

AVL – Proof of Runtime

On Friday, we proved an upper-bound on the height of an AVL tree is $2 * \lg(n)$ or $O(\lg(n))$.

AVL Trees	Red-Black Trees
Balanced BST	Balanced BST <i>Functionally equivalent to AVL trees; all key operations runs in $O(h)$ time.</i>
Max height: $1.44 * \lg(n)$ <i>Q: Why is our proof $2 * \lg(n)$?</i>	Max height: $2 * \lg(n)$
Rotations: - find:	Rotations: - find:
- insert:	- insert:
- remove:	- remove:

In CS 225, we learned **AVL trees** because they're intuitive and I'm certain we could have derived them ourselves given enough time. A red-black tree is simply another form of a balanced BST that is also commonly used.

Summary of Balanced BSTs:

(Includes both AVL and Red-Black Trees)

Advantages	Disadvantages

Using a Red-Black Tree in C++

C++ provides us a balanced BST as part of the standard library:

```
std::map<K, V> map;
```

The map implements a dictionary ADT. Primary means of access is through the overloaded operator[]:

```
V & std::map<K, V>::operator[]( const K & )
This function can be used for both insert and find!
```

Removing an element:

```
void std::map<K, V>::erase( const K & );
```

Range-based searching:

```
iterator std::map<K, V>::lower_bound( const K & );
iterator std::map<K, V>::upper_bound( const K & );
```

Iterators and MP4

Three weeks ago, you saw that you can use an iterator to loop through data:

```
1 DFS dfs(...);
2 for ( ImageTraversal::Iterator it = dfs.begin();
3     it != dfs.end(); ++it ) {
4     std::cout << (*it) << std::endl;
5 }
```

You will use iterators extensively in MP4, creating them in Part 1 and then utilizing them in Part 2. Given the iterator, you can use the for-each syntax available to you in C++:

```
1 DFS dfs(...);
2 for ( const Point & p : dfs ) {
3     std::cout << p << std::endl;
4 }
```

The exact code you might use will have a generic ImageTraversal:

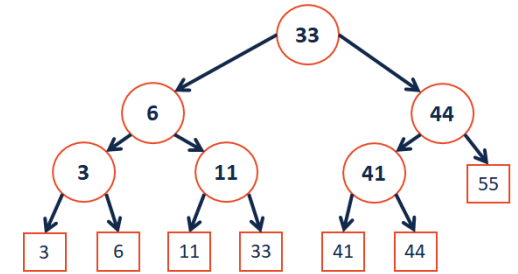
```
1 ImageTraversal & traversal = /* ... */;
2 for ( const Point & p : traversal ) {
3     std::cout << p << std::endl;
4 }
```

Running Time of Every Data Structure So Far:

	Unsorted Array	Sorted Array	Unsorted List	Sorted List
Find				
Insert				
Remove				
Traverse				

	Binary Tree	BST	AVL
Find			
Insert			
Remove			
Traverse			

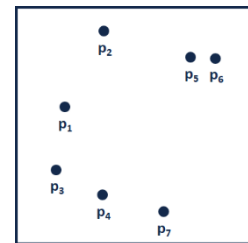
Range-based Searches:



Running Time:

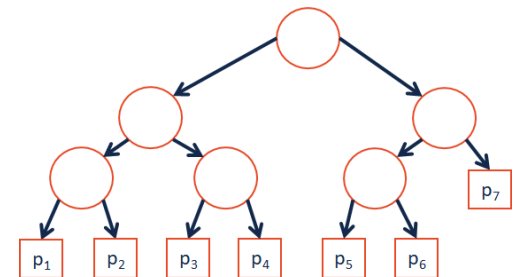
Extending to k-dimensions:

Consider points in 2D: $\mathbf{p} = \{p_1, p_2, \dots, p_n\}$:



...what points are inside a range (rectangle)?
...what is the nearest point to a query point \mathbf{q} ?

Tree Construction:

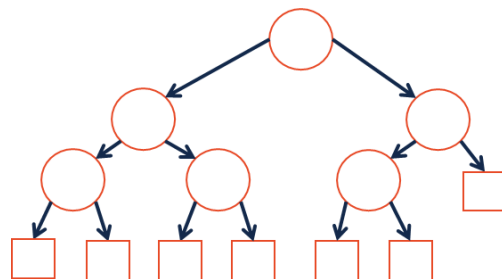


Range-based Searches:

Q: Consider points in 1D: $p = \{p_1, p_2, \dots, p_n\}$.
...what points fall in $[11, 42]$?



Tree Construction:



CS 225 – Things To Be Doing:

1. Programming Exam B starts in 10 days (*grab your time slot!*)
2. MP4 extra credit +7 due tonight
3. lab_avl released this week; details on Wednesday
4. Daily POTDs are ongoing!