CS 225

Data Structures

October 15 – AVL Applications

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AVL Runtime Proof

On Friday, we proved an upper-bound on the height of an AVL tree is **2 × lg(n)** or **O(lg(n))**:

N(h) := Minimum # of nodes in an AVL tree of height h

N(h) = 1 + **N(h-1)** + **N(h-2)**

> **N(h-1)** + **N(h-2)**

> 2 × **N(h-2)**

> $2^{h/2}$

Theorem #1:

Every AVL tree of height h has at least $2^{h/2}$ nodes.

AVL Runtime Proof

On Friday, we proved an upper-bound on the height of an AVL tree is **$2 \times \lg(n)$** or **$O(\lg(n))$**:

$\# \text{ of nodes } (n) \geq N(h) > 2^{h/2}$

$n > 2^{h/2}$

$\lg(n) > h/2$

$2 \times \lg(n) > h$

$h < 2 \times \lg(n)$ ***, for $h \geq 1$***

Proved: The maximum number of nodes in an AVL tree of height h is less than $2 \times \lg(n)$.

Summary of Balanced BST

AVL Trees

- Max height: 1.44 * lg(n)
- Rotations:

Summary of Balanced BST

AVL Trees

- Max height: 1.44 * lg(n)
- Rotations:

  Zero rotations on find
  One rotation on insert
  O(**h**) == O(**lg(n)**) rotations on remove

Red-Black Trees

- Max height: 2 * lg(n)
- Constant number of rotations on insert (max 2), remove (max 3).

Why AVL?

Summary of Balanced BST

Pros:

- Running Time:

  - Improvement Over:

- Great for specific applications:

Summary of Balanced BST

Cons:

- Running Time:

- In-memory Requirement:

Red-Black Trees in C++

C++ provides us a balanced BST as part of the standard library:

```
std::map<K, V> map;
```

Red-Black Trees in C++

```
V & std::map<K, V>::operator[]( const K & )
```

Red-Black Trees in C++

```
V & std::map<K, V>::operator[]( const K & )
```

```
std::map<K, V>::erase( const K & )
```

Red-Black Trees in C++

```
iterator std::map<K, V>::lower_bound( const K & );
iterator std::map<K, V>::upper_bound( const K & );
```

Mattox Monday

CS 225 -- Course Update

Over the next two days, your grades will be uploaded into Compass for our first “grade update” where we will calculate your current course grade for you.

We will discuss the grades for the course as a whole (ex: average, etc) in lecture on Wednesday.

Iterators

Why do we care?

```
DFS dfs(...);
for ( ImageTraversal::Iterator it = dfs.begin(); it != dfs.end(); ++it ) {
  std::cout << (*it) << std::endl;
}
```

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```
DFS dfs(...);
for ( const Point & p : dfs ) {
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Iterators

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DFS dfs(...);
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```
DFS dfs(...);
for ( const Point & p : dfs ) {
  std::cout << p << std::endl;
}
```

```
ImageTraversal & traversal = /* ... */;
for ( const Point & p : traversal ) {
  std::cout << p << std::endl;
}
```

Iterators

```
ImageTraversal *traversal = /* ... */;
for ( const Point & p : traversal ) {
  std::cout << p << std::endl;
}
```

Every Data Structure So Far

	Unsorted Array	Sorted Array	Unsorted List	Sorted List	Binary Tree	BST	AVL
Find							
Insert							
Remove							
Traverse							

Range-based Searches

Q: Consider points in 1D: $\mathbf{p = \{p_1, p_2, \ldots, p_n\}}$.

…what points fall in [11, 42]?

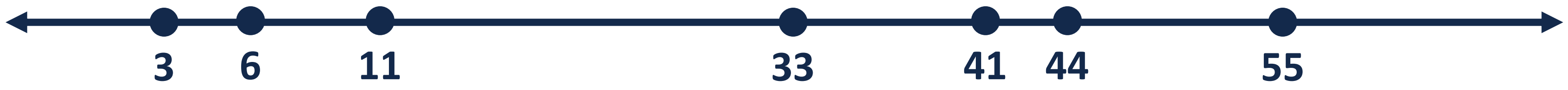
Tree construction:

Range-based Searches

Balanced BSTs are useful structures for range-based and nearest-neighbor searches.

Q: Consider points in 1D: $\mathbf{p} = \{p_1, p_2, \ldots, p_n\}$.
…what points fall in [11, 42]?

Ex:



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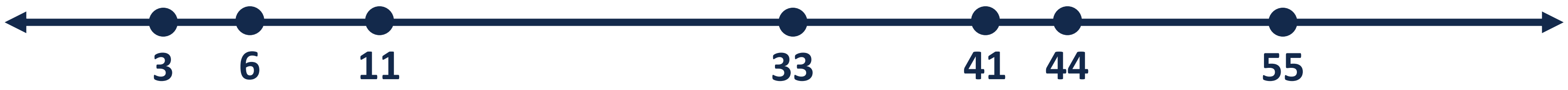

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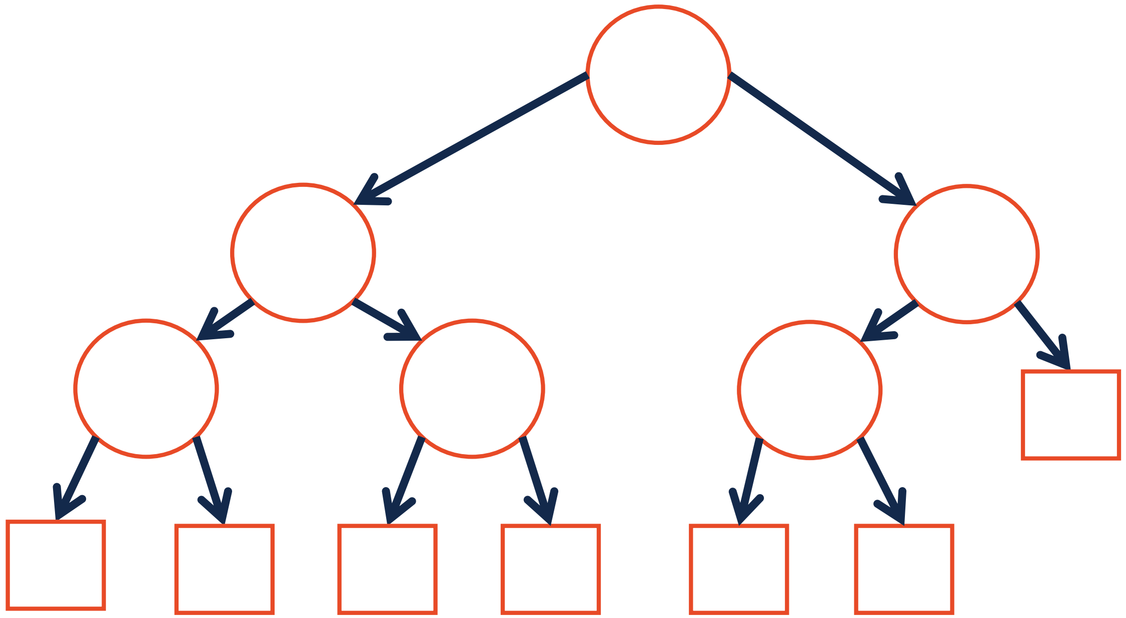

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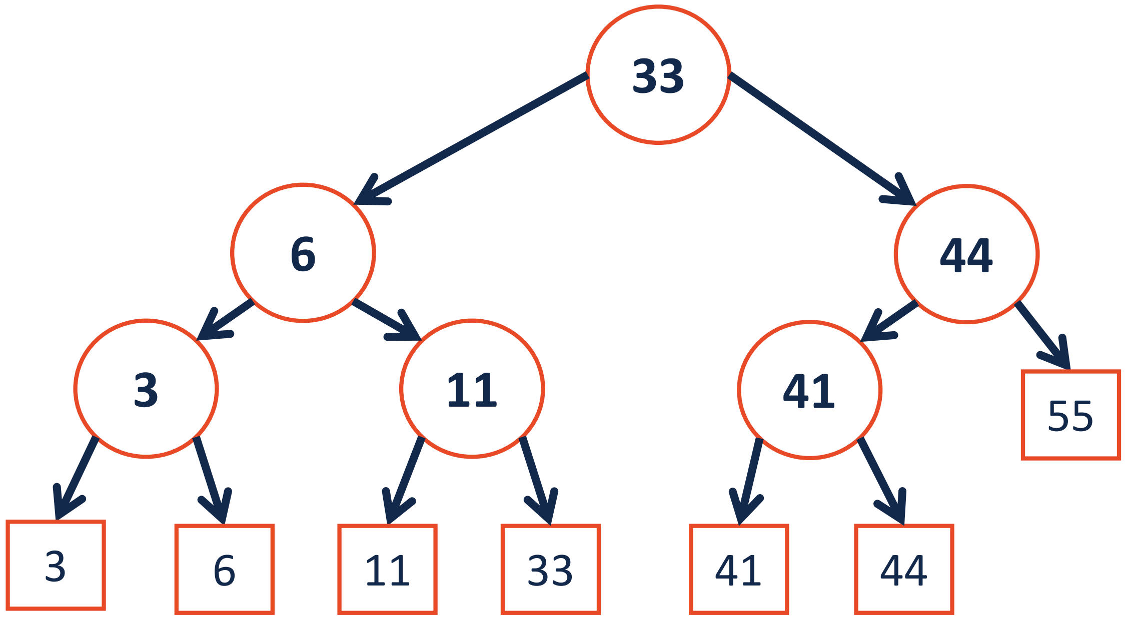

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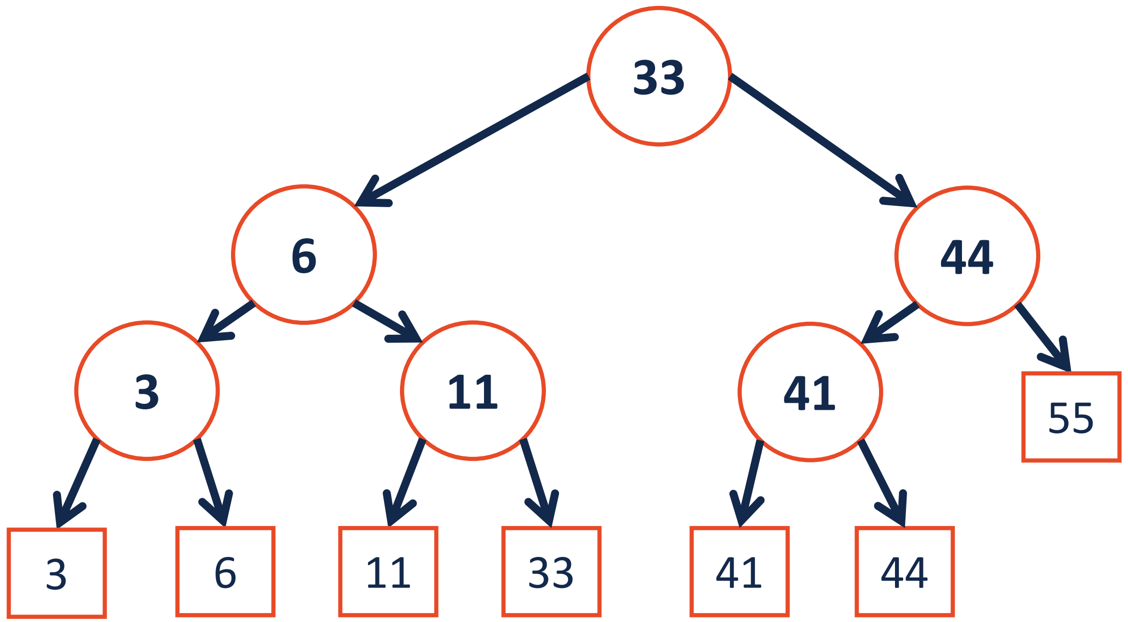
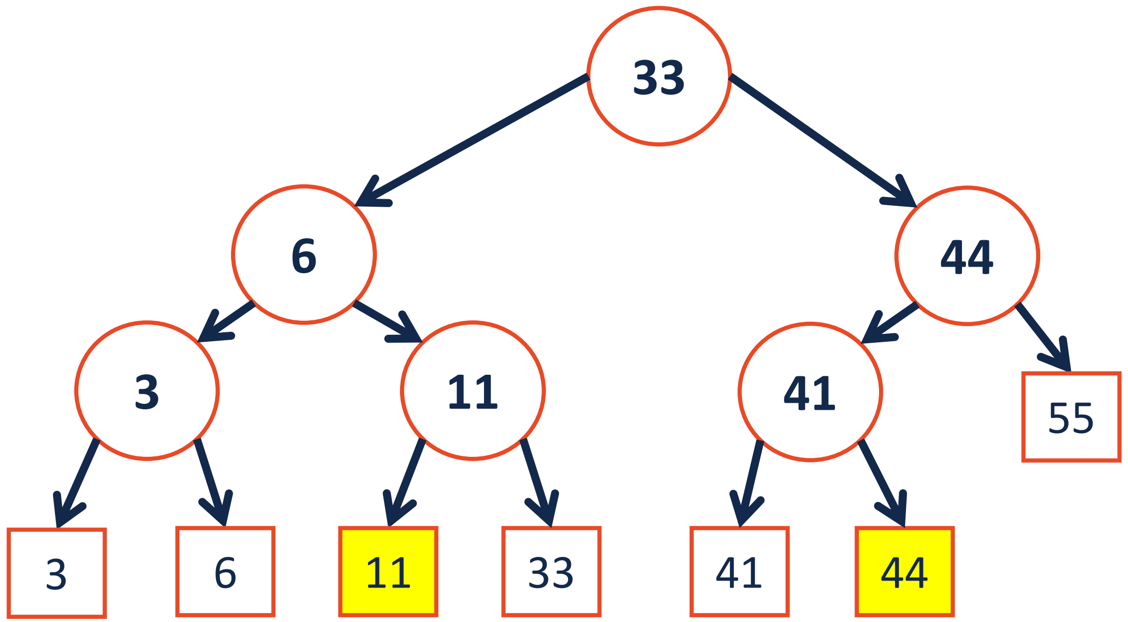
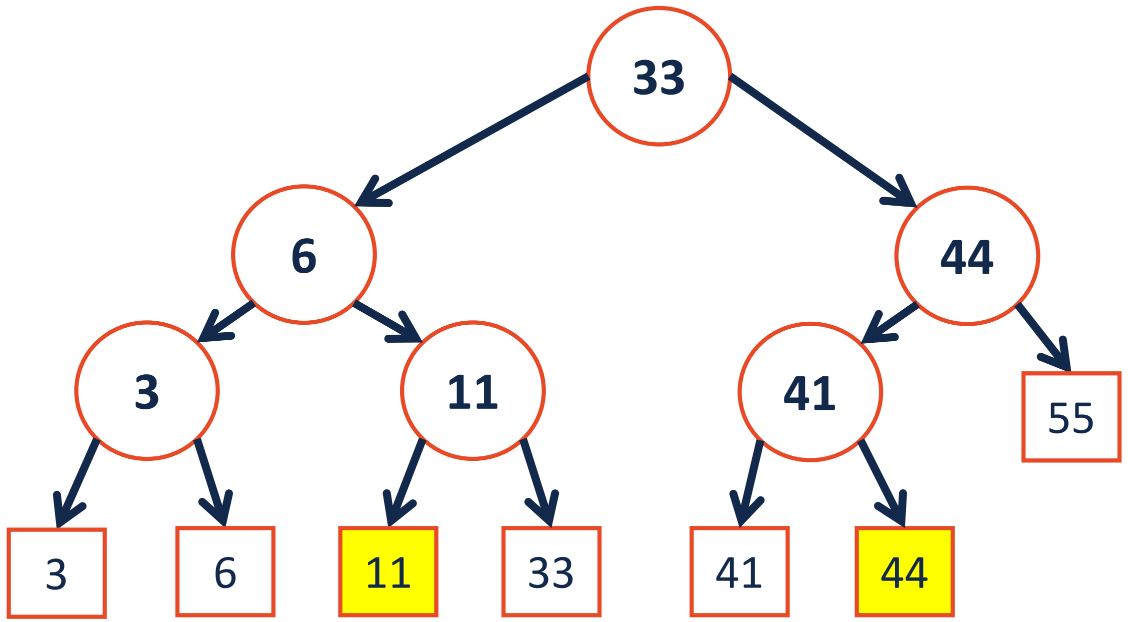


Range-based Searches



Running Time



Range-based Searches

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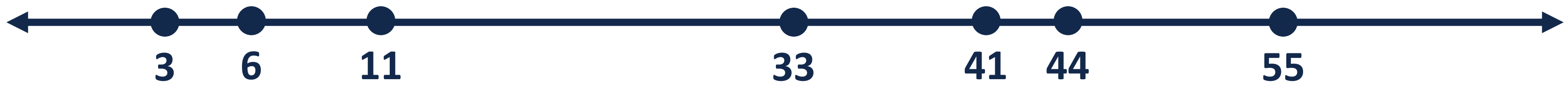
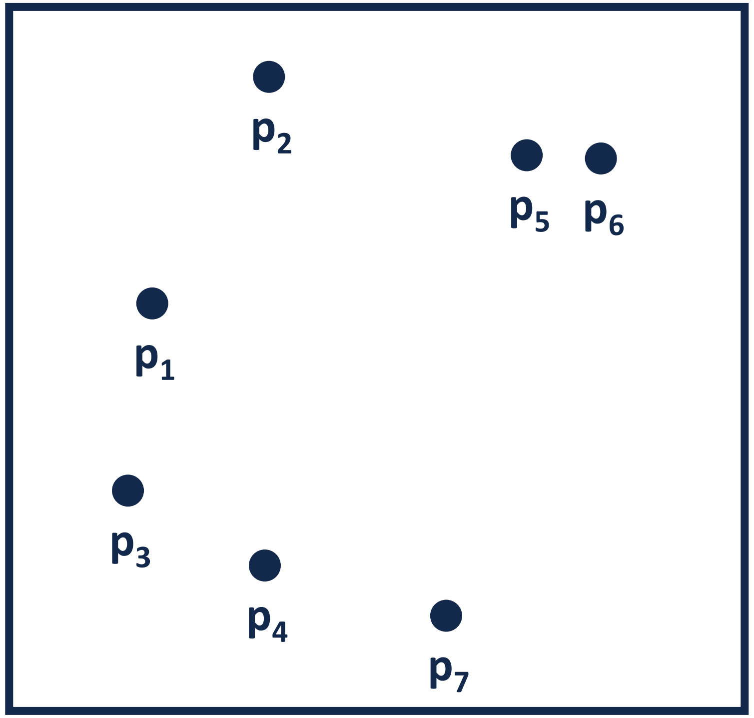


Range-based Searches

Consider points in 2D: $\mathbf{p} = \{p_1, p_2, \ldots, p_n\}$.

Q: What points are in the rectangle:
$[\ (x_1, y_1),\ (x_2, y_2)\ ]$?

Q: What is the nearest point to (x_1, y_1)?



Range-based Searches

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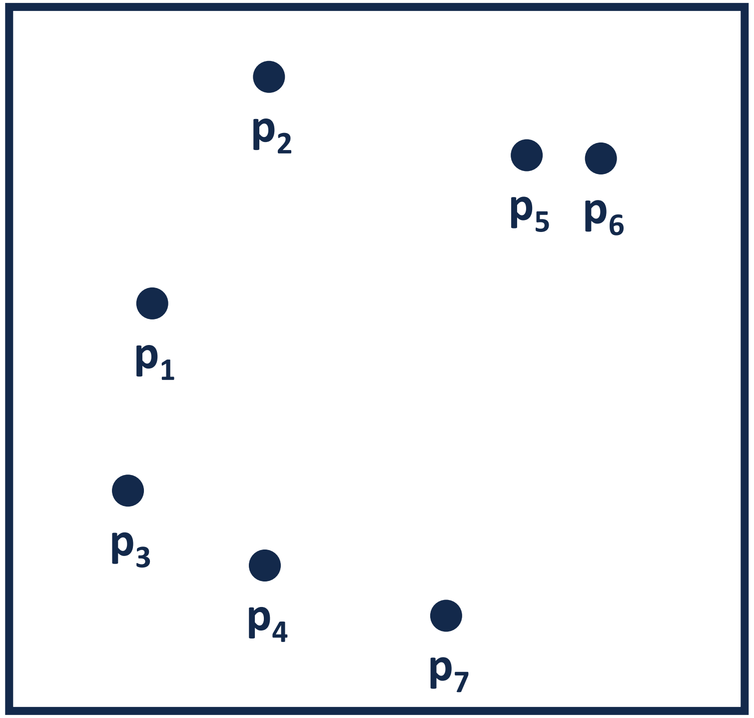
Tree construction:



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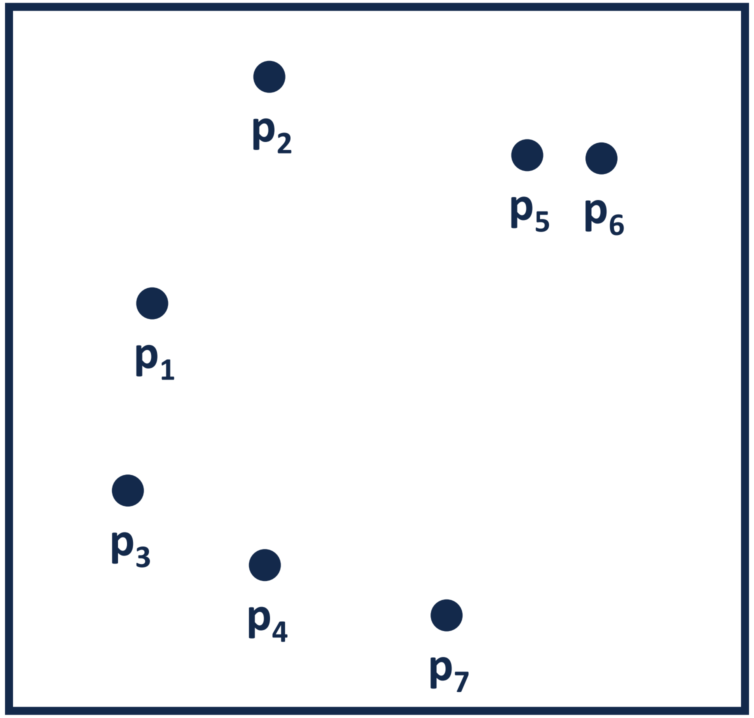
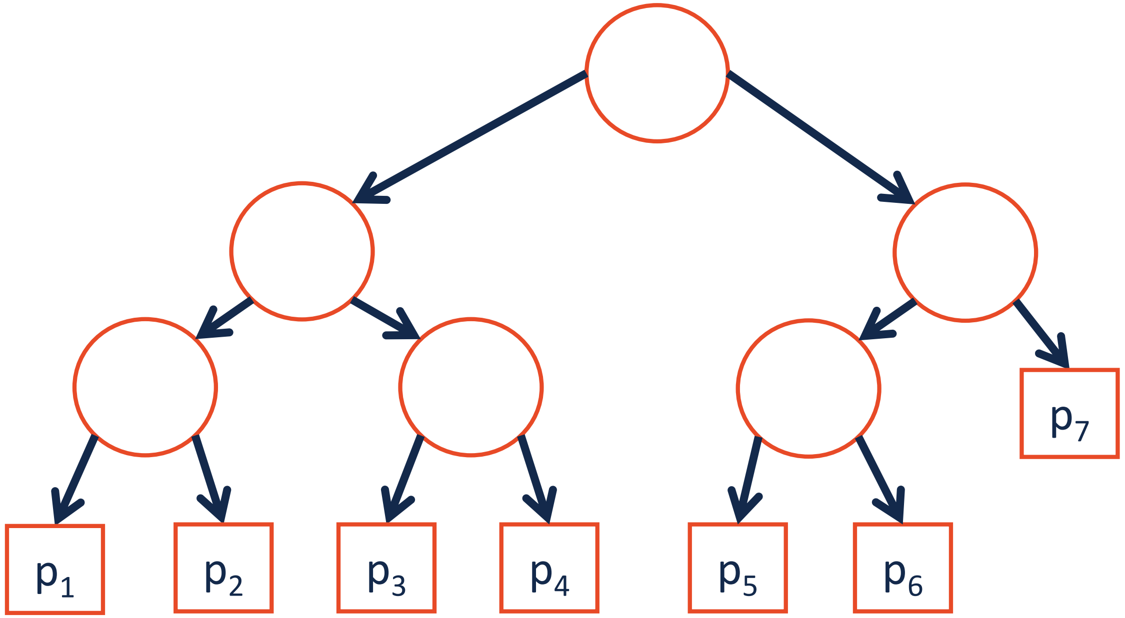
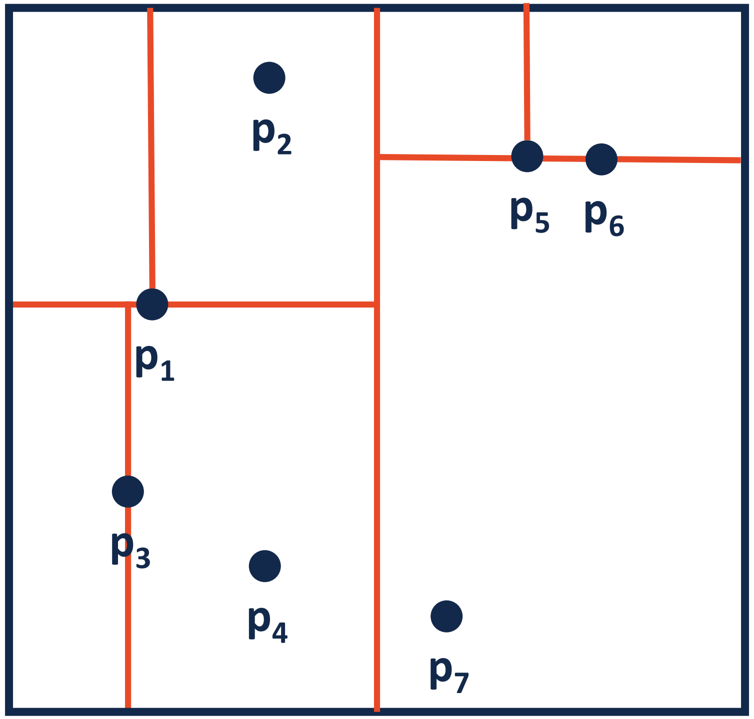
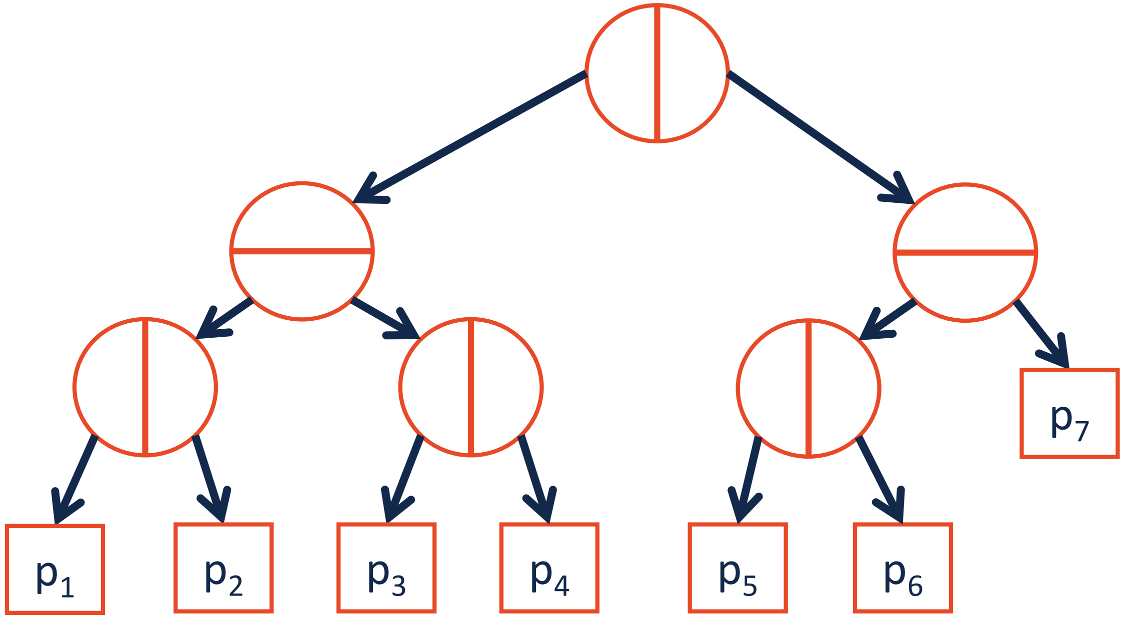




kD-Trees





kD-Trees