, \mathbf{k}) + path(\mathbf{k}, v)$

Big Idea: \_\_\_\_\_

- Store intermediate results to improve build towards an optimal solution.
- Example application of memorization and **dynamic programming (DP)** more in CS 374!

# **Running Time:**

```
Pseudocode for Floyd-Warshall's Algorithm
   FlovdWarshall(G):
      Input: G, Graph;
3
      Output: d, an adjacency matrix of distances between
4
                 All vertex pairs
 5
     Let d be an adj. matrix (2d array) initialized to +inf
 6
7
     foreach (Vertex v : G):
8
        d[v][v] = 0
9
     foreach (Edge (u, v) : G):
10
        d[u][v] = cost(u, v)
11
12
     foreach (Vertex u : G):
13
        foreach (Vertex v : G):
14
          foreach (Vertex k : G):
15
            if d[u, v] > d[u, k] + d[k, v]:
              d[u, v] = d[u, k] + d[k, v]
16
17
18
     return d
```

## **Overview of Graphs:**

Implementations

- Edge List
- Adjacency Matrix
- Adjacency List

#### Traversals

- Breadth First
- Depth First

Minimum Spanning Tree (MST)

- Kruskal's Algorithm
- Prim's Algorithm

### **Shortest Path**

- Dijkstra's Algorithm (Single Source)
- Floyd-Warshall's Algorithm (All Pairs)

#### Maximum Flow

- Ford-Fulkerson (DFS paths) Algorithm
- Edmonds-Karp (BFS paths) Algorithm

...and this is just the beginning. The journey continues to CS 374!

### **End of Semester:**(

### CS 225 Final Exam

- The final exam is a 3 hour CBTF exam, is a cumulative exam, and has the format of theory exam + programming exam
  - Expect ~25-30 theory questions
  - Expect ~2-3 programming problems
- Exam begins on Thursday (December 13<sup>th</sup>), last day of office hours is December 12<sup>th</sup> (no office hours once finals begin)
- In-lecture review w/ TAs on Wednesday (December 12<sup>th</sup>)

# "Pre-Final" Grade Update

• As soon as possible after the MP7 deadline, I'll provide a "Pre-Final" grade update in Compass 2g with all grades except for your final exam.

#### End of Semester Grade Review

- Did we miss something that impacts your final grade? I want to be absolutely sure you get the grade you earned!
- After final grades are posted, I will provide a Google Sheet that allows you to submit a **Grade Review** if you believe the grade review will change your final letter grade.
  - You will have the chance to justify why you received an incorrect grade and how it impacts your letter grade in the course.
  - Instructions on Piazza at the same time as that the final grades are posted.

# CS 225 – Things To Be Doing:

- 1. MP7 due tonight (December 10); two-day grace period applies!
- 2. Final Exam starts Thursday, December 14